THE BEHAVIOUR OF PLAYERS IN GAMES WITH A MIXED STRATEGY NASH EQUILIBRIUM: EMPIRICAL EVIDENCE FROM PENALTY KICKS

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Abstract

I have constructed a data set of penalty kicks in professional football matches that has allowed me to conduct empirical test of the main implications of the Minimax theorem. In this natural situation professionals football players play a one-shot two-person zero-sum game. I find that the results are consistent with the first implication of the Minimax theorem (winning probabilities are statistically identical across strategies for players) when I only take into account two strategies (Left and Right), but they become inconsistent when I expand the set of possible actions. In contrast, I find that the second implication of the Minimax theorem (player's choices are serially independent) is not rejected in my data set.

1. INTRODUCTION

In many economic and non-economic situations, agents behave strategically: they take into account how their behaviour affects that of others and vice versa. Game theory is a formal approach to analysing strategic interactions (see Osborne (2003) and Osborne and Rubinstein (1998) for textbook treatments).

A Nash equilibrium (NE) is defined as the situation where each player chooses the strategy that maximizes his expected payoff given the strategies chosen by other players, so no one has an incentive to deviate.

A pure strategy is defined as a specific action that a player will follow in every possible attainable situation in a game.

A player, rather than choosing a pure strategy, could choose his strategy by randomizing between his pure strategies, assigning probability p1 to the first pure strategy, p2 to the second pure strategy and so on, with: p1 + p2 +…+ pk = 1. These are mixed strategies. When we allow for mixed strategies, we can often find a NE for a game which has no NE in pure strategies.

For example, in the Matching Pennies game, there are two players and each player has a penny. The rules of the game are the following: the players must simultaneously display their pennies with either heads (H) or tails (T) facing up. If the two pennies match, then player 1 wins player 2's penny. If the pennies do not match, then player 2 wins player 1's penny. This game has no pure strategy NE. Taking what player 2 does as given, player 1 will only play a mixed strategy if he is indifferent between playing H and playing T. The same reasoning applies to player 2. There is a mixed strategy NE, where each player chooses H with probability ½ and T with probability ½.

In the last few decades game theory has helped to reshape vital parts of Economics and other social sciences. The assumption that players are rational helps us to understand the behaviour of real individuals, firms and other agents. However, testing the implications of the theory has been very difficult to do.

In this paper, I test if the empirical results obtained by Palacios Huerta (2002) are still valid with recent data.

To do so, I have collected an up to date data set of 549 penalty kicks in professional football games that include very detailed information on all relevant aspects of the play, specifically actions and outcomes.

Moreover, I have expanded the actions of the players including Left down, Left up, Centre down, Centre up, Right down and Right up, for a tougher test on the predictions of von Neumann's Minimax theorem.

The first testable implication I check is that the scoring probabilities for a player are identical across strategies. I will test the null hypothesis using a Linear Probability Model, a testing procedure which is simpler and more intuitive than the one used by Palacios Huerta (2002). The second testable implication is that a player's mixed strategy is the same at each penalty kick regardless of his previous actions. I will test this implication using a test for serial independence, which is also simpler and more informative than the one used by Palacios Huerta (2002).

The paper is organised as follows. Section 2 contains a literature review. Section 3 discusses the theoretical model and the equilibrium. Next, in section 4 I explain the econometric methodology while section 5 provides a description of the data base that I have compiled. The results of the empirical analysis are presented in section 6. This is followed by the conclusions and an appendix where additional details can be found.

2. LITERATURE REVIEW

The main purpose of this introduction is to discuss the existing empirical evidence on strategic interactions. I focus mostly on two person zero sum games in professional sports. A game is called zero-sum if the payoff of one player is always the negative payoff of the other player.

O'Neil (1987) is the first paper to test the Minimax theory for two-person zero sum experimental games (the Minimax theory is a decision rule used in game theory for minimizing the possible loss in the worst case scenario). He conducted an experiment based on the following two person game. Each player chooses one of four cards: Ace, Two, Three, or Joker. In order to determine the winner of the game, player 1 wins if there is a match of Jokers or a mismatch of number cards (Ace and Three, for example). Player 2 wins if there is a match of number cards or a mismatch of Joker.

The game has a unique NE in mixed strategies: each player chooses the Joker with probability 0.4 and chooses each number card with probability 0.2. For simplicity, O'Neil groups the cards Ace, Three and Two into one, called Non-Joker.

In O'Neil's (1987) experiment, 25 pairs of subjects played a simple two-person game 105 times, where each subject always playing against the same opponent. There were 5250 moves in total. He awarded the winner of each play a nickel (5 cents) and the loser nothing (a total of $5.25 per pair of subjects).

The players who were considered as player 1 chose the Joker 36.2% of the time, while those considered as player 2 chose it 43%. Using a t-test for the equality between the sample proportions to its population value (40%), O'Neil (1987) could not reject the null hypothesis.

However, the standard deviation across players of the proportion of Jokers chosen is greater than the predicted by the Minimax theory. Another difference from the theoretical predictions is found in the numbers of runs, where a run happens when the same card is chosen consecutively. O'Neil (1987) observed that many subjects tended to switch moves more often than if their choices were independent. In particular, a surprising result was that the players tended to avoid repeating a move after a win. This indicates that there is negative serial correlation.

An advantage of this experiment is that players understood the game. Moreover, the theoretical predictions are identical regardless of the utilities for money of the subjects. In summary, evidence in favour of the theory was the correct proportion of strategies used and the correct proportions of wins. Negative evidence was the dependence among successive moves and the high variance in proportion of jokers from subject to subject.

There have been many other experiments trying to test the theory of mixed strategies. However, it may be more informative to analyse strategic situations with real life incentives to see if players behave consistently with the theory.

With this objective, Walker and Wooders (2001) tested whether professional tennis players played Minimax strategies when serving and receiving. They represent the game using a simple 2×2 matrix with a server and a receiver. Both server and receiver have two strategies. The server can serve right or left; and the receiver can receive right or left. In practice, they both play simultaneously. The game has a unique NE in mixed strategies.

During a tennis match, it is not possible to observe which strategy the receiver is choosing. The only outcomes observable are the server's action and the winner of the point. Moreover, the majority of the points do not determine who the winner of the match is. However, in an earlier paper, Walker and Wooders (2000) proved that in equilibrium each player regards the point played as if it was the last one. Still, they only take into account the advantage and deuce court points. As a consequence, they defined the winner of the game as the winner of the point regardless if the receiver returns the ball or not.

