Circulation of Bank Liability, Interbank Exposures and Lending Efficiency

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Abstract

When bank liability circulates as a means of payment, a fraction of one bank's liability naturally flows out to another, generating interbank exposures. This paper studies how these exposures affect bank lending efficiency and how their structure is determined by the resource distribution and the Input-Output network of the real economy. We capture the general equilibrium effect of bank lending on banks' deposits. We find that banks with a smaller outflow fraction are less responsive to monetary policy and that a positive interbank interest rate causes lending inefficiency.

Key words: circulation of bank liability, interbank exposures, outflow fraction, resource distribution, Input-Output network

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1 Introduction

Bank liability, in the form of banknotes historically and bank deposit nowadays, is widely accepted as a means of payment. This acceptance is a privilege to banks because it is rarely granted to the IOU of a non-bank business or a natural person. Due to this privilege, when banks lend "money" to the real economy, they do not hand out sacks of currency, but credit the borrowers' account with the agreed value; that is, the money that they lend out is not in the form of currency, but of bank deposit.[1] As money, bank liability naturally circulates. This circulation results in interbank exposures. For example, suppose one firm borrows from the HSBC one million pounds – which are the bank's liability – and uses the money to buy a machine from another firm that banks only with the Natwest. Then, after the trading, one million pounds of the HSBC's liability is held by the Natwest (which accordingly credits as much to the seller' account), that is, the HSBC owes one million pounds to the Natwest. How will the interbank positions thus formed affect the efficiency of bank lending? Moreover, these interbank positions are typically subject to immediate bilateral netting and the net positions knit banks into a network.[2] Considering that this interbank network results from circulation of bank liability through the real economy, how does its structure depend on the interactions between the real sectors and of them with the banking sector? In this paper we study these questions. Our exercise also identifies a new bank characteristic that affects bank lending and its responsiveness to monetary policy.

In the model, the real economy consists firms and households, the former endowed with production technology, the latter with resources, such as land, labor, human capital and political connection. We present our analysis in two steps. Only the interaction of firms with households is considered in the baseline model, while the interaction between the production sectors is incorporated in the extension. More specifically, in the baseline

[1] This way of banks making lending by creating deposits is recognized by Keynes (1914) in Chapter 2, "BANK-MONEY".

[2] This network is in fluid because the net interbank positions are in a continuous process of clearing, which sometimes involves debtor banks borrowing reserve from a third-party bank, that is, new interbank liabilities being created to clear old ones. In this paper, we focus on the origination stage and abstract from the clearing stage that follows it.

model, firms use only resources to produce one and the only consumption good. In the extension, multiple consumption goods are produced and their production uses the goods themselves as an input as well, which leads to an Input-Output (I-O hereafter) network. The aforementioned privilege of banks is modeled by the assumption that firms' promise to pay is not accepted as a means of payment for them to obtain resources or intermediate goods, but banks' liability is. To fund their production, therefore, firms have to borrow bank liability first and then use it as a means of payment to buy resources from households and to order intermediate goods from other firms. Finally, the sales revenue of these inputs is deposited by the sellers with their banks. Hence interbank positions are formed and bilaterally netted, resulting in an interbank liability network.

We find that the structure of this network is determined by the distribution of resources between banks' depositors and the I-O network. While we derive the general formula for this relationship, we illustrate it here with special cases. First, consider the resource distribution. Assume that there are two types of resources. The sales revenue of type 1 resources is evenly deposited into all the banks, whereas that of type 2 resources is deposited only into Bank 1. What this assumption captures is that some resources, such as land or labor, are ubiquitous throughout the economy and hence evenly distributed between different banks' depositors, while other resources are concentrated in the hands of relatively few people, their incomes deposited with few banks, such as transportation convenience or political connection. In equilibrium, liabilities between banks other than Bank 1 are netted out and only the liabilities between bank 1 and another bank remain. That is, the equilibrium interbank liability network is a star, with Bank 1 at the hub. Second, consider the I-O network. Assume that the production of good 1 uses only good 2 as an input, that of good 2 using only good 3, and so on; and that good 1's producers bank with Bank 1, good 2's with Bank 2, and so on. Then, in equilibrium Bank 1's liability is used by producers of good 1 to buy good 2 and then deposited with Bank 2, the liability of which, similarly, is deposited with Bank 3, and so on. Hence, the interbank liability network is a chain: Bank 1 owes to Bank 2, Bank 2 to Bank 3, and so on.

In reality, lending of the banking system has a non-negligible effect on banks' deposits:

Large part of bank deposits comes from economic agents' earnings, which are affected by the levels of business activity, which, in turn, depend on bank lending to a substantial degree. We capture this general equilibrium effect by tracking the full circulation of banks' liabilities: They are first originated as loans to firms, then portioned by the market mechanics into sales incomes, and finally flow back to the banking system as deposits. In this paper, hence, banks' deposits ultimately originate from the lending of the banking system.

By tracking the full circulation of bank liability, this paper identifies a new bank characteristic that matters for banks' lending decisions. Of the liability that a bank lends out, a fraction is deposited back, and the rest flows out to other banks, becoming interbank liabilities and incurring the expense of interbank interest to the issuing bank. Therefore, this outflow fraction determines the degree to which the interbank interest rate affects the bank's funding cost. The fraction tends to be smaller for a bank that has extensive branches than for one that has only sparse branches; it takes value 0 if what the bank lends out always flows back, in which case the bank's lending cost is insulated from the influence of the interbank interest rate. One corollary of this heterogeneity is that banks with a smaller outflow fraction is less responsive to a monetary policy that moves the interbank interest rate. Another is that a positive interbank interest rate is a source of inefficiency if banks have heterogeneous outflow fractions: If this rate is positive, banks with a smaller outflow fraction have a lower lending cost and charge a lower lending rate, whereby their borrower firms obtain cheaper funds and gather too much of the resources. In the special case where the interbank network is a star, this effect causes the resources over-concentrated at the place where the hub bank operates, which is usually an economy center.

The interbank exposures that we consider passively result from circulation of bank liability as a means of payment. How important are they relative to the exposures formed by banks actively borrowing reserve on the interbank markets? While this empirical question has not been directly examined so far, lights can be shed from the empirical studies that look into the flows of bank reserve. When indebted in the passive way that we consider, a debtor bank will eventually use reserve to settle the debt, in which case a flow of reserve funds is generated. Observe that such flows are unpaired because the creditor banks need not to pay funds back. In contrast, active borrowing of reserve generates flows in pairs (unless the debtor bank is default): A flow of funds to the borrower now must be paired with an inverse flow of repayment to the lender in the future. Therefore, the frequency of unpaired flows of reserve funds is indicative of the importance of passively formed interbank debts. Regarding this frequency, Furfine (2003), using the Fedwire funds data, finds that during February and March 1998 there are on average about $15,000$ flows of Fed funds per day, but identifies only about $3,000$ paired transactions of overnight borrowing. If typically the majority of active interbank borrowings is of overnight, then each day about $15,000 - 2 \times 3,000 = 9,000$ flows of Fed funds are not due to active interbank borrowing. While there must be a variety of reasons behind these flows, their number still suggests the importance of the passively formed interbank exposures.

This paper contributes to the growing literature that studies financial networks; for a survey see Allen and Babus (2009), Bougheas and Kirman (2014), Cabrales et al (2015), and Glasserman and Young (2015). While most of the studies in the literature consider an exogenous network, exceptions include Freixas et. al. (2000) and recently Acemoglu et al (2014), Allen et al (2012), Babus (2016), Farboodi (2015) and Zawadowski (2013). These studies have shed many important insights. Freixas et. al. (2000) show the vulnerability of the banking network to mis-coordinated withdraw in the manner of Diamond and Dybvig (1983). In their paper, the interbank network is formed by depositors moving around, which is similar to the way of this paper. Allen et. al. (2012) show that the systemic risks critically depend on the banks' funding maturity. Both Acemoglu et al (2014) and Farboodi (2015) underline that an interbank link can bring about both an opportunity of investment and a chance of contagion.[3] Both Acemoglu et al (2014) and Zawadowski (2013) demonstrate that inefficiency is caused by financial-network externalities, namely that a bank fails to internalize the implication of its decision for banks with which it is not directly linked. Babus (2016), based on Allen and Gale (2000), shows that the mutual

[3]This trade-off, in a reduced form, is also studied by Blume et al (2013) and Erol and Vehra (2014). Moreover, Glasserman and Young (2015) survey the studies on a similar trade-off, between the benefit of diversification and the cost of possible contagion.

insurance network bears a small or even nil systemic risk.

This literature models banks as intermediaries of loanable funds. It is not concerned with banks' privilege that their liabilities are accepted as a means of payment, whereas this privilege is the foundation of the present paper. Because of this privilege, in this paper, banks are the issuers of means of payment and their liabilities naturally circulate resulting in the interbank network that we study. Relative to the existing literature, this paper makes three innovations. First, it captures the general equilibrium effect of bank lending on banks' deposits. Second, this paper accommodates richer features of the real economy; no existing study has considered the effect of the I-O network, or that of the resource distribution, for the structure of the interbank network. Lastly, this paper identifies a new source of inefficiency – which is a positive interbank interest rate – and a new bank characteristic that matters for bank lending, which is the outflow fraction.

A nascent and growing strand of literature examines the acceptance and circulation of bank liability as a means of payment in general equilibrium; see Donaldson, Piacentino and Thakor (forthcoming), Faure and Gersbach (2016), Jakab and Kumhof (2015), Parlour, Rajan and Walden (2017), and Wang (forthcoming). Unlike the present paper, however, these studies are not concerned with interbank networks, especially how their structures map the real economies'.

The rest of the paper proceeds as follows. Section 2 sets up the baseline model which includes the resource distribution only. This model is studied in Section 3 and is extended in Section 4 to incorporate the I-O network. Section 5 concludes. Proofs of technical importance are relegated in Appendix.

2 The Baseline Model

The economy lasts for two dates, $t = 0$ for contracting and production, and $t = 1$ for yielding and consumption. There are N banks and N sectors. Bank $i \in \mathbf{N} := \{1, 2, ..., N\}$ is specialized to lending to sector i. Each sector consists of continuum $[0, 1]$ of firms which are managed by their owners. Firms use J types of resources (or factors of production) to produce the consumption good, corn, which is used as the numeraire. The production technology of a firm in sector i is as follows:

$$y_i = A_i h^{1-\alpha_i} \left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}} \right)^{\alpha_i},$$

where h is the human capital of the firm's manger-owner, $\alpha_i \in (0,1)$, $\beta_{ij} \geq 0$ and $\sum_{j=1}^{J} \beta_{ij} = 1$. Without loss of generality, we normalize $h = 1$. There are X_j units of type j resources for $j \in \mathbf{J} := \{1, 2, ..., J\}$. All resources are owned by households.

Households are willing to give up their resources at $t = 0$ only in the hope of being repaid with corn at $t = 1$. That is, at $t = 0$ they exchange resources for a promise to be paid back.

Assumption 1: households do not accept firms' promise to pay, but accept banks', as a means of paying for their resources.

This assumption captures banks' privilege that their liabilities are widely accepted as a means of payment, whereas rarely so are the liabilities of non-bank firms. A micro foundation of this assumption is provided by Kiyotaki and Moore (2001), who argue that this difference between banks and non-bank firms arises because the former has stronger commitment power than the latter.

