# PDEs @ Essex 2026

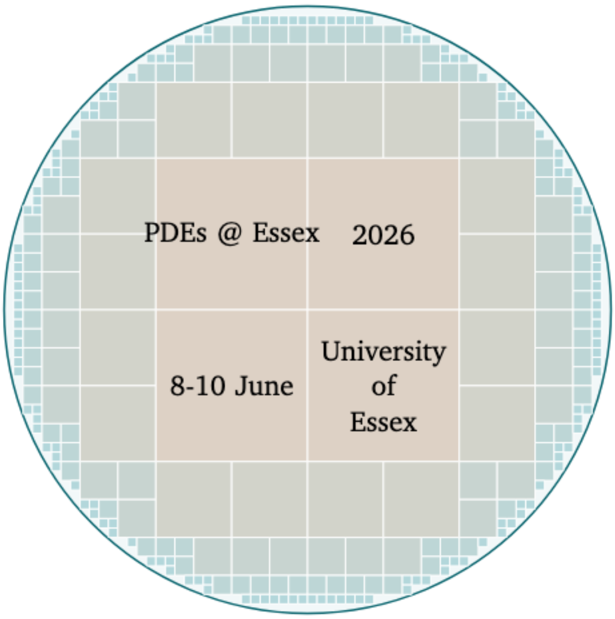
A conference on Analysis of PDEs and Calculus of Variations with Applications

Organizer: Murat Akman - murat.akman@essex.ac.uk

8-10 June 2026

University of Essex, Colchester

https://www.essex.ac.uk/Events/2026/06/08/PDEsatEssex2026

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# Some Practical Information

The talks will be at **CTC.2.01** (Causeway Teaching Centre). `https://maps.app.goo.gl/UedraE3z7wg3zybK7`.

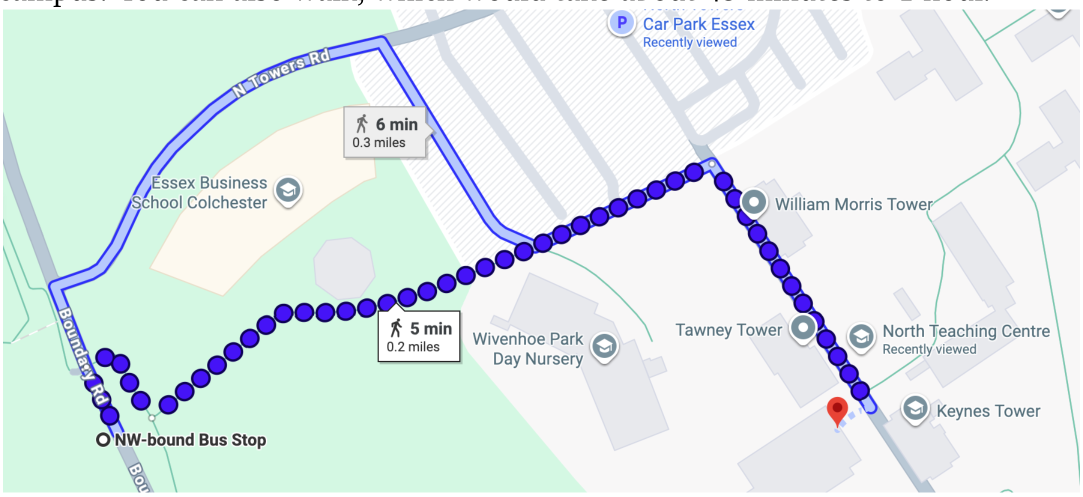
Taxi companies in Colchester (from university to town or train station, it would cost about č12 and you can pay cash, with contactless credit cards, Apple/Google Wallet):

- Five sevens taxi (01206 577777) | Panther Cabs (01206 525525)

They also have Android and Apple Apps that you can use.

If you are taking a taxi from town or train station to the university, **North Towers Car Park at the university** is the closest drop-off point to room CTC.2.01. All taxi drivers should be familiar with North Towers Car Park.