Their data came from ten matches between high ranked tennis players. The data set contains detailed information on the direction of the serve (Left, Right) and if the server won the point.

Walker and Wooders (2001) tested if the server's probability of winning the point was the same for left and right serves. The reason is that if a player is playing according to mixed strategies, he must have the same probability of success irrespective of the direction. They considered several tests. The first one tests that the probabilities of winning for left and right serve are the same for each match. The results at the 5% level of significance showed that in only one of the 40 point-games the probability of serving left was not equal to the probability of serving right. Also, only one other point game is rejected at the ten percent level. Under the null, there should be approximately two rejections at the 5% level and four at the 10% level. Thus, it appears that the data is consistent with the theory.

Then, they used Pearson chi-squared test to see if the probabilities of winning after a left or right serve were the same within each match for all matches simultaneously. They cannot reject the null hypothesis at any of the common levels of significance. Moreover, Walker and Wooders (2001) conducted Monte Carlo simulations to evaluate the power of the Pearson test. They concluded that this test has high power, so their lack of rejection seems genuine.

They also tested the serial independence of the player´s moves based on the number of runs. If there are too many or too few runs, the null is rejected. Too few runs mean that the server changes direction too many times. Too many runs is the opposite. Only five times is the null rejected at the five percent level. In three of the five cases the rejection is because there are too many runs (negative serial correlation) and in two because too few runs (positive serial correlation). If there was serial independence, there should be only two rejections out of 40 point games.

The finding that even professional tennis players sometimes switch actions too often is not surprising, especially in view of O'Neil (1987) results. There is a lot of experimental evidence that shows that when people try to generate random sequences, they avoid repeating numbers because they believe doing so is more consistent with random generated choices.

Walker and Wooders (2001) also carried out lab experiments in which they strongly established that inexperienced players will not generally mix in the equilibrium proportions when there is uncertainty about the actions of the opponents. In summary, their paper contains promising results on how to understand human behaviour in strategic situations.

Another sport which is even more popular than tennis is football. In fact, football is one of the most important sports in the world. More than a century after it was invented, the media claimed that about 50% of humans who were alive in 2010 watched the final of the World Cup between Holland and Spain.

Chiappori, Levitt and Groseclose (2002) offered evidence on the application of mixed strategies to penalty kicks in football. A penalty kick can be also regarded as a zero sum game between two players, one kicker and one goalkeeper, because the rules of the game forbid any other player to intervene. Given that the kicker cannot touch the ball a second time until it has been touched by another player, Chiappori et al. (2002) assumed for simplicity that the kicker's actions are simply to shoot left, right or centre. In turn, given that if the goalie decides the action after the kick, he will not be able to stop the shot, both players must choose their strategies simultaneously. In this regard, Chiappori et al. (2002) also assumed that the goalie has three strategies: jump to the left, remain in the middle or jump to the right. In this game, the kicker's main objective is to maximize the scoring probability while the goalie wants to minimize it. Theoretically, the game can be defined by a 3×3 matrix. This game has no pure strategy NE. Taking what the goalkeeper does as given, the kicker will only play a mixed strategy if he is indifferent between playing Left, Centre and playing Right. The same reasoning applies to the goalkeeper.

Chiappori et al. (2002) made three assumptions to solve the game. The first assumption, "Sides and Centre" (SC), states that if the goalkeeper action was known, the kicker would kick somewhere else. The "Natural Side" (NS) assumption, where the natural side for a right-footed kicker is his left side and vice versa, implies that the kicker has a preference to his natural side regardless of the action of the opponent. Finally, the "Kicker's Side" (KS) assumption states that kicks to the natural side score more often.

The empirical evidence presented in the paper justifies these assumptions. In particular, when the goalie action differs from the kicker's, the scoring probability range is between 89 to 95 percent. In contrast, when the goalie makes the correct action, the range falls to 43 to 64 percent. Similarly, the scoring probability between kicking to one side versus the centre is 92 to 84 percent respectively. Finally, the scoring probability in the kicker's natural side is systematically higher. In addition, the difference is even greater when the goalie makes the correct choice.

Using their three assumptions, Chiappori et al. (2002) showed that there are two types of NE, depending on the values of the parameters. In one of them (known as the restricted randomization (RR) equilibrium) there are only two actions: left or right, because neither kicker nor goalkeeper will choose centre. In the other equilibrium (known as the general randomization (GR) equilibrium) there are three actions: left, centre or right. In both equilibriums, the scoring probabilities are equal.

The authors collected their data set by watching videotapes that contained penalty kicks in the French and Italian premier leagues over a period of three years, gathering a total of 459 kicks. The data, which is extremely detailed, contains the name of the player, whether he shot with his left or right foot, and the actions taken by him and the goalie.

To test the assumption that both players moved simultaneously, Chiappori et al (2002) tested that the strategy chosen by the rival did not forecast the other player's action on the penalty kick. Using a linear probability model, they did not reject this null hypothesis. However, the actions of both players coincide 2.7 percent more than would be expected when using data for all kickers. Nevertheless, for kickers with multiple penalties, there seems to be no relationship between the kicker´s actions in a penalty and the actions of the goalkeeper.

Unfortunately, unlike tennis matches, in football the combination of specific players and goalies rarely repeats. Therefore, testing the prediction of this model is complicated as there are limited observations for each player. Consequently, it is not easy to observe whether the player is playing mixed strategies or not. For that reason, Chiappori et al. (2002) test those predictions of the model that are robust to aggregation across heterogeneous players. They also focused on players with repeated observations.

Under the assumption that kickers randomize their actions, it is easy to predict the number of kickers with at least two penalties who always use the same strategy. They find 16 kickers using the same strategy, which compares favourably with the predicted number of 14 kickers. They get similar results on goalkeepers.

Finally, like O'Neil (1987) and Walker and Wooders (2001), Chiappori et al. (2002) also tested if there is serial correlation in actions. Conditional on the probability of choosing left, centre or right, the actions previously chosen by a player should not predict the action he will take next. They find absence of serial correlation.

Chiappori et al. (2002) model predicts that the action centre is more often used by the kicker than the goalie. The data supports the prediction as the kicker chooses centre 79 times while the goalie only 11 times. Another prediction from the model is that the goalkeeper should choose the action of the kicker's natural side more frequently than the kicker himself. As seen from the data, the goalie plays action natural side 56.6% of the times, while the kicker only does it 44.9%. Given that there are about 70-80 percent of right footed footballers, the model predicts that the kickers are more likely to shot left than right. The data confirms the prediction as 260 times the goalie jumps to the left, and only 188 to the right.