In the model economy, therefore, firms have to acquire banks' promise to pay as a means of payment for resources. We define *one unit of liability* as a promise to pay one unit of corn. To acquires a bank's promise to pay, a firm enters the following loan contract: At $t = 0$, the firm receives m units of the bank's liability; in exchange at $t = 1$,the firm pays mR units of corn to the bank, where R is the gross interest rate of lending. Using this payment of corn, then, at $t = 1$ the bank pays m units of corn to the bearers of the m units of promises to pay that it has issued to the firm at $t = 0$. The bank's gain from this loan contract is thus $m(R-1)$. The firm pays this to the bank because it cannot use its own promise to pay to exchange for resources. Using the above argument of Kiyotaki and Moore (2001), what the firm does is essentially lease the bank's commitment power and the interest payment $m(R-1)$ is the rent.

Assume that banks need to screen borrowers before lending to them. Moreover, it is costless for a bank to screen firms within the sector to which it is specialized to lending, but it is prohibitively costly for it to screen firms without. Indeed, this difference in the screening cost might be what drives the specialization of banks, which is well documented in the empirical studies.[4] As a result, an firm borrows only from the bank specialized to its sector. After borrowing, firms use borrowed bank liability to buy resources from households. We assume that households deposit all their sales revenue with banks. For type $j \in \mathbf{J}$ resources, a fraction d_{ji} of them is owned by depositors of bank i, which, therefore, receives fraction d_{ji} of the total sales revenue of type j resources as deposit. Then, for each $j \in \mathbf{J}$,

$$\sum_{i \in \mathbf{N}} d_{ji} = 1.$$

Together, the matrix of $\{d_{ji}\}_{j \in \mathbf{J}, i \in \mathbf{N}}$ represents the distribution of resources between depositors of different banks and is referred to as the *resource distribution matrix*.

In this economy, there is no risk of bank default and banks' liabilities are one-to-one exchangeable. Hence, when a household deposit F units of bank n' liability with bank i, what bank i does is as follows. It adds F units of credit to the depositor's account on the liability side. On the asset side, given that it holds bank n's promise to pay F units of corn, bank n owes as much to bank i, namely, an interbank liability link is formed. Assume that the interbank interest rate is $\rho \geq 0$. With the deposit, bank i's balance sheet is as follows.

Assets	Liabilities
Old assets: X	Old liabilities: X
Debt owed by bank n: $F \times (1 + \rho)$	The account of the depositor: F
	Gain to the equity: $F \times \rho$

Table 1: The balance sheet of bank i with the deposit of F units of bank n's liability

At $t = 0$, the circulation of banks' liabilities can be illustrated as follows.

[4] See among others Jonghe et. al. (2016), Liu and Pogach (2016), Ongena and Yu (2017), and Paravisini et. al. (2014).

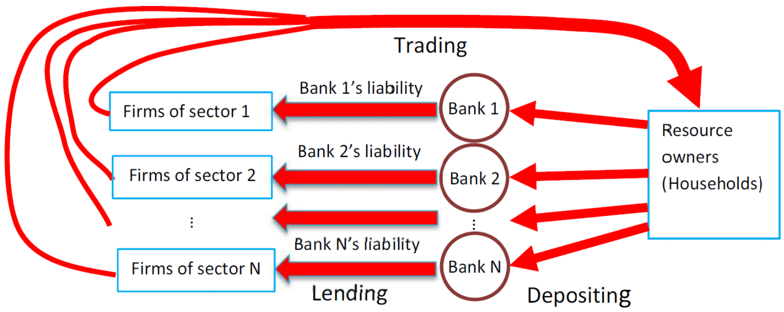

Figure 1: The circulation of bank liability: First, it are originated and lent to firms, and then used by them to exchange for resources from households, and lastly deposited back to banks

The timing at $t = 0$ is as follows.

1. Bank $i \in \mathbf{N}$ post the gross interest rates they charge, R_i.

2. Firms of sector $i \in \mathbf{N}$ each borrow m_i units of bank i's liability and use it as a means of payment to buy type j resources at price p_j for $j \in \mathbf{J}$.

3. Households deposit their sales proceeds. A fraction d_{ji} of type j resources' sales revenue is deposited into bank i.

4. Interbank liabilities are bilaterally netted.

The timing at $t = 1$ is as follows.

1. Firms produce corn. Firms of sector $i \in \mathbf{N}$ repay $m_i R_i$ units of corn to bank i and settle their debts.

2. Banks use corn to settle their liabilities to other banks and to the depositors.

3. Households, firms' manager-owners and bank shareholders consume the corn that they have obtained.

Equilibrium is defined as follows, where $\mathbf{P} := \{p_j\}_{j \in \mathbf{J}}$ denotes the profile of resource prices.

Definition 1 *A profile* $\left(\{m_i, R_i\}_{i \in \mathbf{N}}, \mathbf{P}\right)$ *forms an equilibrium, if*

(a) given $(R_i, \mathbf{P})$, m_i *is the optimal demand of bank* i*'s liability by firms of sector* i*;*

(b) given all other banks' choices $\{R_n\}_{n \in \mathbf{N}/\{i\}}$ *and the firms' demand function* $m_i(R_i; \mathbf{P})$*,* R_i *is the optimal interest rate charged by the bank* i*; and*

(c) for each $j \in \mathbf{J}$, p_j clears the market for type j resources.

In the next section, we characterize the equilibrium and demonstrate how the resource distribution matrix $\{d_{ji}\}_{j\in\mathbf{J},i\in\mathbf{N}}$ determines the equilibrium interbank liability network.

3 The Interbank Liability Network Determined by the Resource Distribution

We start with firms' decision problem, then banks' decision problem and lastly the market clearing conditions.

In this economy, because banks do not default, a unit of bank liability, that is, a bank's promise to pay one unit of corn at $t = 1$, is worth one unit of corn at $t = 0$. If an firm of sector i acquires m units of bank i's liability at gross interest rate R, then at $t = 0$, his budget constraint is

$$\sum_{j\in\mathbf{J}} p_j x_{ij} = m. \tag{1}$$

At $t = 1$, he pays back mR units of corn to the bank. Hence, the firm's decision problem is

$$\max_m A_i \left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}}\right)^{\alpha_i} - mR, \text{ s.t. } (1), \tag{2}$$

where he takes the interest rate R and the resource prices $\mathbf{P}$ as given. Considering that the objective is a Cobb-Douglas function, at the optimum, β_{ij} of the income is spent in buying type j resources, that is,

$$x_{ij} = \frac{\beta_{ij} m}{p_j}.$$

Hence, the firm's problem is equivalent to

$$\max_m m^{\alpha_i} A_i \prod_{j=1}^{N} \left(\frac{\beta_{ij}}{p_j}\right)^{\alpha_i \beta_{ij}} - mR,$$

which leads to the following demand of bank i's liability by the firms of sector i:

$$m_i = \left(\frac{\alpha_i A_i}{R}\right)^{\frac{1}{1-\alpha_i}} \left(\prod_{j=1}^{N} \left(\frac{\beta_{ij}}{p_j}\right)^{\beta_{ij}}\right)^{\frac{\alpha_i}{1-\alpha_i}} := m_i\left(R; \mathbf{P}\right). \tag{3}$$

Now we consider banks' decision problem. If bank i charges interest rate R_i, then it lends $M_i = m_i(R_i; \mathbf{P})$ units of its liability to firms of sector i at $t = 0$ and will obtain $M_i R_i$ units of corn at $t = 1$. At the end of $t = 0$, bank i's balance sheet is as follows.

Assets	Liabilities
Loans to firms ($M_i R_i$)	Deposit of its own liability by households (D_{own})
	Deposit of other banks' liability by households (D_{other})
Credit to other banks whose liabilities are deposited with bank i ($D_{other}(1+\rho)$)	Debt owed to the banks with which bank i's liability is deposited ($D_{out}(1+\rho)$)
	Equity (Π_i)

Table 2: bank i's balance sheet at the end of $t = 0$

The bank's total deposit from households $D_i = D_{own} + D_{other}$. The bank's net liability position to other banks, denoted by Υ_i, equals $D_{out} - D_{other} = (D_{own} + D_{out}) - (D_{own} + D_{other})$, where $D_{own} + D_{other} = D_i$ we have known. $D_{own} + D_{out} = M_i$ because the liability M_i issued by the bank flows either to other banks or back to itself. Hence, bank i's net liability position is:

$$\Upsilon_i = M_i - D_i. \tag{4}$$

This equation is also intuitive if we follow the loanable-funds approach. With this approach, M_i is the quantity of funds that bank i lends to firms, and D_i the quantity of funds available to the bank. The deficit, $M_i - D_i$, has to be borrowed from other banks and becomes its interbank liability position.

The bank's value $\Pi_i = M_i R_i + D_{other}(1 + \rho) - [D_{own} + D_{other} + D_{out}(1 + \rho)]$. With a little rearrangement,[5]

$$\Pi_i = M_i (R_i - 1) - \Upsilon_i \rho. \tag{5}$$

[5] $\Pi_i = M_i R_i + D_{other}(1+\rho) - D_{own} - D_{other} - D_{out}(1+\rho) = M_i R_i - (D_{out} - D_{other})\rho - (D_{own} + D_{out}) = M_i (R_i - 1) - \Upsilon_i \rho$.

Intuitively, the equation says that what bank i earns is equal to the net interest of its loans – that is $M_i (R - 1)$ – subtracting the interest on its interbank liability position – that is $\Upsilon_i \rho$. Substitute for Υ_i from (4), bank i's value

$$\Pi_i = M_i (R_i - 1 - \rho) + D_i \rho \tag{6}$$

We saw $M_i = m_i (R_i; \mathbf{P})$. To calculate Π_i, we need only to find the size of the bank's deposit D_i. For this purpose, note that firms at any industry n spend β_{nj} fraction of bank n's liability M_n that they acquire on buying type j resources. It follows that the total sales revenue of this type of resources is

$$E_j = \sum_{n \in \mathbf{N}} M_n \beta_{nj}. \tag{7}$$

Fraction d_{ji} of this revenue is deposited with bank i. Hence,

$$\begin{aligned} D_i &= \sum_{n \in \mathbf{N}, j \in \mathbf{J}} M_n \beta_{nj} d_{ji} \\ &= \sum_{n \in \mathbf{N}} M_n \times f_{ni} \\ &= M_i \times f_{ii} + \sum_{n \in \mathbf{N}/\{i\}} M_n \times f_{ni}, \end{aligned}$$

where

$$f_{ni} := \sum_{j \in \mathbf{J}} \beta_{nj} d_{ji}, \tag{8}$$

that is, f_{ni} is the fraction of bank n's liability that flows into bank i: out of one unit of liability that bank n lends out, fraction β_{nj} is spent on type j resources, out of which fraction of d_{ji} is deposited into bank i; and hence in total fraction f_{ni} of bank n's liability flows into bank i. The portions of funds $\{f_{ni}\}_{n \in \mathbf{N}, i \in \mathbf{N}}$ are determined by the market mechanics: In formula (8), $\{\beta_{nj}\}_{n \in \mathbf{N}, j \in \mathbf{J}}$ represents the effect due to trading and $\{d_{ji}\}_{j \in \mathbf{J}, i \in \mathbf{N}}$ depositing.

Symmetrically, a fraction $f_{in} = \sum_{j \in \mathbf{J}} \beta_{ij} d_{jn}$ of bank i's liability is deposited with bank n. As a result, the net liability that bank i owes to bank n, denoted by Υ_{in}, is

$$\Upsilon_{in} = M_i f_{in} - M_n f_{ni}. \tag{9}$$

It is straightforward to show that $\sum_{n\in\mathbf{N}/\{i\}} \Upsilon_{in} = \Upsilon_i$, that is, summing bank i's liability to each of the other banks finds its total interbank liability. The matrix $\boldsymbol{\Upsilon} = \{\Upsilon_{in}\}_{i\in\mathbf{N},n\in\mathbf{N}}$ characterizes the interbank liability network that passively results from the circulation of banks' liabilities as a means of payment. Obviously, $\Upsilon_{ii} = 0$ for any $i \in \mathbf{N}$, that is a bank owes nothing to itself; and $\Upsilon_{in} = -\Upsilon_{ni}$, that is, bank i's debt to bank n is exactly bank n's credit to bank i. It follows that $\sum_{i\in\mathbf{N}} \Upsilon_i = 0$,[6] that is, the aggregation of all interbank positions must be zero because one bank's credit position must be another bank's liability.