You can take bus 87 (University/Great Horkesley), 51 (Wivenhoe/University), or 74 (Clacton) to come to campus. (The same buses go back to town). The closest bus stop is **Subway** and it should take 3 to 5 minutes to walk to CTC.2.01. You can pay in cash (£2.5) or use your Apple/Google Wallet or contactless card. It may take about 30 minutes to arrive on campus. You can also walk, which would take about 45 minutes to 1 hour.




All shops and cafés on campus are cashless they only accept card payments or mobile payments (Apple Pay, Google Pay, etc.).

You can use eduroam if your home institution supports it. For those without eduroam access, temporary guest Wi-Fi credentials can be obtained through Guest Wifi Network.

The workshop dinner will be at the Wivenhoe House Hotel on the campus on Tuesday (09/06) at 6:30pm. `https://maps.app.goo.gl/DAgAZVnLqGrCKPah8`.

Google Maps: Here are a few places of interest on the Google Maps: `https://maps.app.goo.gl/a21kFbtE4FHFUNWP8`

**If you incur any reimbursable expenses (e.g. travel, meals not provided), please keep your receipts, as these might be required for reimbursement.**

# Conference Schedule

**8 June 2026 (Monday)**, Room: CTC.2.01

- 9:00 - 9:30 Registration
- 9:30 - 10:30 **Zhuolin Li** (Max Planck Institute)
  Title of the talk: Regularity for variational problems with PDE constraints
- 10:30 - 11:00 Coffee break
- 11:00 - 12:00 **Matthew Thorpe** (University of Warwick)
  Title of the talk: Discrete-To-Continuum Limits in Graph-Based Semi-Supervised Learning
- 12:00 - 12:30 **Yunus Cobanoglu** (University of Warwick)
  Title of the talk: Implicit Bias of Deep Linear Networks
- 12:30 - 14:00 Lunch Break
- 14:00 - 15:00 **Georgi Grahovski** (University of Essex)
  Title of the talk: On the $N$-wave hierarchy with constant boundary conditions
- 15:00 - 15:30 Coffee Break
- 15:30 - 16:30 **Vahideh Vahidifar** (University of Sussex)
  Title of the talk: Analytic and Geometric Features of Solutions to Nonlinear Diffusion Equations on Static and Evolving Weighted Manifolds
- 16:30 - 17:00 **Steven Flynn** (University College London)
  Title of the talk: Refined Strichartz estimates for sub-Laplacians on Heisenberg and H-type groups

**9 June 2026 (Tuesday)**, Room: CTC.2.01

- 9:30 - 10:30 **Martin Dindos** (University of Edinburgh)
  Title of the talk: The $L^p$ Neumann problem for parabolic operators with coefficients satisfying a small Carleson condition

- 10:30 - 11:00 Coffee Break

- 11:00 - 12:00 **Joseph Feneuil** (Université Paris-Saclay)
  Title of the talk: Stability of the solvability of the Dirichlet problem under small bi-Lipschitz domain transformations

- 12:00 - 12:30 **idem Susuzlu** (izmir Institute of Technology)
  Title of the talk: Stabilisation of damped and viscoelastic linear wave equations exposed to external Neumann manipulations

- 12:30 - 14:00 Lunch Break

- 14:00 - 15:00 **Phoebe Valentine** (University of Warwick)
  Title of the talk: Characterising 1-rectifiable metric spaces via connected tangent spaces

- 15:00 - 15:30 Coffee Break

- 15:30 - 16:30 **Nikos Katzourakis** (University of Reading)
  Title of the talk: A measure-theoretic generalisation of Danskin's theorem and semi-differentiability of supremal functionals

- 16:30 - 17:00 **Frederick Temple** (University of Reading)
  Title of the talk: On vectorial PDE constrained optimisation problems in $L^\infty$ arising from aquifer modelling

- 17:00 - 17:30 **Simone Carano** (University of Reading)
  Title of the talk: Energy Maximum Principle for Absolute Minimisers