Finally, assuming that all players were identical, the theory implies that the average success rate for kickers should be the same for all actions, and similarly for goalies. But one could argue that some players are better than others. For that reason, Chiappori et al. (2002) use a regression framework to test the assumption that goalkeepers are homogeneous. To increase the power of the test, they restrict the sample to only goalkeepers with at least four penalties. Chiappori et al. (2002) rejected the hypothesis that all goalies were identical.

Although this paper represents one of the first attempts to test mixed strategy behaviour using data from real life situations, Chiappori et al. (2002) look at aggregate behaviour rather than individual players. The contribution of Palacios Huerta (2002) is to construct a bigger database, which allows him to observe repeatedly many individual players. In particular, his data set contains 1417 penalty kicks collected during five years from football games from the major European Leagues.

This allows him to contrast the two fundamental implications of von Neumann's Minimax theorem. The first testable prediction is that the winning probabilities should be equal regardless of the strategy for both players. The second is that the actions taken by the players must not be serially dependent.

To test the first implication, he uses Pearson's chi square goodness of fit test, which compares the success rates across actions. Of the 42 players with repeated penalty kicks, the null hypothesis is rejected for two kickers and one goalkeeper at the 5% level of significance and for three kickers and two goalkeepers at the 10% level. Since 5% of 42 is 2.1 and 10% is 4.2, these estimates suggest that at the individual level the hypothesis that scoring probabilities are identical across strategies cannot be rejected for most players at the usual significance levels.

Then he examined whether actions at the aggregate level could be considered to be generated from equilibrium play by testing the joint hypothesis that each one of the penalty kicks is consistent with equilibrium play. For kickers the Pearson statistic is 24.14 whereas for goalies the Pearson statistic is 19.81. As a result, the hypothesis of equality of winning probabilities cannot be rejected for kickers or goalies. Therefore, the empirical evidence on professional penalty kicks is consistent with the first implication of the Minimax Theorem.

As mentioned earlier, the second testable implication of this theorem is that player's strategies are serially independent. Using a serial correlation test, Palacios Huerta (2002) finds that the null hypothesis is rejected for one kicker and one goalkeeper at the 5% significance level and two kickers and two goalkeepers at the 10% level. Given the expected number of rejections, the number of actual rejections is remarkably consistent with the theory. Therefore, it seems that professional football players generate random sequences.

In summary, the results in Palacios Huerta (2002) results represent the first time that both implications of von Neumann's Minimax theorem are successfully tested under natural conditions.

3. PENALTY KICKS IN FOOTBALL

According to Federation Internationale de Football Association (FIFA) in the Official Laws of the Game (FIFA, 2000) "in football, a penalty kick is awarded against a team which commits one of the ten punishable offenses inside its own penalty area while the ball is in play."

In a penalty kick, the ball takes about 0.3 seconds to travel the distance between the penalty mark and the goal line; thus, if the goalie decides the action after the kick, he will not be able to stop the shot, so both players must choose their strategies simultaneously. This is observed by Miller (1998) where he reports evidence on ball speed, reaction times, and movement times from all the penalty kicks in World Cups.

The penalty kick has only two possible outcomes: goal or miss. The outcome is immediately decided after both players choose their strategies.

3.1 The model

One goalkeeper and one kicker are facing each other at a penalty kick. The kicker prefers scoring to not scoring, while the goalkeeper has the opposite preferences. The kicker may choose to kick to the goalkeeper's right (R), to his left (L), or to the centre (C). Similarly, the goalkeeper may choose to jump to his left, to the right or remain at the centre. When the kicker and the goalie choose the same side (L, C, or R) the outcome is less likely to be a goal. For simplicity let's assume that in this theoretical model the probability of scoring a goal when the actions coincide is in fact zero. Otherwise, when the goalie and the kicker choose different sides, the goal is scored.

The payoff matrix is:

Goalkeeper

		Left	Centre	Right
Kicker	Left	0,1	1,0	1,0
	Centre	1,0	0,1	1,0
	Right	1,0	1,0	0,1

where the first payoff corresponds to the kicker and the second payoff corresponds to the goalkeeper.

There are no pure strategies Nash Equilibrium. Therefore, the players must play mixed strategies to maximize their gain. In mixed strategies, suppose that the kicker believes that the goalkeeper plays L with probability q_L, R with probability q_R and C with probability $(1-q_L- q_R)$. Similarly, the goalkeeper believes that the kicker plays L with probability p_L, R with probability p_R and C with probability $(1-p_L- p_R)$.

For the kicker to play mixed strategies, we need his expected payoff regardless of the strategy chosen (L, C or R) to be exactly the same. The same reasoning applies to the goalkeeper. In this special game, the game has a unique mixed strategy Nash Equilibrium where both the kicker and the goalkeeper choose L, R and C with probability 1/3.

4. ECONOMETRIC METHODOLOGY

4.1 <u>Tests of equal scoring probabilities</u>

4.1.1 Individual tests

For the first implication of the Minimax theorem, Palacios Huerta (2002) uses a Pearson's χ^2 goodness-of-fit test of equality of two distributions. Instead, I use a Linear Probability Model (LPM) to test the null hypothesis that the scoring probabilities for both kickers and goalkeepers should be equal regardless of the strategy chosen (see Wooldridge (2002) chapter 7, section 5 for more details).

The LPM is described as follows: Let Y denote a Bernoulli random variable which takes the value 1 if the penalty is scored and 0 otherwise. Suppose there are three strategies available for each player (Left “L”, Centre “C”, and Right “R”). In that case, the model becomes:

$$Y=\delta_L L+ \delta_C C+ \delta_R R+ u \ ,$$

where L, C and R are dummy variables. L takes the value 1 if the penalty is shot in that direction and 0 otherwise. The same applies to the other dummy variables (R and C). It is easy to see that the regression coefficients have a direct interpretation as conditional scoring probabilities. In particular, δ_L=P(Y=1,L=1)/P(L=1), that is, the proportion of left kicks scored. Thus, the coefficients of the dummy variables are always between 0 and 1, avoiding one common criticism of the Linear Probability Model (see again Wooldridge (2002)).