By (6) bank i's value is thus

$$\Pi_i = M_i \left(R_i - 1 - (1 - f_{ii})\,\rho\right) + \rho \sum_{n\in\mathbf{N}/\{i\}} M_n \times f_{ni}.$$

The bank's problem is hence:

$$\max_{R_i} M_i \left(R_i - 1 - (1 - f_{ii})\,\rho\right) + \rho \sum_{n\in\mathbf{N}/\{i\}} M_n \times f_{ni}$$

$$s.t. M_i = m_i\left(R; \mathbf{P}\right),$$

where the firms' demand of its liability $m_i\left(R; \mathbf{P}\right)$ is given by (3). In this problem the bank takes the other banks' lending sizes $\{M_n\}_{n\in\mathbf{N}/\{i\}}$ as given. It also take as given the resource prices $\mathbf{P}$ because there are a large number of banks and any single bank's influence on these prices is negligible. The optimal interest rate R_i^* of bank i is hence:

$$R_i^* = \frac{1}{\alpha_i}\left[1 + (1 - f_{ii})\,\rho\right]. \tag{10}$$

In this formula, the term $1/\alpha$ is the mark-up factor due to the monopolistic power that bank i has over the firms of sector i and the term in the square brackets is the marginal cost of lending to bank i, denoted by c_{mi}; that is,

$$c_{mi} = 1 + (1 - f_{ii})\,\rho. \tag{11}$$

To see why c_{mi} takes this form, note first that what banks lend out is their liabilities. Hence, when issuing each unit of loans, bank i creates one unit of liability, to redeem which it costs

[6] Because $\sum_{i\in\mathbf{N}} \Upsilon_i = \sum_{i\in\mathbf{N},n\in\mathbf{N}/\{i\}} \Upsilon_{in} = \frac{1}{2}\left(\sum_{i\in\mathbf{N},n\in\mathbf{N}/\{i\}} \Upsilon_{in} + \Upsilon_{ni}\right) = 0.$

1. Second, out of one unit of liability that bank i lends out, fraction f_{ii} is deposited back to the bank, but the rest $1-f_{ii}$ fraction flows out to other banks, incurring an interbank interest cost of $(1-f_{ii})\,\rho$ to bank i.[7] The total marginal cost of lending to the bank is thus $1+(1-f_{ii})\,\rho$. Obviously, the higher the *outflow fraction* $1-f_{ii}$, the higher the marginal lending cost of the bank.

Finally, we come to the market clearing conditions to determine the price of resources $\mathbf{P}$. We have found the the aggregate spending on type j resources E_j in (7). With bank n charges interest rate R_n^*, which is to be found in (10), the size of its issuance is $M_n = m_n(R_n^*;\mathbf{P})$. The aggregate demand for type j resources is E_j/p_j, while the aggregate supply is X_j. The market clearing condition for type j resource is thus:

$$\frac{1}{p_j}\sum_{n\in\mathbf{N}} m_n(R_n^*;\mathbf{P})\,\beta_{nj} = X_j, \tag{12}$$

which holds true each $j \in \mathbf{J}$. These market clearing conditions together determines the resource prices $\mathbf{P}$ in equilibrium.

We prove the existence of a unique equilibrium in the following proposition.

Proposition 1 *There is a unique equilibrium. Hence, the resource distribution matrix* $\{d_{ji}\}_{(j,i)\in\mathbf{J}\times\mathbf{N}}$ *determines a unique interbank liability network* $\boldsymbol{\Upsilon} = \{\Upsilon_{in}\}_{(i,j)\in\mathbf{N}\times\mathbf{N}}$, *where the net liability that bank i owes to bank n is*

$$\Upsilon_{in} = m_i(R_i^*;\mathbf{P})\,f_{in} - m_n(R_n^*;\mathbf{P})\,f_{ni},$$

where function $m_i(R;\mathbf{P})$ is given by (3), R_i^* by (10), and $f_{in}$is to be found with (8).

Proof. See Appendix. ■

Thus far, we have demonstrate how the structure of the interbank liability network that results from the circulation of bank liability (as illustrated in Figure 1) is determined

[7] In reality, part of the outflow liability that is not deposited back does not flow to other banks, but is converted into bank reserve – that is, the bearer of the liability withdraw from his account. However, in this case, the cost to the issuing bank is still ρ per unit of withdraw, because the interbank interest rate ρ is equal to the opportunity cost of bank reserve, given it can be used to redeem interbank liabilities.

by the distribution of resources between banks' depositors. To see this relationship more clearly, below we consider a special case where the interbank liability network is a star.

3.1 The Special Case of the Interbank Network Being a Star

Assume $J = 2$, that is, there are two types of resources. Type 1 resources are those that are quite evenly distributed in the economy, such as land, labor. As a result, the revenue that they generate is evenly deposited with all banks, that is, $d_{1i} = 1/N$ for any $i \in \mathbf{N}$. By contrast, type 2 resources are those that are concentrated at only few special places, such as the convenience of transportation, which is typically to be found at towns close to big rivers or seas, or political connection, which is mostly to be found at political centers. As a result, the revenue generated by type 2 resources is deposited only in the banks that operate at these special places, which are represented by bank 1 in the model economy. That is, $d_{21} = 1$ and $d_{2n} = 0$ for $n \in \mathbf{N}/\{1\}$. To simplify the exposition, we assume all firms have the same production technology: $A_i = A$, $\alpha_i = \alpha$ and $\beta_{i1} = \beta$ and $\beta_{i2} = 1 - \beta$ for any $i \in \mathbf{N}$. Then, we have the following proposition.

Proposition 2 *If $J = 2$, $d_{1i} = 1/N$ for any $i \in \mathbf{N}$, and $d_{21} = 1$ and the production technology is the same across firms, then in the unique equilibrium the interbank liability network is a star, with bank 1 at the hub. Furthermore, $\Upsilon_{1n} < 0$, that is, the hub bank holds credit claims to all the peripheral banks if and only if*

$$\left(\frac{1 + (1 - \beta/N)\,\rho}{1 + (\beta - \beta/N)\,\rho}\right)^{\frac{1}{1-\alpha}} < \frac{\beta/N + 1 - \beta}{\beta/N}. \tag{13}$$

Proof. See Appendix. ■

Observe that when N goes to infinity, the left hand side of (13) converges to $[(1+\rho)/(1+\beta\rho)]^{\frac{1}{1-\alpha}}$, the right hand side to ∞. Hence, if N is large enough, $\Upsilon_{1n} < 0$ for $n \in \mathbf{N}/\{1\}$. That is,

Corollary 1 *If the hub bank is very well connected, then it holds a credit position rather than a debt position to all the peripheral banks.*

3.2 New Insights Relative to the Existing Literature

This paper is based on banks' privilege that their liabilities are accepted as a means of payment. In this paper, banks are modeled as issuers of means of payment and the interbank liability network results from circulation of bank liability as a means of payment. By contrast, the existing literature on interbank networks models banks as intermediaries of loanable funds. This difference in modeling approach leads to several new insights, which are elaborated below.

A. *The degree of freedom is reduced.*

This paper track the full cycle of the circulation of bank liability: First it is lent out to firms, then portioned by the market mechanics into the sales incomes of households, and finally deposited back to the banking system. Hence, in this paper, banks' deposits are endogenous. Put differently, the quantity of funds available to each bank is ultimately determined by the conditions on banks' asset side. *There is no freedom to assume it to take a convenient value.* By contrast, the assumption on the available funds on banks' liability side, besides that on their asset-side conditions, is often the starting point in the existing studies. This difference bears on some important questions, one of which is systemic stability. We illustrate this point with the special case studied in subsection 3.1, in which the equilibrium interbank network is a star. A star provides us with the probably simplest framework to study the issue of Too Connected To Fail (TCTF): If the hub bank's failure could cause such severe losses to all the peripheral banks as to bring them all down, then it should not be allowed. Observe that a *necessary* condition for this argument to hold is that the hub bank owes a *debt* position to all the peripheral banks. If, instead, it holds a credit position, then its failure will inflict no loss to them and the issue is not there. The question is, which case – the TCTF-prone one or TCTF-immune one – arises in equilibrium? With the loanable-funds approach, given banks' investment opportunities, the TCTF-prone case can always arise, so long as we assume that the hub bank in deficit of funds, the peripheral banks in surplus. In this paper, however, this freedom of making assumptions on banks' liability side is no more. Indeed, we show that the TCTF-prone case can never arise in equilibrium: By Corollary 1, if the hub bank is well connected, then it holds a credit claim to all the peripheral banks.[8]

B. *The interbank interest rate ρ has a heterogeneous effect on banks' lending cost.*

Certainly, a rise in the interbank interest rate ρ increases all bank's lending cost. With the loanable-funds approach, the scale of this increase is homogeneous: A rise in ρ increase all banks' funding cost one-to-one. The reason is that with the loanable-funds approach, what banks lend out is not their liabilities but "funds", a homogenous good; the cost of obtaining this good is therefore also homogeneous, which is the interbank interest rate ρ, one and the same; and hence a rise in ρ increase all banks' cost of funding one by one. By contrast, in this paper, while a rise in ρ increases bank's lending cost also, the scale of this increase is heterogeneous: It is in proportion to the outflow fraction of the bank because by (11), $\partial c_{mi}/\partial \rho = 1 - f_{ii}$. The reason is that in this paper, what banks lend out is their liabilities; to a given bank i, only the part of the liability that flows to other banks, which is of fraction $1 - f_{ii}$, becomes interbank liabilities and incurs the expense of interbank interest to bank i; hence, $\partial c_{mi}/\partial \rho = 1 - f_{ii}$.

This heterogeneity has following two implications.

B1. *A monetary policy which moves the interbank interest rate has a smaller effect on the lending rates of banks that have a smaller outflow fraction.*

In the extreme case, if a bank sees all the money that it lends out flows back and hence its outflow fraction is zero, then its lending rate is insulated from the influence of monetary policy.

B2. *A positive interbank interest rate is a source of inefficiency if banks have heterogeneous outflow fractions.*

According to point B above, if the interbank interest rate ρ is strictly positive, then banks with heterogenous outflow fractions have heterogeneous lending costs and accordingly charge heterogeneous lending rates. This is a source of inefficiency. To explain it in a

[8] A caveat should be raised here: the holding of the corollary is subject to the qualification that only the resource distribution is considered. Later, when the Input-Output network is also factored in, the TCTF-prone case can arise in equilibrium at certain areas of the parameter space.

simple manner, we consider a special case where all the sectors have the same production technology:

$$\left(A_i, \alpha_i, \beta_{ij}\right) = \left(A, \alpha, \beta_j\right)$$

for any $i \in \mathbf{N}$ and $j \in \mathbf{J}$. As a result, in the first-best allocation $\left\{x_{ij}^{FB}\right\}_{i\in\mathbf{N},j\in\mathbf{J}}$, firms of different sectors should obtain an equal quantity of resources; that is,

$$\frac{x_{ij}^{FB}}{x_{nj}^{FB}} = 1 \tag{14}$$

for any $(i, n, j) \in \mathbf{N} \times \mathbf{N} \times \mathbf{J}$. Now look at the equilibrium allocation $\left\{x_{ij}^{*}\right\}_{i\in\mathbf{N},j\in\mathbf{J}}$. All the firms spend borrowed means of payment in the same way, β_j fraction on type j resources for any $j \in \mathbf{J}$. Hence, for any $(i, n, j) \in \mathbf{N} \times \mathbf{N} \times \mathbf{J}$,

$$\frac{x_{ij}^{*}}{x_{nj}^{*}} = \frac{m_i}{m_n}.$$

By (3), the demand m_i of means of payment by firms is in proportion to $(1/R_i)^{\frac{1}{1-\alpha}}$, while the gross lending rate R_i, by (10), is in proportion to the marginal cost $1 + (1 - f_{ii})\,\rho$. Therefore,

$$\frac{x_{ij}^{*}}{x_{nj}^{*}} = \frac{\left(\frac{1}{1+(1-f_{ii})\rho}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{1}{1+(1-f_{nn})\rho}\right)^{\frac{1}{1-\alpha}}}, \tag{15}$$

which is different to the socially optimal allocation given in (14) if $\rho > 0$ and banks have a heterogeneous outflow fraction $1 - f_{ii}$. To characterize which sectors obtain too much of resources relative to the first allocation, and which too little, we define the average marginal lending cost across all banks by

$$c_m^e\left(\rho\right) := \left(\frac{1}{N}\sum_{i\in\mathbf{N}}\left(\frac{1}{1+(1-f_{ii})\,\rho}\right)^{\frac{1}{1-\alpha}}\right)^{-(1-\alpha)}.$$

Then, we have the following proposition.