**Workshop Dinner at 18:30 at Wivenhoe House Hotel.**

**10 June 2026 (Wednesday)**, Room: CTC.2.01

- 9:30 - 10:30 **Jinrong Hu** (TU Wien)
  Title of the talk: The $L_p$-Brunn-Minkowski inequalities for variational functionals with $0 \leq p < 1$
- 10:30 - 11:00 Coffee Break
- 11:00 - 12:00 **Ariel Aguas Barreno** (University of Essex)
  Title of the talk: The p-harmonic Brunn-Minkowski inequality
- 12:00 - 12:30 **Oliver Gough** (University of Bath)
  Title of the talk: Blow-up dynamics and stability in quadratic derivative nonlinear wave equations
- 12:30 - 14:00 Lunch Break
- 14:00 - 15:00 **George Papamikos** (University of Essex)
  Title of the talk: Lie symmetry reductions of the p-Laplace equation and invariant solutions
- 15:00 - 15:30 **Yupei Huang** (Imperial College London)
  Title of the talk: Steady States of the 2D Incompressible Euler Equation and Semilinear Elliptic Equations
- 15:30 - 16:00 Closing remarks

# Titles and Abstracts

## The p-Harmonic Brunn-Minkowski Inequality

10 June
11am

Ariel Aguas Barreno
University of Essex

The Brunn-Minkowski inequality is a cornerstone of the Brunn-Minkowski theory. In this talk we will provide a little history surrounding Minkowski problems, from Minkowski, to Aleksandrov, Fenchel-Jessen, Lewis et. al, etc. We will see how the Brunn-Minkowski Inequality connects to these problems and, for $2 < p < n + 1$, we will provide a local p-Harmonic Brunn Minkowski inequality that is related to and a result of the work done by Akman and Mukherjee on the Minkowski Problem for p-Harmonic measures. We will recall the p-Harmonic Minkowski problem and its setting, then provide a decomposition of the p-laplacian into support functions $h_u$ and its derivatives, necessary to our steps of the proof. We will then show a sketch of the proof of the $p$-harmonic Brunn-Minkowski Inequality.

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## Energy Maximum Principle for Absolute Minimisers

9 June
5pm

Simone Carano
University of Reading

We show that the supremand of absolute minimisers for $L^\infty$ variational problems satisfies an appropriate maximum principle. Although this property is only necessary for general absolute minimisers, it characterises a weaker notion of absolute minimality associated with compactly supported variations.

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8 June
12pm

## Implicit Bias of Deep Linear Networks

Yunus Cobanoglu
University of Warwick

Deep linear neural networks are highly overparameterized models, in the sense that they use more parameters than training samples, and are trained by gradient flow. The resulting optimization problem is nonconvex and admits many global minimizers. Despite this overparameterization and nonconvexity, gradient-flow training often converges to solutions which generalize well. This selection phenomenon is known as implicit bias.

This talk studies deterministic gradient-flow selection in a spectrally reducible regime. We show that the selected singular values are described by a depth-dependent Bregman projection onto the set of global minimizers, with potentials ranging from unnormalized entropy at depth two to Burg entropy at infinite depth. This gives an explicit variational characterization of implicit bias, together with convergence rates.

I will also discuss how this variational picture fits into a broader geometric thermodynamic framework. In this viewpoint, the deep linear parametrization induces a natural Riemannian geometry and volume measure on the space of learned matrices, and the deterministic implicit bias can be interpreted as the zero-temperature limit of a corresponding Gibbs-type selection principle.

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9 June
9:30am

## The $L^p$ Neumann problem for parabolic operators with coefficients satisfying a small Carleson condition

Martin Dindos
University of Edinburgh

We resolve the question of whether the Neumann problem for the parabolic equation

$$-\partial_t u + \operatorname{div}(A\nabla u) = 0$$

on a Lipschitz cylinder $\mathcal{O} \times \mathbb{R}$ is solvable in $L^p$ for some $p \in (1, \infty)$ when the matrix $A$ is real, bounded, measurable, uniformly elliptic, possibly non-symmetric, and depends on *both* the space and time variables, while satisfying a natural Carleson measure condition (the parabolic analogue of the elliptic DKP condition).