Under the null, the probability of Y being equal to 1 should not depend on the strategy. The null hypothesis can then be express as: H_0: $\delta_L=\delta_C=\delta_R= 0$. In practice, it is easier to test this hypothesis by estimating the following model:

$$Y=\beta_0+ \beta_1 L+ \beta_2 R+ u \ ,$$

where $\beta_0= \delta_C$, $\beta_1= \delta_L - \delta_C$ and $\beta_2= \delta_R- \delta_C$. Hence, in this regression, the coefficients of the dummy variables are the differences between the probabilities of the corresponding strategy and the base line. The adjustment of this regression is identical to the adjustment of the regression written in terms of deltas. But it has the advantage that the null hypothesis of equal scoring probabilities can be expressed as: H_0: $\beta_1 = \beta_2 = 0$. This can be tested using an F-test with 2 degrees of freedom in the numerator and n-3 degrees of freedom in the denominator.

As is well known, the formula of this F-test is: $=\frac{R^2/k}{(1-R^2)/n-k-1}$, where R squared measures which proportion of the variability of the dependent variable is explained by the k non-constant explanatory variables. Therefore, the F-statistic would be zero if the scoring probability is exactly the same across strategies and it would be infinity when the regressors provide a perfect fit, which will only happen when the scoring probability for each strategy is either zero or one.

Furthermore, I have also estimated a LPM with only two strategies to compare my results with the results obtained by Palacios Huerta (2002). Similarly, I have also considered six actions. In both cases, the above F-test formula remains valid after adjusting the value of k.

Some players never employ one of the three strategies (Left, Centre or Right). In that case, the F-test will have 1 degree of freedom in the numerator and n-2 degrees of freedom in the denominator.

The LPM has another potentially important disadvantage. Under the alternative, it violates the homoscedasticity assumption, i.e. the variance of the error term will change depending on the values of the explanatory variables (see again Wooldridge (2002)). However, the variance of u given the x’s is constant (β_0*(1-β_0)) under the null hypothesis of equal scoring probabilities. This means that the homoscedasticity assumption holds and the F-test is valid.

4.1.2 Aggregate Test

The large sample justification for the F-statistic suggests an equivalent test when a player has a significantly high number of observations. If I multiply the F-statistic by dfn (degrees of freedom in the numerator), the resulting random variable will converge to a χ^2 with dfn degrees of freedom when dfd (degrees of freedom in the denominator=number of observations-dfn) goes to infinity.

Therefore, given that the observations for different players are independent, I can compute an aggregate test as the sum of those χ^2 across players, which results in another χ^2 with degrees of freedom equal to the sum of the degrees of freedom for each player.

Algebraically,

$$F_A = \sum_{i=1}^{N} (dfn_i * F_i),$$

where dfn_i is the degrees of freedom in the numerator of player i and F_A is the aggregate F-statistic.

The advantage of this aggregate test is that not only does it take into account the number of individuals rejections, but also the p-value of each individual player. For example, if the individual test for one player has a very small p-value, this rejection maybe sufficient to reject at the aggregate level. This notion is not always taken into account by the papers discussed in section 2.

4.2 Test for Serial independence

Palacios Huerta (2002) uses a run test to check the second testable implication of the Minimax theorem, which states that a player's strategy is the same at each penalty kick regardless of his previous actions. In that regard, note that the players' strategies will not be serially independent if they switch actions too often (negative serial correlation) or if they choose not to switch their actions regularly (positive serial correlation).

Instead, I will use a multivariate version of the linear probability model to test if the player's strategies are serially independent. The advantage of my approach is that it simultaneously estimates the probabilities with which each player change strategies.

A multivariate regression is a technique that combines several regression models with the same regressors, one for each dependent variable. The multivariate regression I have used to detect possible departures from serial independence is similar to a first-order vector autoregressive process (see Wooldridge (2002) chapter 18, section 5 for more details). Specifically:

$$L_t = \delta_{LL} L_{t-1} + \delta_{LC} C_{t-1} + \delta_{LR} R_{t-1} + u_{Lt}$$

$$C_t = \delta_{CL} L_{t-1} + \delta_{CC} C_{t-1} + \delta_{CR} R_{t-1} + u_{Ct}$$

$$R_t = \delta_{RL} L_{t-1} + \delta_{RC} C_{t-1} + \delta_{RR} R_{t-1} + u_{Rt} \;,$$

which in matrix form can be written as:

$$\begin{pmatrix} L_t \\ C_t \\ R_t \end{pmatrix} = \begin{pmatrix} \delta_{LL} & \delta_{LC} & \delta_{LR} \\ \delta_{CL} & \delta_{CC} & \delta_{CR} \\ \delta_{RL} & \delta_{RC} & \delta_{RR} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ C_{t-1} \\ R_{t-1} \end{pmatrix} + \begin{pmatrix} u_{Lt} \\ u_{Ct} \\ u_{Rt} \end{pmatrix},$$

where L_t, C_t and R_t are the dependent variables, L_{t-1}, C_{t-1} and R_{t-1} are lagged regressors, δ_{LC} measures the probability of L_t being equal to 1 given that C_{t-1} is equal to 1, etc. In the multivariate regression with three lagged explanatory variables, i.e. with no constant, the coefficients of the lagged variables are the probability of choosing a second strategy conditional on the first action. These are sometimes called transition probabilities. The sum of δ_{LL}, δ_{CL} and δ_{RL} is equal to one, and the same applies to the other columns in the matrix. Therefore, the coefficients in equation C_t can be obtained from the other two equations because C_t=1-L_t- R_t. For that reason, I can eliminate this equation from the system of equations.

The null hypothesis of serial independence implies that $\delta_{LL}=\delta_{CL}= \delta_{RL}$ and $\delta_{LR}=\delta_{CR}= \delta_{RR}$.

In practice, it is easier to test this hypothesis by estimating the following model:

$$L_t= \beta_{L0} + \beta_{LL} L_{t-1}+ \beta_{LR} R_{t-1}+ u_{Lt}$$

$$R_t= \beta_{R0} + \beta_{RL} L_{t-1}+ \beta_{RR} R_{t-1}+ u_{Rt}$$

where $\beta_{L0}= \delta_{LC}$ and $\beta_{R0}= \delta_{RC}$, $\beta_{LL} = \delta_{LL}- \delta_{LC}$, $\beta_{LR} = \delta_{LR}- \delta_{LC}$, $\beta_{RL} = \delta_{RL}- \delta_{RC}$ and $\beta_{RR} = \delta_{RR}- \delta_{RC}$.