Proposition 3 *A sector $i \in \mathbf{N}$ obtains too much of resources if $1+(1-f_{ii})\,\rho < c_m^e\left(\rho\right)$ and too little if $1+(1-f_{ii})\,\rho > c_m^e\left(\rho\right)$. Sectors associated with banks that have the minimum outflow fraction obtain the greatest quantity of resources, those with the maximum outflow* *fraction the smallest. Moreover, the higher the interbank interest rate ρ, the more the former sectors obtain and the less the latter obtains.*

Proof. See Appendix. ■

Intuitively, the smaller the outflow fraction f_{ii} of a bank, the higher the marginal cost of lending, and the higher the lending rate that the bank charges; consequently, its firms borrow a smaller quantity of means of payment and obtain less resources. Moreover, the gap in the lending cost between a bank with the highest (the lowest) outflow fraction and all the rest grows if the interbank interest rate ρ increases, leading even less (more) resources to flow to its firms. In the special case studied in subsection 3.1, in which the equilibrium interbank network is a star with bank 1 at the hub, we can find $1 - f_{11} = \beta - \beta/N$ and $1 - f_{nn} = 1 - \beta/N$ for $n \in \mathbf{N}/\{1\}$. Hence, the outflow fraction of bank 1, the hub bank, is smaller than that of peripheral banks. An immediate corollary of Proposition 3 is thus:

Corollary 2 *In the special case studied in subsection 3.1, the resources are over-concentrated in the hand of the firms associated with the hub bank if the interbank interest rate $\rho > 0$. Moreover, the higher is ρ, the more resources agglomerated at the hub.*

Remark: In the model economy, the monopoly power that a bank has over the firms of its specialized sector is not a source of inefficiency if $\alpha_i = \alpha$ for any $i \in \mathbf{N}$, that is, if all the banks have the same monopoly power. The reason is as follows. Indeed, as demonstrated in a textbook, the monopoly raises the lending rate in the sense that it contributes a factor $1/\alpha_i$ to the lending rate, as shown in (10). However, in the model economy, all the resources are allocated to economic agents that borrow from banks, i.e. firms, which are all subject to banks' monopoly power. Hence, the effect of this monopoly power on the allocation's efficiency cancels each other if banks have the same monopoly power. If there were a sector in no need of borrowing means of payment from banks, then their monopoly power causes inefficiency by raising the borrowing cost and consequently induce too little resources to the sectors that depend on bank lending.

Thus far, we have factored in the resource distribution in considering the effect of the real economy for the structure of the interbank liability network. In the next section, the model is extended to incorporate an Input-Output (I-O) network, where production in one sector needs not only resources, but also other sectors' products, as an input.

Passing on to the extension, we first consider a modification of the present model, which helps us understand the working of the extended model.

3.3 A Modification of the Baseline Model

In the baseline model laid out in section 2, it is assumed that firms settle their debts to the banks with a payment of the consumption good, corn. In reality, firms repay their loans typically with money rather than with real goods. This feature can be incorporated in the baseline model with a slight modification, by allowing firms to use bank liability to settle their debts. More specifically, we modify the timing of events at $t = 1$ as follows.

1. Firms produce corn.

2. Each bank $i \in \mathbf{N}$ issues Q_i units of new liability (i.e. promise to pay corn later within date 1). Then, the corn market opens, where banks use the newly issued liabilities and households use bank liability that they have deposited at $t = 0$ to buy corn from firms.

3. Firms use either corn or bank liability to settle their debts to the banks, with one unit of liability equivalent to one unit of corn in value (recall that one unit of liability is defined as a promise to pay one unit of corn); e.g. an firm of sector i can use $0.5m_iR_i$ units of corn and $0.5m_iR_i$ units of bank liability to clear its debt to bank i. As a result of this debt settlement, one bank's liability might flow to another bank, becoming an interbank liability.

4. The net interbank liabilities (formed at stage 3 of $t = 1$ and also previously at $t = 0$) are cleared, first with netting and then with a payment of corn.

5. Consumption occurs.

At stage 3, firms can use any banks' liabilities to settle their debts. Therefore, at stage 2 the prices of all banks' liabilities in the unit of corn are the same, denoted by p. The market clearing price is found in the following lemma, where $Q := \sum_{i \in \mathbf{N}} Q_i$ denote the aggregate size of the new issuance and $V := \sum_{i \in \mathbf{N}} M_i (R_i^* - 1)$ denote the aggregate lending profit.

Lemma 1 *At stage 2, given Q, the market clearing price of bank liability is*

$$p = \left\{ \begin{array}{c} 1 \text{ if } Q < V \\ [0,1] \text{ if } Q = V \\ 0 \text{ if } Q > V \end{array} \right\}.$$

Proof. Given that banks have issued $M := \sum_{i \in \mathbf{N}} M_i$ units of liability at $t = 0$, the aggregate supply of bank liability at stage 2 of at $t = 1$ is then $Q + M$. Now consider the aggregate demand. At stage 2 of $t = 1$, firms want bank liability solely to use it as a means to settle their debts to the banks. If $p < 1$, i.e., bank liability is cheaper than corn, then firms want to use only bank liability, not corn, for the debt settlement. Their aggregate demand is thus $\overline{D} := \sum_{i \in \mathbf{N}} M_i R_i^*$. If $p = 1$, i.e. bank liability is at par with corn, then firms are indifferent in using bank liability or corn for the debt settlement. Hence the aggregate demand for bank liability can take any value within $\left[0, \sum_{i \in \mathbf{N}} M_i R_i^*\right]$. If $p > 1$, then no firms want to use bank liability for the debt settlement and the aggregate demand is 0. Observe that $Q + M < \overline{D}$ if and only if $Q < V$. When the demand and supply sides meet, as illustrated by the figure below, the lemma follows. ■

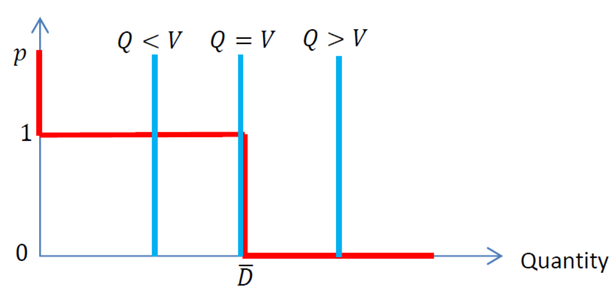

Figure 2: The demand (in red) and supply (in blue) of bank liability on the corn market at stage 2 of $t = 1$

Now consider the banks' decision on the sizes of new issuances. Two observations follow. First, in equilibrium $p = 1$; if $p < 1$, banks would choose not to issue new liabilities and let firms pay back with corn as much as possible, that is, $Q = 0 < V$, but then by Lemma

1 $p = 1$, a contradiction to the supposition that $p < 1$. Second, given $p = 1$, banks are indifferent with any $Q \leq V$. Hence, there are multiple equilibria concerning the sizes Q_i of new issuances. One of them is that in which each bank newly issues a quantity equal to its net profit, that is, $Q_i = \Pi_i = M_i \left(R_i^* - 1\right) - \Upsilon_i \rho$ for any $i \in \mathbf{N}$; because $\sum_{i \in \mathbf{N}} \Upsilon_i = 0$ (namely the aggregate of interbank positions is null), $Q = \sum_{i \in \mathbf{N}} M_i \left(R_i^* - 1\right) - \Upsilon_i \rho = V$, so that $p = 1$ is admissible. This equilibrium is referred to as the *convenient equilibrium*. It is named so because it has the following features: No corn (the real good), but only bank liability, is used for firms to repay all of their loans; bank liability is valued at par; and at stage 4 of $t = 1$ all the interbank liabilities are cleared with netting, with no resort to payment of corn.

In the extension that is laid out in the next section, multiple consumption goods are introduced. Rather than assume that firms pass various goods to banks for loan repayment, it is more convenient, besides more realistic, to assume that at $t = 1$ banks issues new liabilities and use it as a means of payment to buy a variety of goods and firms use bank liability to repay loans, as is in the modified version of the baseline model laid out above. Therefore, multiple equilibria exist concerning the sizes of the new issuances. To simplify exposition, we will focus on the convenient equilibrium in which $Q_i = \Pi_i$ for any $i \in \mathbf{N}$ and $p = 1$.

4 Extension: The I-O Network and the Interbank Network

In order to incorporate the I-O network, certain changes to the baseline model in section 2 need to be made. First, in the baseline model, all sectors produce one and the same good. Now they produce different goods. Sector $i \in \mathbf{N}$ produces good i. These goods can be used both for consumption and as an intermediate good for production. More specifically, in sector $i \in \mathbf{N}$, an firm uses a bundle of resources $\{x_{ij}\}_{j \in \mathbf{J}}$ and a bundle of intermediate goods $\{y_{in}\}_{n \in \mathbf{N}}$ to produce good i, according to the following production function:

$$y_i = A_i h^{1-\alpha_i} \left(\prod_{j \in \mathbf{J}} x_{ij}^{\beta_{ij}} \right)^{\alpha_i \gamma_i} \left(\prod_{n \in \mathbf{N}} y_{in}^{w_{in}} \right)^{\alpha_i (1-\gamma_i)},$$

where all the power coefficients are non-negative, $\sum_{j\in\mathbf{J}}\beta_{ij}=1$ and $\sum_{n\in\mathbf{N}}w_{in}=1$ for any $i\in\mathbf{N}$. γ_i describes the importance of resources and w_{in} the relative importance of intermediate good n in the production of good i. Again, normalize the human capital of the firm's manager-owner $h=1$.

Second, in the baseline model, for each sector $i\in\mathbf{N}$, there is only one bank – that is bank i – that monopolises lending to all firms of the sector. Keeping this assumption would imply a situation where bank i's lending decision, by affecting all the producers of good i, would have a non-negligible effect on the price of the good. This effect seems unrealistic. To avoid it, in the extension, we assume that in each sector there is a large number B of symmetric banks, each of which monopolises lending to fraction $1/B$ of firms in the sector. As a result, a single bank has only negligible influence on any of the prices of the goods and the resources and take these prices as given in its lending decision. To simplify exposition, in our analysis below, for each sector $i\in\mathbf{N}$ we group all the B banks of the sector into one bank and refer to it as bank i.

Third, given there are multiple good, the utility function of households, firm manager-owners and bank shareholders is now:

$$u\left(c_1,c_2,...,c_N\right)=\prod_{n\in\mathbf{N}}c_n^{\theta_n}, \tag{16}$$

where $\theta_n\geq 0$ for any $n\in\mathbf{N}$ and $\sum_{n\in\mathbf{N}}\theta_n=1$. If $\theta_i=0$, then good i is not a consumption good, but an intermediate good solely. The Cobb-Douglas form of the utility function again means that the aggregate consumption spending on good n is fraction θ_n of the aggregate income.

We pick good 1 as the numeraire. As such, bank liability take the form of a promise to pay good 1. As before, one unit of liability is defined as a promise to pay one unit of good 1.

Lastly, in parallel to Assumption 1, banks' privilege that their promise to pay is widely accepted as a means of payment is modeled by the following assumption.

Assumption 2: Firms cannot buy intermediate goods with their promise to pay. To obtain intermediate goods at $t = 1$, firms have to place orders at $t = 0$ with the full payment made with bank liability.

The timing at $t = 0$ is then as follows.

1. Bank $i \in \mathbf{N}$ posts the gross interest rate of lending R_i.

2. firms of sector $i \in \mathbf{N}$ borrow m_i units of bank i's liability and use it as a means of payment to buy type j resources at price p_j for $j \in \mathbf{J}$ and and to order intermediate good n at price q_n for $n \neq i$. In equilibrium, this price will be the same as the good's price at $t = 1$. Hence, each firm of sector i obtains a revenue of $v_i = q_i \sum_{n \in \mathbf{N}/\{i\}} y_{ni}$ from the pre-sale order of its product. To simplify exposition, we assume that the debt that firms owe to their banks is *non-callable*, that is, they cannot use the pre-sale revenue to partly clear their debt at $t = 0$. As a result, firms of sector i, while presently depositing v_i with bank i, are still obligated to pay back $m_i R_i$ to the bank at $t = 1$.

3. Households deposit their sales proceeds, with bank i receives fraction d_{ji} of the sales value of type j resources for $j \in \mathbf{J}, i \in \mathbf{N}$.

4. Interbank liabilities are bilaterally netted.

As was said in subsection 3.3, in the extension, we will allow firms to settle their debts using bank liability. Hence, the timing at $t = 1$ follows that in subsection 3.3 and is as follows.

1. Firms produce the goods.

2. Each bank $i \in \mathbf{N}$ issues Q_i units of new liability. Then, the goods markets open, where banks use the newly issued liabilities and households use bank liability that they have deposited at $t = 0$ to buy goods from firms. Manager-owners of firms in sector $i \in \mathbf{N}$ use bank liability that they have deposited at $t = 0$ – which is of value v_i – and that they receive from selling their products at $t = 1$ to buy goods $n \in \mathbf{N}/\{i\}$ from firms in sector n.

3. Firms use either good 1 or bank liability to settle their debts to the banks. As a result of this debt settlement, one bank's liability might flow to another bank, forming interbank liabilities.

4. Interbank liabilities are cleared, first with netting and then with good 1.

5. Agents consume the goods that they have obtained.

As was said in subsection 3.3, we focus on the convenient equilibrium with $Q_i = \Pi_i$ for any $i \in \mathbf{N}$, in which bank liability is valued at par at the goods markets; and all interbank liabilities are completely netted out at stage 4 of $t = 1$.

4.1 The Interbank Liability Network Determined by the I-O Network and the Resource Distribution Together

As before, we start with the analysis of firms' demand for bank liability. Suppose at $t = 0$, an firm of sector i borrows m_i units of bank liability to buy a bundle of factors of production $\{x_{ij}\}_{j\in\mathbf{J}}$ and pre-order a bundle of intermediate goods $\{y_{in}\}_{n\in\mathbf{N}/\{i\}}$. Then, the budget constraint is

$$\sum_{j\in\mathbf{J}} p_j x_{ij} + \sum_{n\in\mathbf{N}/\{i\}} q_n y_{in} = m_i. \tag{17}$$

Meanwhile, he receives a revenue of

$$v_i = q_i \sum_{n\in\mathbf{N}/\{i\}} y_{ni} \tag{18}$$

from the pre-sale booking of his product and deposit this revenue with bank i. At $t = 1$, out of the total product y_i, y_{ii} has been used for his own production and $\sum_{n\in\mathbf{N}/\{i\}} y_{ni}$ has been pre-sold. Hence, at $t = 1$, the sales revenue is $q_i \left(y_i - \sum_{n\in\mathbf{N}} y_{ni}\right)$, out of which the firm pays back the bank $m_i R$. The firm's objective function is thus

$$\begin{aligned} & q_i \times \left[A_i \left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}} \right)^{\alpha_i\gamma_i} \left(\prod_{n=1}^{N} y_{in}^{w_{in}} \right)^{\alpha_i(1-\gamma_i)} - \sum_{n\in\mathbf{N}} y_{ni} \right] - m_i R + v_i \\ = \;& q_i \times \left[A_i \left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}} \right)^{\alpha_i\gamma_i} \left(\prod_{n=1}^{N} y_{in}^{w_{in}} \right)^{\alpha_i(1-\gamma_i)} - y_{ii} \right] - m_i R, \end{aligned} \tag{19}$$

where the first term represents *the gross revenue of his production*, denoted by s_i – that is, $s_i = q_i (y_i - y_{ii})$ – and the second the total cost. The firm's problem is hence:

$$\max_{m_i, \{x_{ij}\}_{j\in\mathbf{J}}, \{y_{in}\}_{n\in\mathbf{N}}} q_i \times \left[A_i \left(\prod_{j=1}^{J} x_{ij}^{\beta_{ij}} \right)^{\alpha_i\gamma_i} \left(\prod_{n=1}^{N} y_{in}^{w_{in}} \right)^{\alpha_i(1-\gamma_i)} - y_{ii} \right] - m_i R,$$

$$s.t. \text{ the budget constraint } (17),$$

where he takes the lending rate R, the prices $\{p_j, q_n\}_{j\in\mathbf{J},n\in\mathbf{N}}$, and other firms' order for his product v_i as given. We solve this problem in two steps. First, we find the maximum value of the gross revenue s_i under the optimal choice of y_{ii}, given $\{x_{ij}\}_{j\in\mathbf{J}}$ and $\{y_{in}\}_{n\in\mathbf{N}/\{i\}}$, which is as follows.

$$s_i\left(\{x_{ij}\}_{j\in\mathbf{J}}, \{y_{in}\}_{n\in\mathbf{N}/\{i\}}\right) =: q_i\frac{1-\tau_i}{\tau_i}\left[A_i\tau_i\right]^{\frac{1}{1-\tau_i}}\left(\prod_{j=1}^{J}x_{ij}^{\beta_{ij}}\right)^{\frac{\alpha_i\gamma_i}{1-\tau_i}}\left(\prod_{n\in\mathbf{N}/\{i\}}y_{in}^{w_{in}}\right)^{\frac{\alpha_i(1-\gamma_i)}{1-\tau_i}},$$

where

$$\tau_i := \alpha_i\left(1-\gamma_i\right)w_{ii}. \tag{20}$$

Hence, the firm's problem is equivalent to

$$\max_{m_i,\{x_{ij}\}_{j\in\mathbf{J}},\{y_{in}\}_{n\in\mathbf{N}/\{i\}}} s_i\left(\{x_{ij}\}_{j\in\mathbf{J}}, \{y_{in}\}_{n\in\mathbf{N}/\{i\}}\right) - m_iR, \tag{21}$$

$$s.t. \text{ the budget constraint } (17).$$

The Cobb-Douglas form of function s_i means that at the optimum the spending on each type $j\in\mathbf{J}$ resources and intermediate good $n\in\mathbf{N}/\{i\}$ is a fraction of the budget, as follows.

$$p_jx_{ij} = \frac{\gamma_i\beta_{ij}}{1-(1-\gamma_i)w_{ii}}m_i; \tag{22}$$

$$q_ny_{in} = \frac{(1-\gamma_i)w_{in}}{1-(1-\gamma_i)w_{ii}}m_i. \tag{23}$$

To understand these fractions, note that τ_i defined in (20) is the weight of intermediate good i in the production of sector i. Given this good is the sector's own product, the cost of obtaining it is thus self-financed. Hence, the total weight of of the inputs that depend on bank finance is $\alpha_i-\tau_i$ (recall that $1-\alpha_i$ is the weight for the human capital of the firm's manager-owner). Hence, the fraction of spending on type j resources is thus $\alpha_i\gamma_i\beta_{ij}/(\alpha_i-\tau_i) = \gamma_i\beta_{ij}/(1-(1-\gamma_i)w_{ii})$ and that on intermediate good n is thus $\alpha(1-\gamma_i)w_{in}/(\alpha_i-\tau_i) = (1-\gamma_i)w_{in}/(1-(1-\gamma_i)w_{ii})$.

Based on (22) and (23), it is straightforward to find that the optimal demand of bank liability by firms of sector i is

$$m_i = \left(\frac{1}{R}\right)^{\frac{1-\tau_i}{1-\alpha_i}} \left(\frac{\alpha_i - \tau_i}{1-\tau_i}\xi_i \prod_{j\in\mathbf{J}} p_j^{-\frac{\alpha_i\gamma_i\beta_{ij}}{1-\tau_i}} \prod_{n\in\mathbf{N}} q_n^{-\frac{\alpha_i(1-\gamma_i)w_{in}}{1-\tau_i}} q_i^{\frac{1}{1-\tau_i}}\right)^{\frac{1-\tau_i}{1-\alpha_i}} := m_i\left(R;\mathbf{P},\mathbf{Q}\right), \tag{24}$$

where exogenous parameter

$$\xi_i := \frac{1-\tau_i}{\tau_i}\left[A_i\tau_i\right]^{\frac{1}{1-\tau_i}} \prod_{j=1}^{N}\left(\frac{(1-\gamma_i)\,w_{in}}{1-(1-\gamma_i)\,w_{ii}}\right)^{\frac{\alpha_i\gamma_i\beta_{ij}}{1-\tau_i}} \prod_{n\in\mathbf{N}/\{i\}}\left(\frac{(1-\gamma_i)\,w_{in}}{1-(1-\gamma_i)\,w_{ii}}\right)^{\frac{\alpha_i(1-\gamma_i)w_{in}}{1-\tau_i}}, \tag{25}$$

τ_i is given in (20), $\mathbf{P} = (p_1, p_2, ..., p_J)$ and $\mathbf{Q} := (q_1, q_2, ..., q_N)$ denote the price vectors. At this demand, the gross revenue of the firm in sector i is:

$$s_i = \left[\frac{1}{R}\right]^{\frac{\alpha_i-\tau_i}{1-\alpha_i}} \left[\left(\frac{\alpha_i-\tau_i}{1-\tau_i}\right)^{\frac{\alpha_i-\tau_i}{1-\tau_i}} \xi_i \prod_{j=1}^{N} p_j^{-\frac{\alpha_i\gamma_i\beta_{ij}}{1-\tau_i}} \prod_{n\in\mathbf{N}} q_n^{-\frac{\alpha_i(1-\gamma_i)w_{in}}{1-\tau_i}} q_i^{\frac{1}{1-\tau_i}}\right]^{\frac{1-\tau_i}{1-\alpha_i}} := s_i\left(R;\mathbf{P},\mathbf{Q}\right). \tag{26}$$

Now we move to consider banks' decision problem. If bank $i \in \mathbf{N}$ charges R_i, it lends out $M_i = m_i\left(R_i;\mathbf{P},\mathbf{Q}\right)$. Then, similar to Table 2 of the preceding section, the bank's balance sheet is as follows.