In the regression with only two lagged variables, the coefficients of the lagged variables are the differences between the probabilities of the corresponding strategy and the base line, which is the lagged variable C_{t-1}.The adjustment of these regressions is identical to the adjustment of the regressions written in terms of deltas. But they have the advantage that the null hypothesis of serial independence can be expressed as $\beta_{LL} = \beta_{LR} = \beta_{RL} = \beta_{RR} = 0$. In addition, homoscedasticity will again hold under the null, so the F-test remains valid.

5 DATA

I test the assumptions and predictions of the model using a novel data set I have constructed, which contains 549 penalty kicks.

The penalty data I have collected covers the seasons 2005-2006 until the current one (2014-2015) from professional games in Spain, Italy, England and other European countries. The information for the data comes from the following Spanish TV programs and internet pages: Estudio Estadio (TVE), GOL TV, Canal Plus Liga, El Dia Despues (Canal + futbol), Deportes Cuatro, As.com and Marca.com. These programs and internet pages review all of the best games played during the weekend, including all penalty kicks that take place in those games.

The data include the names of the teams involved in the match, the date of the match, the names of the kicker and goalkeeper for each penalty kick, the choices taken (Left down, Left up, Centre down, Centre up, Right down and Right up), the time within the match at which the penalty takes place, the score at the time of the penalty, the final score of the game, the foot used by the kicker (left or right) and the outcome of the kick (goal or miss).

Table 1 offers a basic description of the data with three actions.

(TABLE 1)

In particular, it shows the relative proportions of different choices made by both kickers and goalkeepers (Left (L), Centre (C), or Right (R)), with the total number of observations in the second column starting from the left. The first letter refers to the choice made by the kicker and the second to the choice made by the goalkeeper, both from the point of view of the goalkeeper. For instance, "LL" means that the kicker chooses to kick to the left hand side of the goalie and the goalie chooses to jump to this left. The first column starting from the right shows the scoring rate for a given score difference. Score difference is defined as the number of goals scored by the kicker's team minus the number of goals scored by the goalkeeper's team at the time the penalty takes place. For example, a "1" means that the kicker's team was ahead by one goal at the time of the penalty kick.

The strategy followed by goalkeepers coincides with that followed by kickers in 42.8% of all penalties in the data set. Kickers do not usually kick to the centre (7.1% of all kicks), whereas goalies remain in the middle even less often (4.55%). The percentage of kicks where none of both players coincide is mostly divided between LR (26.78) and RL (20.95). A goal is scored in 86.34% of all penalty kicks. The scoring rate is over 90% when the kicker choice differs from the goalie, and it is just over 65% when it coincides.

According to the Official Laws of the Game Rule 7 (FIFA, 2000), football matches have two 45 minutes halves and an approximately 15 minute break in-between. The scoring rate of penalties is lower in the second half (84.34%) than in the first half (87.45%), and slightly lower given the matches tend to be balanced in the last 10 minutes (85.58%) than the overall average (86.34%).

Moreover, it is not surprising that in most penalty kicks the existing score difference is 0, 1 and -1. In this regard, the score rate at the time of the penalty is greater in tied matches (89.08%), followed by matches where the kicker's team is ahead by one goal (83.33%), and finally matches where the kicker's team is behind by one goal (83.12%).

6 EMPIRICAL ANALYSIS

In the data set, there are 12 kickers with more than 20 penalty kicks and 11 kickers with at least 10 penalties. Similarly, there are 10 goalkeepers with more than 10 observations. I focus on those subjects who were involved in a relative large number of penalties. The identities of goalies and kickers are shown in the appendix. For each of these players the observations in the data set include all the penalties they have participated between seasons 2005-2006 until 2014-2015, in chronological order.

6.1 Tests of equal scoring probabilities

As I mentioned in section 3, I use a Linear Probability Model test of the null hypothesis that the scoring probabilities for both kickers and goalkeepers are identical across strategies.

6.1.2 Individual tests

First, I test the hypothesis that the scoring probabilities are equal regardless of the strategy chosen with three actions only: Left, Right and Centre (see section 3.1.1). The results of the tests are described in Table 2.

(TABLE 2)

Of the 22 players in the sample, the null hypothesis is rejected for 5 players (three kickers and two goalkeepers). This means that these estimates suggest that at the individual level the hypothesis that scoring probabilities are identical across strategies cannot be rejected for most players, but not all.

Then, I decided to carry out a tougher test on the first implication of the Minimax Theorem, so I expanded the actions of the players to left down, left up, center down, center up, right down and right up. I only do it for kickers because goalkeepers apparently do not jump left up, right up or centre up, so that they only follow the three strategies already considered. This may happen because it is virtually impossible for a goalkeeper to jump sufficiently high when a penalty takes place.

The results are shown in Table 3.

(TABLE 3)

These results show that the p-values of the F-statistics for kickers typically increase compared to the ones I obtained with three actions. But qualitatively, the results obtained with six and three actions are identical, i.e. rejections occur for the same three kickers.

Hence, the hypothesis stating that the scoring probabilities for both kickers and goalkeepers should be equal regardless of the strategy chosen cannot be rejected for most players but it is again inconsistent with the theory for a few of them.

To compare my results with the results obtained by Palacios Huerta (2002), I consider only the actions he took into account (Left and Right) to test if the results he obtained are still consistent with my more recent data set.

(TABLE 4)

The results in Table 4 show that the number of individual rejections has decreased. In fact, I do not reject the null for any of the kickers in the sample. Thus, it seems that kickers have the same probability of scoring regardless of the strategy chosen (Left or Right, in this case).

Similarly, for all the goalkeepers except one, the probability of scoring does not depend on the strategy chosen.

Therefore, the results are consistent with the theory for all individual kickers but not for all goalkeepers at the individual level.

In the case of three actions, my tests can be regarded as tests of what Chiappori et al. (2002) called the general randomization equilibriums. In contrast, Palacios Huerta (2002) effectively focused on what Chiappori et al. (2002) called the restricted randomization equilibrium, in which the players have only two actions: Left or Right, because neither kickers nor goalkeeper will choose centre.

6.1.3 Aggregate Test

The rejections I have found for a few players does not necessarily imply the rejection of the null hypothesis. For that reason, I also examined whether players considered at the aggregate level could have the same scoring probabilities regardless of the strategy chosen.

As already indicated in section 3.1.2, the test statistic for this joint test is the sum of each individual χ^2 for players with larges samples. Under the null hypothesis, this test is distributed as a $\chi^2$with degrees of freedom being equal to the sum of the number of strategies each player has minus one.

The results are shown in Table 5.

(TABLE 5)

Panel A describes the result from the aggregate test for individual players with three actions (Left "L", Right "R" and Centre "C"). For kickers the test statistic is 56.86 and its p-value is 1.2E-05 and for goalkeepers it is 36.75 with a p-value of 0.002274. This means that the null hypothesis is rejected for both types of players.

Panel B shows the results from aggregate tests for individual players with six actions (Left down "LD", Left up "LU", Centre down "CD", Centre up "CU", Right down "RD" and Right up "RU"). For kickers, the test statistic is 91.48 and its p-values is 1.56E-05 whereas for goalies the test statistic is 36.75 with a p-value of 0.002274. As a result, the hypothesis of equality of scoring probabilities is rejected for kickers and goalies.

Finally, Panel C includes the results I obtain with the two strategies Palacios Huerta (2002) considers (Left and Right). The results show that the test statistic is 32.88 and its associated p-value is 0.0635 for all 22 players. Hence the hypothesis of equality of scoring probabilities cannot be rejected at the aggregate level at the usual 5% significance level. Focusing only on kickers, the test statistic is 10.73 and its p-value is 0.5521, while for goalkeepers it is 22.15 with a p-value of 0.014. Thus, the hypothesis of equality of scoring probabilities cannot be rejected for kickers but it is rejected for goalies.

As a result, the empirical evidence on professional penalty kicks is consistent with the first implication of the Minimax Theorem for kickers only when I eliminate the centre strategy for all players. Nevertheless, given that some players still choose the centre strategy, I would argue that the general randomization equilibrium with three strategies is empirically more relevant.

6.1.4 Interpretation of the results

For the individual tests with three actions, I found that three kickers and two goalkeepers did not have the same scoring probability regardless of the strategy chosen. For example, kicker 2 misses 40% of the time when he shoots to the centre, which is 10% of all his penalties (see Table 2). In contrast, when he kicks to the left (34% of his penalties) he never misses and when he decides to kick to the right (56% of his penalties) he misses only 4% of the times. Presumably, in the future kicker 2 will never choose to kick to the centre because he has learned that the probability of scoring is lower than when he chooses the other two strategies. The same reasoning applies to kicker 6 and 9 who have never scored when they chose strategy C.

Additionally, goalkeeper 4 has a probability of scoring equal to 1 when he jumps to the left, 1 when he jumps to the right and 0.5 when he remains in the middle. This means that when a penalty shot occurs, and goalkeeper 4 decides to remain in the middle, he saves 50% of the penalties; but when he jumps in any other direction, the outcome will be a goal. Something similar occurs to goalkeeper 10, which only saves penalties when he jumps to the left.

In contrast, when only two actions are taken into account (L and R); the null is not rejected for any of the kickers while it is only rejected for goalkeeper 10. This is because his probability of scoring when he jumps to the left is 0.33 and 1 when he jumps to the right.

Given the type 1 error, one expects the F-test for individual kickers and goalkeepers to reject some of the time by mere chance when the null is true. For that reason it is important to consider a joint test that makes allowances for this type 1 error.

In the aggregate tests, for kickers and goalkeepers with three and six actions the null is rejected as the test statistic for this aggregate test is the sum of each individual χ^2. This occurs because the three kickers and the two goalkeepers have a very small p-value, so when I add them up with the rest of the players, the result is still a rejection.

On the contrary, when only two actions are available (L and R), the aggregate test do not reject for kickers but rejects for goalkeepers because of goalkeeper 10. In addition, the aggregate test for all players is not rejected as only one player out of 22 rejects the null.

6.2 Serial Correlation

As mentioned earlier, the second testable implication of the Minimax theorem states that the actions taken by the players must not be serially dependent. The test is implemented with a multivariate version of the Linear Probability Model, as explained in section 3.2. The results are shown in Table 6.

(TABLE 6)

The main result in this analysis is that the null hypothesis that all the explanatory variables are jointly insignificant is not rejected for any of the players, which is perhaps not surprising because the penalty kicks take place days apart. These findings suggest that professional football players are indeed able to generate random sequences; they neither appear to switch strategies too often or too seldom. Therefore, the results obtained are consistent with the second implication of the theory.

7. CONCLUSION

I have constructed a data set of 549 penalty kicks in professional football matches that has allowed me to conduct empirical test of the main implications of the Minimax theorem under natural conditions, i.e. penalty kicks in professional football matches.

I find that the results are consistent with the first implication of the Minimax theorem (winning probabilities are statistically identical across strategies for players) when I only take into account two strategies (Left and Right), but they become inconsistent when I expand the set of possible actions (Left, Centre and Right or Left down, Left up, Centre down, Centre up, Right down and Right up). In contrast, I find that the second implication of the Minimax theorem (player's choices are serially independent) is not rejected in my data set.

Nevertheless, my analysis is only one attempt to test mixed strategy behaviour under natural conditions, but there is still ground breaking research to be made in proving the validity of the Minimax theorem in real life situations.

APPENDIX: KICKERS AND GOALKEEPERS

Players are divided between kickers and goalkeepers. In brackets is the identification number used in table 2, and in parentheses it appears the teams they play for.

KICKERS

[2] Cristiano Ronaldo (Real Madrid/Manchester United), [1] Messi *(Barcelona), [5] Falcao (Atlético de Madrid/Monaco), [6] Gerrard (Liverpool), [7] Guiseppe Rossi (Villareal/Fiorentina), [8] Hulk* (Oporto/Zenit), [9] Ibrahimovic (Inter Milan/Milan/PSG), [10] Kanoute (Sevilla), [3] Negredo *(Almería/Sevilla), [11] Soldado (Getafe/Tottenham), [12] Villa (Valencia/Atlético de Madrid), [4] Xabi Prieto (Real Sociedad)

GOALKEEPERS

[1] Aouate (Deportivo La Coruña/Mallorca), [2] Diego Alves (Almería/Valencia), [3] Diego López (Villareal/Real Madrid), [4] Iraizoz (Athletic Club Bilbao), [5] Moya (Mallorca/Getafe/), [6] Palop (Sevilla), [7] Ricardo (Osasuna), [8] Roberto (Granada), [9] Ruben (Rayo Vallecano), [10]Tono (Racing Santander/Granada/Rayo Vallecano).

*kickers are left-footed. All others are right footed.