Assets	Liabilities
Loans to firms (M_iR_i)	Deposit of its own liability by households (D_{own})
	Deposit of other banks' liability by households (D_{other})
	Deposit of other banks's liability by firms (v_i)
Credit to other banks whose liabilities are deposited with bank i ($(D_{other}+v_i)(1+\rho)$)	Debt owed to the banks with which bank i's liability is deposited ($D_{out}(1+\rho)$)
	Equity (Π_i)

Table 3: a bank's balance sheet at the end of $t = 0$

The bank's total deposit $D_i = D_{own} + D_{other} + v_i$, of which $D_i^H := D_{own} + D_{other}$ is from households, v_i from firms. The bank's net liability position $\Upsilon_i = D_{out} - (D_{other} + v_i)$;

we can find $\Upsilon_i = M_i - D_i$, the same as (4) in the preceding section. The bank's value $\Pi_i = M_i R_i - \Upsilon_i(1+\rho) - [D_{own} + D_{other} + v_i]$; we can find $\Pi_i = M_i (R_i - 1) - \Upsilon_i \rho$, the same as (5) in the preceding section. Given $\Upsilon_i = M_i - D$, it follows that

$$\Pi_i = M_i (R_i - 1 - \rho) - D_i \rho. \tag{27}$$

To calculate D_i, we first find D_i^H, the deposit from households of the sales revenue of resources, and then v_i, the deposit from the firms. Regarding the former, similar to the preceding section, by (22), the aggregate spending E_j on type j resources is

$$E_j = \sum_{n \in \mathbf{N}} M_n \frac{\gamma_n \beta_{nj}}{1 - (1 - \gamma_n) w_{nn}}. \tag{28}$$

Of this spending, fraction d_{ji} is deposited into bank i. Hence, the deposit into the bank from households is

$$D_i^H = M_i \times f_{ii}^H + \sum_{n \in \mathbf{N}/\{i\}} M_n \times f_{ni}^H,$$

where for any bank $n \in \mathbf{N}$,

$$f_{ni}^H := \frac{\gamma_n}{1 - (1 - \gamma_n) w_{nn}} \sum_{j \in \mathbf{J}} \beta_{nj} d_{ji} \tag{29}$$

is the fraction of its liability deposited into bank i by households. Observe that f_{ni}^H equals the fraction in the preceding section (given by 8) multiplied by $\gamma_n / [1 - (1 - \gamma_n) w_{nn}]$, because here not all but only fraction $\gamma_n / [1 - (1 - \gamma_n) w_{nn}]$ of bank n's liability is spent on resources.

The deposit from the firms with bank i is the pre-sale revenue v_i of their product, good i. By (23) the spending $q_i y_{ni}$ of sector n on intermediate good i, for $n \neq i$, is $M_n f_{ni}^E$, where

$$f_{ni}^E := \frac{(1 - \gamma_n) w_{ni}}{1 - (1 - \gamma_n) w_{nn}} \tag{30}$$

denote the fraction of bank n's liability that flows into bank i due to the former's borrowers ordering good i as an input for their production. Hence, the total deposit into bank i due to this channel is

$$v_i = \sum_{n \in \mathbf{N}/\{i\}} M_n f_{ni}^E. \tag{31}$$

Put together, the total deposit of bank i is thus

$$\begin{aligned} D_i &= D_i^H + v_i \\ &= M_i \times f_{ii}^H + \sum_{n \in \mathbf{N}/\{i\}} M_n f_{ni}, \end{aligned}$$

where for $n \in \mathbf{N}/\{i\}$, $f_{ni} := f_{ni}^H + f_{ni}^E$ is the total fraction of bank n's liability flowing to bank i, that is,

$$f_{ni} = \frac{1}{1-(1-\gamma_n)\,w_{nn}} \left(\gamma_n \sum_{j \in \mathbf{J}} \beta_{nj} d_{ji} + (1-\gamma_n)\,w_{ni} \right). \tag{32}$$

By (27), bank i's value equals thus

$$\Pi_i \left(M_i; \{M_n\}_{n \in \mathbf{N}/\{i\}} \right) := M_i \left[R_i - 1 - \left(1 - f_{ii}^H\right) \rho \right] + \rho \sum_{n \in \mathbf{N}/\{i\}} M_n f_{ni}.$$

The bank's problem is thus

$$\max_{M_i, R_i} \Pi_i \left(M_i; \{M_n\}_{n \in \mathbf{N}/\{i\}} \right), s.t. M_i = m_i \left(R_i; \mathbf{P}, \mathbf{Q}\right),$$

where the demand function $m_i(R; \mathbf{P}, \mathbf{Q})$ is given by (24). As was said, a single bank, like bank i here, has negligible effect on prices $(\mathbf{P}, \mathbf{Q})$. Hence, the bank takes $(\mathbf{P}, \mathbf{Q})$ as given in solving the above decision problem. The optimum lending rate of bank i, denoted by R_i^*, is:

$$R_i^* = \frac{1-\tau_i}{\alpha_i - \tau_i} \left(1 + \left(1 - f_{ii}^H\right) \rho\right). \tag{33}$$

Similar to formula (10) for the optimal lending rate in the preceding section, the first term is the mark-up factor due to the bank's monopolistic power over the firms; and the term in the square parentheses is the marginal cost of lending.

The lending size of bank i as function of prices is thus

$$M_i \left(\mathbf{P}, \mathbf{Q}\right) = m_i \left(R_i^*; \mathbf{P}, \mathbf{Q}\right), \tag{34}$$

and the gross revenue of a firm in sector i is thus

$$s_i \left(\mathbf{P}, \mathbf{Q}\right) = s_i \left(R_i^*; \mathbf{P}, \mathbf{Q}\right),$$

where functions $m_i(R; \mathbf{P}, \mathbf{Q})$ and $s_i(R; \mathbf{P}, \mathbf{Q})$ are respectively given by (24) and (26).

Now we determine prices $(\mathbf{P}, \mathbf{Q})$ using market clearing conditions. The total spending E_j on type j resources is given by (28) and their total supply in value is $p_j X_j$. The market clearing condition for each type $j \in \mathbf{J}$ resources is thus:

$$\sum_{n \in \mathbf{N}} M_n(\mathbf{P}, \mathbf{Q}) \frac{\gamma_n \beta_{nj}}{1 - (1 - \gamma_n) w_{nn}} = p_j X_j. \tag{35}$$

We have picked good 1 as the numeraire. Hence, $q_1 = 1$. For any good i, it is either used for consumption or as an intermediate good. Thus,

$$c_i + \sum_{n \in \mathbf{N}} y_{ni} = y_i.$$

Recall that the revenue of a firm producing good i is $s_i = q_i(y_i - y_{ii})$ and the proceeds it obtains from selling the good to other producers $v_i = q_i \sum_{n \in \mathbf{N}/\{i\}} y_{ni}$. Hence, the market clearing for good i is

$$q_i c_i + v_i = s_i, \tag{36}$$

that is, the firm obtains revenue by selling good i either to consumers or to producers. We have found $s_i = s_i(\mathbf{P}, \mathbf{Q})$. By (31),

$$v_i = \sum_{n \in \mathbf{N}/\{i\}} M_n(\mathbf{P}, \mathbf{Q}) f_{ni}^E := v_i(\mathbf{P}, \mathbf{Q}).$$

Because of the the Cobb-Douglas form of the utility function in (16), the aggregate consumption spending on good i – that is $q_i c_i$ – is θ_i fraction of the aggregate income, $\sum_{n \in \mathbf{N}} (s_n - v_n)$. Hence, for any good $i \in \mathbf{N}$, the market clearing condition is

$$\theta_i \sum_{n \in \mathbf{N}} (s_n(\mathbf{P}, \mathbf{Q}) - v_n(\mathbf{P}, \mathbf{Q})) = s_i(\mathbf{P}, \mathbf{Q}) - v_i(\mathbf{P}, \mathbf{Q}). \tag{37}$$

Only $N - 1$ of these N equations are independent.[9] Pick any $N - 1$ of them and these $N - 1$ equations and the J resource-market clearing equations (given by 35) determine all the other $N + J - 1$ prices than q_1 (which is 1). Thus price profile $(\mathbf{P}, \mathbf{Q})$ is determined.

[9] As is well known, if the markets for $N - 1$ goods clear, then the market for the remained good clears too. A straight way to see this is to note that summing up both sides of (37) over $i \in \mathbf{N}$ reaches an identity.

The whole equilibrium is thus characterized. In the same way in which Proposition 1 is proved, it is straightforward to prove that there is a unique equilibrium in the extension. Hence the following proposition.

Proposition 4 *Given the resource distribution matrix* $\{d_{ji}\}_{(j,i)\in\mathbf{J}\times\mathbf{N}}$, *the Input-Output network* $\{w_{in}\}_{(i,n)\in\mathbf{N}\times\mathbf{N}}$ *determines a unique interbank liability network* $\{\Upsilon_{in}\}_{(i,n)\in\mathbf{N}\times\mathbf{N}}$, *where* $\Upsilon_{ii}=0$ *and for any* (i,n) *with* $i\neq n$, *the net liability position that bank* i *owes to bank* n *is*

$$\Upsilon_{in}=M_i\left(\mathbf{P},\mathbf{Q}\right)f_{in}-M_n\left(\mathbf{P},\mathbf{Q}\right)f_{ni},$$

where $M_i\left(\mathbf{P},\mathbf{Q}\right)$, *given by (34), is bank* i*'s lending size, and* f_{in}, *to be found with (32), is the fraction of bank* i*'s lending that is deposited with bank* n.

By this proposition, given the resource distribution, an I-O network is one-to-one mapping to an interbank liability network. To see this mapping more clearly, in what follows, we first abstracts away the influence of the resource distribution by assuming that for any $i\in\mathbf{N}$,

$$\gamma_i=0. \tag{38}$$

As a result, firms do not need resources for production and hence the resource distribution bears no effect on the interbank network. Indeed, by (29), the fraction of flow due to household depositing $f_{in}^H=0$ for any $(i,n)\in\mathbf{N}\times\mathbf{N}$. Second, we consider two special cases to illustrate how the structure of the interbank liability network reflects the structure of the I-O network.

4.2 The Interbank Network Is a Chain if the I-O Network is a Chain

Assume that the I-O network is a chain: Sector 2 uses only sector 1' product as the intermediate good, sector 3 uses sector 2's only, and so on; and sector 1 uses only its own good as the intermediate good. That is, $w_{11}=1$ and for $n\in\mathbf{N}/\left\{1\right\}$ and $w_{n,n-1}=1$. Firms of sector 1 do not need to borrow means of payment and hence, $M_1=0$. For sector $n\in\mathbf{N}/\left\{1\right\}$, firms need to borrow bank n's liability to buy intermediate good $n-1$ and consequently bank n's liability flows into bank $n-1$. Hence, the interbank liability network is a chain, as the I-O network: Bank N owes to bank $N-1$, bank $N-1$ to bank $N-2$,..., and bank 2 to bank 1.

Formally, by (32), for $n \in \mathbf{N}/\{1\}$ and $i \in \mathbf{N}$,

$$f_{ni} = \left\{ \begin{array}{c} 1 \text{ if } i = n-1 \\ 0 \text{ otherwise} \end{array} \right\}.$$

Hence, $\Upsilon_{n,n-1} = M_n f_{n,n-1} - M_{n-1} f_{n-1,n} = M_n > 0$ for $n \in \mathbf{N}/\{1\}$, and if $|n-i| > 1$, $\Upsilon_{ni} = M_n f_{ni} - M_i f_{in} = M_n \times 0 - M_i \times 0 = 0$.