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TABLES

Table 1

Distribution of strategies and scoring rates

Score difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	Scoring rate
0	238	23.95	3.36	27.31	3.36	0.84	2.10	18.91	0.00	20.17	89.08
1	120	15.83	1.67	27.50	3.33	1.67	1.67	23.33	2.50	22.50	83.33
-1	77	19.48	3.90	28.57	1.30	2.60	1.30	23.38	2.60	16.88	83.12
2	45	31.11	0	20.00	4.44	0	2.22	24.44	0	17.78	86.67
-2	26	11.54	0	26.92	3.85	0	7.69	26.92	0	23.08	88.46
3	29	6.90	3.45	20.69	10.34	0	6.90	17.24	0	34.48	86.21
-3	5	20	0	20	20	0	0	0	0	40	100
4	4	25	0	50	0	0	0	25	0	0	100
-4	1	100	0	0	0	0	0	0	0	0	0
Others	4	0	0	50	0	0	0	0	0	50	75
Penalties Shot in:											
First half	247	23.08	3.64	24.29	2.43	1.62	2.02	23.48	0.40	19.03	87.45
Second half	198	19.19	2.53	31.82	5.05	0.51	1.52	17.17	1.01	21.21	84.34
Last 10 min	104	17.31	0	22.12	5.77	0.96	4.81	22.12	1.92	25.00	85.58
All penalties	549	20.58	2.55	26.78	3.64	1.09	2.37	20.95	0.91	21.13	86.34
Scoring rate	86.34	69.91	92.86	97.96	100	0	92.31	95.65	100.00	78.45	

Note: The first letter of the strategy denotes the Kicker's action and the second letter the goalkeeper's action. "L" denotes the left hand side of the goalkeeper, "R" denotes the right hand side of the goalkeeper, and "C" denotes the centre.

TABLE 2

Test for equality of scoring probabilities with 3 actions

Player	#Obs.	Mixture L	Mixture C	Mixture R	Scoring Rate L	Scoring Rate C	Scoring Rate R	F-Test	P-value
Kicker 1	36	0.58	0.11	0.31	0.90	1.00	0.82	0.53	0.5963
Kicker 2***	50	0.34	0.10	0.56	1.00	0.60	0.96	7.12	0.002
Kicker 3	28	0.68	0.11	0.21	0.68	1.00	1.00	1.85	0.1774
Kicker 4	20	0.55	0.20	0.25	1.00	1.00	0.80	1.59	0.2321
Kicker 5	21	0.43	0.19	0.38	0.89	1.00	0.88	0.23	0.7946
Kicker 6**	33	0.58	0.03	0.39	0.95	0	0.85	4.97	0.0136
Kicker 7	22	0.50	0	0.50	0.91	0	0.82	0.36	0.5568
Kicker 8	26	0.54	0.15	0.31	0.79	1.00	0.75	0.54	0.5896
Kicker 9***	42	0.33	0.02	0.64	0.93	0	0.96	9.22	0.0005
Kicker 10	20	0.55	0.20	0.25	0.80	1.00	1.00	1.43	0.2657
Kicker 11	20	0.25	0	0.75	1.00	0	0.80	1.12	0.3029
Kicker 12	20	0.70	0	0.30	0.93	0	1.00	0.42	0.5274
All kickers	338	0.49	0.09	0.42	0.88	0.80	0.88		
Goalkeeper 1	14	0.57	0.07	0.36	0.75	1.00	1.00	0.79	0.4798
Goalkeeper 2	18	0.50	0.11	0.39	0.56	0.50	0.57	0.01	0.9867
Goalkeeper 3	12	0.50	0.08	0.42	0.67	1.00	0.57	1.12	0.3664
Goalkeeper 4***	18	0.67	0.11	0.22	1.00	0.50	1.00	6.67	0.0085
Goalkeeper 5	15	0.13	0.27	0.60	1.00	0.75	0.89	0.35	0.7145
Goalkeeper 6	10	0.40	0	0.60	0.50	0	0.83	1.16	0.3122
Goalkeeper 7	10	0.50	0	0.50	1.00	0	0.80	1	0.3466
Goalkeeper 8	13	0.46	0	0.54	1.00	0	0.86	0.85	0.3774
Goalkeeper 9	11	0.73	0	0.27	0.88	0	1.00	0.48	0.5059
Goalkeeper 10***	13	0.23	0.08	0.69	0.33	1.00	1.00	7.69	0.0095
All goalkeepers	134	0.47	0.08	0.45	0.79	0.73	0.88		

Note: *Indicates we reject the null at the 10% significance level, **5% level, ***1% level.

TABLE 3

Test for equality of scoring probabilities with 6 actions

	Mixture							Scoring Rate							
Player	#Obs.	LD	LU	CD	CU	RD	RU	LD	LU	CD	CU	RD	RU	F-Test	P-value
Kicker 1	36	0.36	0.22	0.03	0.08	0.28	0.03	0.85	1.00	1.00	1.00	0.80	1.00	0.48	0.7885
Kicker 2***	50	0.22	0.12	0.06	0.04	0.50	0.06	1.00	1.00	0.33	1.00	0.96	1.00	6.46	0.0001
Kicker 3	28	0.64	0.04	0.07	0.04	0.21	0	0.67	1.00	1.00	1.00	1.00	0	1.03	0.4145
Kicker 4	20	0.35	0.20	0.10	0.10	0.20	0.05	1.00	1.00	1.00	1.00	0.75	1.00	0.75	0.6018
Kicker 5	21	0.43	0	0	0.19	0.38	0	0.89	0	0	1.00	0.88	0	0.23	0.7946
Kicker 6**	33	0.55	0.03	0	0.03	0.39	0	0.94	1.00	0	0	0.85	0	3.22	0.0371
Kicker 7	22	0.32	0.18	0	0	0.45	0.05	1.00	0.75	0	0	0.80	1.00	0.62	0.6141
Kicker 8	26	0.50	0.04	0.08	0.08	0.23	0.08	0.77	1.00	1.00	1.00	0.67	1.00	0.44	0.8177
Kicker 9***	42	0.24	0.10	0.02	0	0.55	0.10	1.00	0.75	0	0	0.96	1.00	5.85	0.0009
Kicker 10	20	0.50	0.05	0.20	0	0.25	0	0.80	0	1.00	0	1.00	0	3.17	0.0532
Kicker 11	20	0.25	0	0	0	0.70	0.05	1.00	0	0	0	0.79	1.00	0.7	0.5125
Kicker 12	20	0.70	0	0	0	0.30	0	0.93	0	0	0	1.00	0	0.42	0.5274
All kickers	338	0.40	0.09	0.04	0.04	0.38	0.04	0.88	0.90	0.80	0.80	0.88	0.87		

Note: *Indicates we reject the null at the 10% significance level, **5% level, ***1% level.