4.3 The Case of the Interbank Network Being a Star

Consider now under which I-O networks, the resultant interbank network is a star. Obviously, if the I-O network is a star, that is, if all the sectors use one and the same good – say good 1 – as the only intermediate good for production, then the interbank network is a star, because for any $n \in \mathbf{N}/\{1\}$, bank n's liability is used by firms of sector n to buy good 1 and then deposited into bank 1. In this subsection, we consider a more general scenario where the resultant interbank network is a star. In this scenario, one sector – say sector 1 – is special and all the other sectors are symmetric; and moreover, to simplify the exposition, we assume all the sectors have the same production technology. Hence,

$$\begin{aligned} (A_i, \alpha_i, w_{ik}) &= (A, \alpha, w_k) \text{ for any } i \in \mathbf{N}, k \in \mathbf{N} \\ (w_n, \theta_n) &= (w_{-1}, \theta_{-1}) \text{ for any } n \in \mathbf{N}/\{1\}. \end{aligned}$$

We identify conditions under which the hub bank owes a debt position to the peripheral banks. Such a network, as we saw, could potentially suffer the issue of "Too Connected to Fail" (TCTF).

As a result of the symmetry, all the goods other than good 1 have the same price: $q_n = q_{-1}$ for $n \in \mathbf{N}/\{1\}$, while $q_1 = 1$. Similarly, all the banks other than bank 1 issue the same quantity of liability: By (24) and (33) (also τ_i given by 20 and f_{ii}^H by 29), for $n \in \mathbf{N}/\{1\}$

$$M_n = \varphi q_{-1}^{\frac{1}{1-\alpha}} (1-w_{-1}) \left(\frac{\alpha(1-w_{-1})}{(1-\alpha w_{-1})(1+\rho)} \right)^{\frac{1-\alpha w_{-1}}{1-\alpha}} := M_{-1}, \tag{39}$$

while

$$M_1 = \varphi (1 - w_1) \left(\frac{\alpha (1 - w_1)}{(1 - \alpha w_1)(1 + \rho)} \right)^{\frac{1 - \alpha w_1}{1 - \alpha}}, \tag{40}$$

where φ is a constant to banks:

$$\varphi := [A\alpha]^{\frac{1}{1-\alpha}} \left[\prod_{n \in \mathbf{N}} \left(\frac{w_n}{q_n} \right)^{w_n} \right]^{\frac{\alpha}{1-\alpha}}.$$

By (32), for $(n, i) \in \mathbf{N}/\{1\} \times \mathbf{N}/\{1\}$

$$f_{ni} = \frac{w_{-1}}{1 - w_{-1}} = f_{in}.$$

As a result, $\Upsilon_{ni} = M_n f_{ni} - M_i f_{in} = M_{-1} \times \frac{w_{-1}}{1 - w_{-1}} - M_{-1} \times \frac{w_{-1}}{1 - w_{-1}} = 0$. That is, the liability between any two banks other than bank 1 are netted out; all the interbank liability links exist only between bank 1 and bank $n \in \mathbf{N}/\{1\}$. Namely, the interbank liability network is a star, bank 1 at the hub. Use subscription "-1" to denote a peripheral bank, that is bank n for any $n \in \mathbf{N}/\{1\}$. Then, the net liability position of bank 1 to a peripheral bank is

$$\Upsilon_{1,-1} = M_1 f_{1,-1} - M_{-1} f_{-1,1}.$$

As was said in Point A of subsection 3.2, the sign of $\Upsilon_{1,-1}$ matters for the stability of the banking system. If $\Upsilon_{1,-1} > 0$, that is, if the hub bank – i.e. bank 1 – owes a debt to all the peripheral banks, then its failure would have a chance to inflict a severe loss to their asset values; that is, bank 1 could potentially be TCTF. On the other hand, if $\Upsilon_{1,-1} < 0$, bank 1's failure would inflict no loss to the peripheral banks and no issue of TCTF afflicts the interbank network.

By (32) $f_{1,-1} = w_{-1}/(1 - w_1)$ and $f_{-1,1} = w_1/(1 - w_{-1})$, one factor that affects the sign of $\Upsilon_{1,-1}$ is the production technology represented by w_1 and w_{-1}. For example, if $w_1 << w_{-1}$, then $f_{1,-1} >> f_{-1,1}$ and as a result, $\Upsilon_{1,-1} > 0$. Intuitively, if w_1 is sufficiently small relative to w_{-1}, that is, if good 1 is much less heavily used than any of the other goods, then way less money (i.e. bank liability) is spent on good 1 than any of the other goods. As a result, the flow of a peripheral bank's liability to bank 1 is thinner than the flow of bank 1's liability to the peripheral bank. Hence, bank 1 holds a debt claim to the peripheral banks. This intuitive argument is confirmed by the proposition below.

Proposition 5 $\Upsilon_{1,-1} > 0$, *and hence the issue of TCTF could potentially exist, if and only if*

$$\frac{w_{-1}}{1-w_{-1}} > \frac{w_1}{1-w_1} \times \frac{\theta_{-1}\left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho) + w_{-1}\theta_1(1-w_1)}{\theta_1\left(\frac{1-\alpha w_{-1}}{\alpha}\right)^2(1+\rho) + [w_1 - \theta_1(1-w_{-1})](1-w_{-1})}. \tag{41}$$

Proof. See Appendix. ■

Besides the production technology, condition (41) suggests that another factor that matters for the sign of $\Upsilon_{1,-1}$ is the demand side of the economy, described by parameters θ_1 and θ_{-1} (which are not independent because $\theta_1 + (N-1)\theta_{-1} = 1$).

Corollary 3 *If* $\theta_1 = 0$, *that is, if good 1 is not a consumption good but purely an intermediate good, then* $\Upsilon_{1,-1} \leq 0$, *so long as* $w_1 > 0$.

Proof. See Appendix. ■

Observe that Corollary 3 holds true no matter how small w_1 is. If w_1 is very small, the effect of production technology alone, as was said above, would command that $\Upsilon_{1,-1} > 0$. The corollary therefore shows that this effect is dominated by the effect of the demand side if good 1 is purely an immediate good. Intuitively, the latter is driven by the following general equilibrium effect. If good 1 is not a consumption good, then it is not much demanded and the quantity of its production is limited. As a result, funding for its production is small relative to that for the production of any of the other goods and hence M_1 is small relative to M_{-1}. Hence, $\Upsilon_{1,-1} < 0$.

5 Conclusion

Banks have the privilege that their liabilities are accepted as a means of payment. Due to this privilege, they make loans to the real economy by issuing liability. Given bank liability is used as a means of payment, it circulates after being issued. Naturally a fraction of one bank's liability flows into another bank, then an interbank exposure formed. Bilateral netting of these exposures leads to an interbank liability network. This paper characterizes how its structure is determined by the resource distribution and the Input-Output network. In general, this paper pictures the interweaving of the real sectors and the banking sector. Special attention is paid to star-structured interbank networks. We derive the conditions of the real economy under which the equilibrium interbank network is a star and the conditions under which the hub bank owes a debt claim to the peripheral banks and could potentially be Too Connected to Fail.

Moreover, we find that the outflow fraction, namely the fraction of which the liability that a bank lends out is not deposited back, is an important bank characteristic: The effect of the interbank interest rate on a bank's lending cost is in proportion to its outflow fraction; if this fraction is zero, that is, if the money that the bank lends out is always deposited back, then the bank's lending cost is insulated from the influence of the interbank interest rate. As a result, banks with a smaller outflow fraction have a smaller funding cost and charge a lower lending rate, which induces their borrower firms to inefficiently obtain too much of the resources. Moreover, such banks are less responsive to monetary policy that moves the interbank interest rate.

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Appendix

Proof of Proposition 1:

The objective function in the firm's problem (2) is equal to

$$\left[A_i\left(\prod_{j\in\mathbf{J}} x_{ij}^{\beta_{ij}}\right)^{\alpha_i} - \sum_{j\in\mathbf{J}} p_j x_{ij}\right] - m\left(R-1\right),$$

where the term in the square brackets is the firm's profit if it faces no friction of payment and could use its own IOU as a means of payment, and the second term $m\left(R-1\right)$ represents the cost of this friction to the firm; indeed, as was said above, $m\left(R-1\right)$ is the cost that this firm pays for renting the bank's commitment power. As a result, if $R_i = 1$ for any $i \in \mathbf{N}$, then the equilibrium is reduced to the (perfect) competitive equilibrium. By (10), R_i^* is independent of $\mathbf{P}$. By (3), if we define

$$\widehat{A}_i := \frac{A_i}{R_i^*},$$

then the demand function $m_i\left(R_i^*;\mathbf{P}\right)$ is the same as $m_i\left(1,\mathbf{P};\widehat{A}_i\right)$, that is, the same as the firm's total demand for resources under the (perfect) competitive equilibrium. Considering that the demand for each type $j \in \mathbf{J}$ of resources equals $\frac{1}{p_j}\sum_{i\in\mathbf{N}} m_i\beta_{ij}$, then the aggregate demand for each type of resources in the model economy is equal to that in the competitive equilibrium with $A_i = \widehat{A}_i$. Therefore, the equilibrium allocation $\{x_{ij}\}_{i\in\mathbf{N},j\in\mathbf{J}}$ is the same as that of the competitive equilibrium with $A_i = \widehat{A}_i$ for any $i \in \mathbf{N}$. According to Welfare Theorem 1, then, the equilibrium allocation $\{x_{ij}\}_{i\in\mathbf{N},j\in\mathbf{J}}$ is the one that maximizes the aggregate product, that is, it is the solution to the following social planner's problem:

$$\max_{\{x_{ij}\}_{i\in\mathbf{N},j\in\mathbf{J}}} \sum_{i\in\mathbf{N}} \widehat{A}_i \left(\prod_{j\in\mathbf{J}} x_{ij}^{\beta_{ij}}\right)^{\alpha_i}$$

$$s.t. \sum_{i\in\mathbf{N}} x_{ij} = X_j \text{ for each } j \in \mathbf{J}.$$

The objective function is strictly concave. Hence, there exists a unique solution to the maximisation problem. Hence, a unique equilibrium exists.

Q.E.D.

Proof of Proposition 2:

For the first part of the proposition, by (8), for any $i \in \mathbf{N}$, $f_{in} = \sum_{j \in \mathbf{J}} \beta_{ij} d_{jn} = \beta \times 1/N + (1-\beta) \times 0 = \beta/N$ for $n \in \mathbf{N}/\{1\}$; and $f_{i1} = \sum_{j \in \mathbf{J}} \beta_{ij} d_{j1} = \beta \times 1/N + (1-\beta) \times 1 = \beta/N + 1 - \beta$. In particular, $f_{ni} = f_{in}$ for any $(i,n) \in \mathbf{N}/\{1\} \times \mathbf{N}/\{1\}$. Also it is obvious that all banks other than bank 1 are symmetric and issue the same quantity of liability: there exists some $M_{-1} > 0$ such that $M_i = M_{-1}$ for any $i \in \mathbf{N}/\{1\}$. Therefore, if $(i,n) \in \mathbf{N}/\{1\} \times \mathbf{N}/\{1\}$, then by (9), $\Upsilon_{in} = M_{-1} f_{in} - M_{-1} f_{ni} = 0$. Hence, all the interbank liabilities are between bank 1 and bank $i \in \mathbf{N}/\{1\}$, that is, the interbank liability network is a star and at the hub is bank 1. To find out the sign of Υ_{1n} for $n \in \mathbf{N}/\{1\}$, observe that by (3),

$$M_i = \left(\frac{\alpha A}{R_i^*}\right)^{\frac{1}{1-\alpha}} \left(\prod_{j \in \mathbf{J}} \left(\frac{\beta_j}{p_j}\right)^{\beta_j}\right)^{\frac{\alpha}{1-\alpha}}$$

for any $i \in \mathbf{N}$. Hence, $n \in \mathbf{N}/\{1\}$,

$$\begin{aligned}
\Upsilon_{1n} &< 0 \Leftrightarrow \\
M_1 f_{1n} &< M_n f_{n1} \Leftrightarrow \\
\left(\frac{\alpha A}{R_1^*}\right)^{\frac{1}{1-\alpha}} \left(\prod_{j \in \mathbf{J}} \left(\frac{\beta_j}{p_j}\right)^{\beta_j}\right)^{\frac{\alpha}{1-\alpha}} f_{1n} &< \left(\frac{\alpha A}{R_n^*}\right)^{\frac{1}{1-\alpha}} \left(\prod_{j \in \mathbf{J}} \left(\frac{\beta_j}{p_j}\right)^{\beta_j}\right)^{\frac{\alpha}{1-\alpha}} f_{n1} \Leftrightarrow \\
\left(\frac{R_n^*}{R_1^*}\right)^{\frac{1}{1-\alpha}} &< \frac{f_{n1}}{f_{1n}} = \frac{\beta/N + 1 - \beta}{\beta/N}. \qquad (42)
\end{aligned}$$

To find R_i^*, first note that we have found $f_{11} = \beta/N + 1 - \beta$ and $f_{nn} = \beta/N$ for $n \in \mathbf{N}/\{1\}$. Hence, by (10), $R_1^* = \frac{1}{\alpha}\left[1 + (1 - (\beta/N + 1 - \beta))\rho\right]$ and $R_n^* = \frac{1}{\alpha}\left[1 + (1 - \beta/N)\rho\right]$. Substitute these into (42) and we find $\Upsilon_{1n} < 0$ if and only if

$$\left(\frac{1 + (1 - \beta/N)\rho}{1 + (1 - (\beta/N + 1 - \beta))\rho}\right)^{\frac{1}{1-\alpha}} < \frac{\beta/N + 1 - \beta}{\beta/N},$$

which is equivalent to (13).

Q.E.D.

Proof of Proposition 3:

We have shown that the quantity of resources of any type that a sector i obtains is in proportion to $(1/\left[1+(1-f_{ii})\,\rho\right])^{\frac{1}{1-\alpha}}$ and therefore takes γ_i fraction of the aggregate supply, where

$$\begin{aligned}\gamma_i \quad : \quad &= \frac{\left(\frac{1}{1+(1-f_{ii})\rho}\right)^{\frac{1}{1-\alpha}}}{\sum\limits_{n\in\mathbf{N}}\left(\frac{1}{1+(1-f_{nn})\rho}\right)^{\frac{1}{1-\alpha}}} \\ &= \frac{\left(\frac{1}{1+(1-f_{ii})\rho}\right)^{\frac{1}{1-\alpha}}}{N\left(\frac{1}{c_m^e(\rho)}\right)^{\frac{1}{1-\alpha}}} \\ &= \frac{1}{N}\times\left(\frac{c_m^e(\rho)}{1+(1-f_{ii})\,\rho}\right)^{\frac{1}{1-\alpha}}.\end{aligned}$$

Recall that in the first best allocation $\gamma_i^* = 1/N$. Hence, $\gamma_i > \gamma_i^*$ if and only if $1 + (1-f_{ii})\,\rho < c_m^e(\rho)$. Moreover, $\gamma_i = \max\{\gamma_n| \in \mathbf{N}\}$ if $f_{ii} = \max\{f_{nn}|n\in\mathbf{N}\}$, $\gamma_i = \min\{\gamma_n| \in \mathbf{N}\}$ if $f_{ii} = \min\{f_{nn}|n\in\mathbf{N}\}$. Hence the first part of the proposition is proved. For the second part, note that

$$\gamma_i = \frac{1}{\sum\limits_{n\in\mathbf{N}}\left(\frac{1+(1-f_{ii})\rho}{1+(1-f_{nn})\rho}\right)^{\frac{1}{1-\alpha}}}$$

and $\frac{1+(1-f_{ii})\rho}{1+(1-f_{nn})\rho}$ is decreasing (increasing) with ρ if $f_{ii} > f_{nn}$ ($f_{ii} < f_{nn}$). Therefore, if $f_{ii} = \max\{f_{nn}|n\in\mathbf{N}\}$, then γ_i is increasing with ρ and if $f_{ii} = \min\{f_{nn}|n\in\mathbf{N}\}$ then γ_i is decreasing with ρ.

Q.E.D.

Proof of Proposition 5:

By (32), $f_{-1,1} = w_1/\,(1-w_{-1})$ and $f_{1,-1} = w_{-1}/\,(1-w_1)$, and we have found M_{-1} in

(39) and M_1 in (40). Then,

$$\begin{aligned}
\Upsilon_{1,-1} &= M_1 f_{1,-1} - M_{-1} f_{-1,1} > 0 \Leftrightarrow \\
\varphi \left(1-w_1\right) \left(\frac{1}{R_1^*}\right)^{\frac{1-\alpha w_1}{1-\alpha}} \frac{w_{-1}}{1-w_1} &> \varphi q_{-1}^{\frac{1}{1-\alpha}} \left(1-w_{-1}\right) \left(\frac{1}{R_{-1}^*}\right)^{\frac{1-\alpha w_{-1}}{1-\alpha}} \frac{w_1}{1-w_{-1}} \Leftrightarrow \\
w_{-1} \left(\frac{1}{R_1^*}\right)^{\frac{1-\alpha w_1}{1-\alpha}} &> w_1 q_{-1}^{\frac{1}{1-\alpha}} \left(\frac{1}{R_{-1}^*}\right)^{\frac{1-\alpha w_{-1}}{1-\alpha}} \Leftrightarrow \\
w_{-1} &> w_1 q_{-1}^{\frac{1}{1-\alpha}} \frac{\left(R_1^*\right)^{\frac{1-\alpha w_1}{1-\alpha}}}{\left(R_{-1}^*\right)^{\frac{1-\alpha w_{-1}}{1-\alpha}}}. \qquad (43)
\end{aligned}$$

To find the price q_{-1} of other goods (than good 1), we use the market clearing condition for good 1. Let $i = 1$ in (37), and we have

$$\begin{aligned}
\theta_1 \left(N-1\right) \left(s_{-1} - v_{-1}\right) &= \left(1-\theta_1\right) \left(s_1 - v_1\right) \Leftrightarrow \\
\theta_1 \left(N-1\right) \left(s_{-1} - v_{-1}\right) &= \left(N-1\right) \theta_{-1} \left(s_1 - v_1\right) \Leftrightarrow \\
\theta_1 \left(s_{-1} - v_{-1}\right) &= \theta_{-1} \left(s_1 - v_1\right). \qquad (44)
\end{aligned}$$

From (26), we find

$$\begin{aligned}
s_1 &= \varphi \times \frac{1-\alpha w_1}{\alpha} \left(\frac{1}{R_1^*}\right)^{\frac{\alpha}{1-\alpha}\left(1-w_1\right)} \\
s_{-1} &= \varphi q_{-1}^{\frac{1}{1-\alpha}} \times \frac{1-\alpha w_{-1}}{\alpha} \left(\frac{1}{R_{-1}^*}\right)^{\frac{\alpha}{1-\alpha}\left(1-w_{-1}\right)}.
\end{aligned}$$

From (31),

$$\begin{aligned}
v_1 &= \left(N-1\right) M_{-1} f_{n1}^E \\
&= \left(N-1\right) \varphi q_{-1}^{\frac{1}{1-\alpha}} w_1 \left(\frac{1}{R_{-1}^*}\right)^{\frac{1-\alpha w_{-1}}{1-\alpha}} \\
v_{-1} &= \left(N-2\right) M_{-1} f_{-1,-1}^E + M_1 f_{1,-1}^E \\
&= \varphi w_{-1} \left[\left(N-2\right) q_{-1}^{\frac{1}{1-\alpha}} \left(\frac{1}{R_{-1}^*}\right)^{\frac{1-\alpha w_{-1}}{1-\alpha}} + \left(\frac{1}{R_1^*}\right)^{\frac{1-\alpha w_1}{1-\alpha}}\right].
\end{aligned}$$

Substitute these into (44) and with some rearrangement, and the equation is equivalent to

$$\begin{aligned}
& \theta_1 q_{-1}^{\frac{1}{1-\alpha}} \times \frac{1-\alpha w_{-1}}{\alpha} \left(\frac{1}{R_{-1}^*}\right)^{\frac{\alpha}{1-\alpha}\left(1-w_{-1}\right)} + \left[\left(N-1\right)\theta_{-1} w_1 - w_{-1}\left(N-2\right)\theta_1\right] q_{-1}^{\frac{1}{1-\alpha}} \left(\frac{1}{R_{-1}^*}\right)^{\frac{1-\alpha w_{-1}}{1-\alpha}} \\
= \; & \theta_{-1} \frac{1-\alpha w_1}{\alpha} \left(\frac{1}{R_1^*}\right)^{\frac{\alpha}{1-\alpha}\left(1-w_1\right)} + w_{-1}\theta_1 \left(\frac{1}{R_1^*}\right)^{\frac{1-\alpha w_1}{1-\alpha}},
\end{aligned}$$

which, given $(N-1)\theta_{-1}w_1 - w_{-1}(N-2)\theta_1 = (1-\theta_1)w_1 - w_{-1}(N-2)\theta_1 = w_1 - \theta_1(1-w_{-1})$, is further equivalent to

$$q_{-1}^{\frac{1}{1-\alpha}} \frac{(R_1^*)^{\frac{1-\alpha w_1}{1-\alpha}}}{(R_{-1}^*)^{\frac{1-\alpha w_{-1}}{1-\alpha}}} = \frac{\theta_{-1}\frac{1-\alpha w_1}{\alpha}\frac{1-\alpha w_1}{\alpha(1-w_1)}(1+\rho) + w_{-1}\theta_1}{\theta_1\frac{1-\alpha w_{-1}}{\alpha}\frac{1-\alpha w_{-1}}{\alpha(1-w_{-1})}(1+\rho) + [w_1 - \theta_1(1-w_{-1})]}.$$

Substitute this equation into (43), which then is equivalent to

$$w_{-1} > w_1 \frac{\theta_{-1}\frac{1-\alpha w_1}{\alpha}\frac{1-\alpha w_1}{\alpha(1-w_1)}(1+\rho) + w_{-1}\theta_1}{\theta_1\frac{1-\alpha w_{-1}}{\alpha}\frac{1-\alpha w_{-1}}{\alpha(1-w_{-1})}(1+\rho) + [w_1 - \theta_1(1-w_{-1})]} \Leftrightarrow (41).$$

Q.E.D.

Proof of Corollary 3:

If $\theta_1 = 0$, then $\theta_{-1} = /(N-1)$ and condition (41) is then equivalent to

$$\begin{aligned}
\frac{w_{-1}}{1-w_{-1}} &> \frac{w_1}{1-w_1} \times \frac{\theta_{-1}\left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho)}{w_1(1-w_{-1})} \Leftrightarrow \\
w_{-1} &> \frac{1}{1-w_1} \times \theta_{-1}\left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho) \Leftrightarrow \\
\frac{w_{-1}}{\theta_{-1}} &> \frac{1}{1-w_1} \times \left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho) \Leftrightarrow \\
\frac{\frac{1-w_1}{N-1}}{\frac{1}{N-1}} &> \frac{1}{1-w_1} \times \left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho) \Leftrightarrow \\
1-w_1 &> \frac{1}{1-w_1} \times \left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho) \Leftrightarrow \\
(1-w_1)^2 &> \left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho),
\end{aligned}$$

which can never hold true because $(1-w_1)^2 < \left(\frac{1}{\alpha} - w_1\right)^2 \leq \left(\frac{1-\alpha w_1}{\alpha}\right)^2(1+\rho)$. Hence $\Upsilon_{1,-1} > 0$ is not true. Therefore, $\Upsilon_{1,-1} \leq 0$.

Q.E.D.