"LU" denotes action left up, "LD" denotes action left down, "CD" denotes action centre down, "CU" denotes action centre up, "RD" denotes action right down, and "RU" denotes action right up. All this actions are seen from the goalkeeper perspective.

TABLE 4

Test for equality of scoring probabilities with 2 actions

Player	Mixture			Scoring Rate			
	#Obs.	L	R	L	R	F-Test	P-value
Kicker 1	32	0.66	0.34	0.90	0.82	0.47	0.50
Kicker 2	45	0.38	0.62	1.00	0.96	0.60	0.44
Kicker 3	25	0.76	0.24	0.68	1.00	2.55	0.12
Kicker 4	16	0.69	0.31	1.00	0.80	2.41	0.14
Kicker 5	17	0.53	0.47	0.89	0.88	0.01	0.93
Kicker 6	32	0.59	0.41	0.95	0.85	0.90	0.35
Kicker 7	22	0.50	0.50	0.91	0.82	0.36	0.56
Kicker 8	22	0.64	0.36	0.79	0.75	0.03	0.86
Kicker 9	41	0.34	0.66	0.93	0.96	0.22	0.64
Kicker 10	16	0.69	0.31	0.73	1.00	1.64	0.22
Kicker 11	20	0.25	0.75	1.00	0.80	1.12	0.30
Kicker 12	20	0.70	0.30	0.93	1.00	0.42	0.53
All kickers	308	0.54	0.46	0.88	0.90		
Goalkeeper 1	13	0.62	0.38	0.75	1.00	1.41	0.26
Goalkeeper 2	16	0.56	0.44	0.56	0.57	0	0.95
Goalkeeper 3	11	0.55	0.45	0.67	1.00	2.05	0.19
Goalkeeper 4	16	0.75	0.25	1.00	1.00	0	1.00
Goalkeeper 5	11	0.18	0.82	1.00	0.89	0.20	0.66
Goalkeeper 6	10	0.40	0.60	0.50	0.83	1.16	0.31
Goalkeeper 7	10	0.50	0.50	1.00	0.80	1.00	0.35
Goalkeeper 8	13	0.46	0.54	1.00	0.86	0.85	0.38
Goalkeeper 9	11	0.73	0.27	0.88	1.00	0.48	0.51
Goalkeeper 10***	12	0.25	0.75	0.33	1.00	15.00	0.00
All goalkeepers	123	0.51	0.49	0.79	0.88		

Note: *Indicates we reject the null at the 10% significance level, **5% level, ***1% level.

TABLE 5

Test for equality of scoring probabilities for aggregate distributions

	Sum F-Test	Degrees of Freedom	P-value
Panel A			
Test for equality with three actions (L,C,R)			
	Sum F-Test	Degrees of Freedom	P-value
All players***	93.61	35	3.03E-07
All kickers***	56.86	19	1.2E-05
All goalkeepers***	36.75	16	0.002274
Panel B			
Test for equality with six actions (LU,LD,CD,CU,RD,RU)			
	Sum F-Test	Degrees of Freedom	P-value
All players***	128.23	58	3.21E-07
All kickers***	91.48	42	1.56E-05
All goalkeepers***	36.75	16	0.002274
Panel C			
Test for equality with two actions (L,R)			
	Sum F-Test	Degrees of Freedom	P-value
All players*	32.88	22	0.063582
All kickers	10.73	12	0.552188
All goalkeepers**	22.15	10	0.014358

Note: *Indicates we reject the null at the 10% significance level, **5% level, ***1% level.

TABLE 6

Test of serial independence of choices

		Mixture										
Player	#Obs.	L|L	C|L	R|L	L|C	C|C	R|C	L|R	C|R	R|R	F-test	P-values
Kicker 1	36	0.60	0.15	0.25	0.75	0	0.25	0.45	0.45	0.10	0.53	0.7121
Kicker 2	50	0.47	0	0.53	0.60	0	0.40	0.21	0.18	0.61	2.04	0.1041
Kicker 3	28	0.60	0.10	0.30	1.00	0	0	0.83	0.17	0	0.87	0.4945
Kicker 4	20	0.42	0.25	0.33	1.00	0	0	0.60	0.20	0.20	0.66	0.6222
Kicker 5	21	0.40	0.10	0.50	0.67	0	0.33	0.38	0.37	0.25	0.9	0.4862
Kicker 6	33	0.53	0.05	0.42	0	0	1.00	0.69	0	0.31	0.7	0.5959
Kicker 7	22	0.36	0	0.64	0	0	0.00	0.64	0	0.36	1.61	0.2195
Kicker 8	26	0.50	0.07	0.43	0.50	0	0.50	0.62	0.38	0.00	2.35	0.0844
Kicker 9	42	0.20	0.07	0.73	0	0	1.00	0.42	0	0.58	1.04	0.3972
Kicker 10	20	0.42	0.16	0.42	0.75	0.25	0	0.75	0.25	0.00	1.22	0.3378
Kicker 11	20	0.17	0	0.83	0	0	0	0.29	0	0.71	0.29	0.5966
Kicker 12	20	0.77	0	0.23	0	0	0	0.57	0	0.43	0.8	0.3839
Goalkeeper 1	14	0.62	0	0.38	1.00	0	0	0.40	0.20	0.40	0.65	0.6406
Goalkeeper 2	18	0.56	0	0.44	0.50	0	0.50	0.43	0.28	0.29	0.93	0.4709
Goalkeeper 3	12	0.33	0.17	0.50	0	0	1.00	0.80	0	0.20	1.06	0.4276
Goalkeeper 4	18	0.73	0.09	0.18	0.50	0.50	0	0.60	0	0.40	1.27	0.3268
Goalkeeper 5	15	0.50	0	0.50	0	0.60	0.40	0.12	0.13	0.75	1.93	0.1707
Goalkeeper 6	10	0.33	0	0.67	0	0	0	0.43	0	0.57	0.06	0.8067
Goalkeeper 7	10	0.60	0	0.40	0	0	0	0.40	0	0.60	0.33	0.5796
Goalkeeper 8	13	0.33	0	0.67	0	0	0	0.57	0	0.43	0.66	0.4334
Goalkeeper 9	11	0.56	0	0.44	0	0	0	1.00	0	0	2	0.1877
Goalkeeper 10	13	0	0	1.00	0	0	1.00	0.30	0.10	0.60	0.38	0.8149