We prove that for every $1 < p < \infty$ the $L^p$ Neumann problem is solvable provided the Carleson norm of the coefficients and the Lipschitz constant of the domain are both sufficiently small (with dependence on $p$). The result complements our recent work on the parabolic Regularity problem and brings the parabolic theory in alignment with its elliptic counterpart in the small Carleson regime. The question of solvability in the large-Carleson / large-Lipschitz regime remains open — even for elliptic operators it has only been settled in two dimensions.

After describing the result and its place in the landscape of Dirichlet, Regularity and Neumann problems (elliptic and parabolic), I will outline the main ideas of the proof. I will also discuss a surprising result from the appendix: nontangential bounds on $D_t^{1/2}u$ hold for $1 < p \leq 2$ when the spatial base $\mathcal{O}$ is an unbounded graph domain, but *fail* for every $1 < p \leq \infty$ when $\mathcal{O}$ is bounded.

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## Stability of the solvability of the Dirichlet problem under small bi-Lipschitz domain transformations

9 June
11am

Joseph Feneuil
Université Paris-Saclay

We show that small bi-Lipschitz deformations of a Lipschitz domain (with possibly large Lipschitz constant) preserve the solvability of the Dirichlet problem for the Laplacian with boundary data in $L^p$, for the same value of $p > 1$. As a consequence, for all $p > 1$, we obtain the solvability of the $L^p$ Dirichlet problem for $C^1$ perturbations of convex domains, thereby unifying two fundamentally different settings in which such results were previously known: convex and $C^1$domains. This is a joint work with Linhan Li and Jinping Zhuge.

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## Refined Strichartz estimates for sub-Laplacians on Heisenberg and H-type groups

8 June
4:30pm

Steven Flynn
University College London

We obtain refined (non end-point) Strichartz estimates for the sub-Riemannian Schrodinger equation on H-type Carnot groups using Fourier restriction theorems. In particular, we extend the previously known Strichartz estimates obtained for the Heisenberg group also to non radial initial data. Our new argument is based on estimates for the spectral projectors for sub-Laplacians. The same arguments permits to obtain refined Strichartz estimates also for the wave equation on H-type groups. This is a joint work with Davide Barilari.

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## On the $N$-wave hierarchy with constant boundary conditions

8 June
2pm

Georgi Grahovski
University of Essex

In this talk, we will present the direct scattering transform for the $N$-wave resonant interaction equations with non-vanishing boundary conditions. For special choices of the boundary values $Q_{\pm}$, we outline the spectral properties of $L$, the direct scattering transform and construct its fundamental analytic solutions. Then, we generalise Wronskian relations for the case of constant boundary conditions.

Finally, using the Wronskian relations we derive the dispersion laws for the $N$-wave hierarchy and describe the NLEE related to the given Lax operator. The results are illustrated by an example of 4-wave resonant interaction system related to the algebra $sp(4, \mathbb{C})$.

Based on a joint work with Vladimir S. Gerdjikov.

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10 June
12pm

## Blow-up dynamics and stability in quadratic derivative nonlinear wave equations

Oliver Gough
University of Bath

This talk focuses on finite-time blow-up for nonlinear wave equations with quadratic derivative nonlinearities. A key objective is to understand the precise asymptotic structure and mechanism of singularity formation, as well as the stability of these blow-up dynamics under small perturbations to the initial data. I will discuss recent joint work with Manuel del Pino and Monica Musso addressing these questions.

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10 June
9:30am

## The $L_p$-Brunn-Minkowski inequalities for variational functionals with $0 \leq p < 1$

Jinrong Hu
TU Wien

In *Convex Geometry*, volume serves as a fundamental geometric quantity, the classical Brunn-Minkowski inequality for volume lies at the heart of the Brunn-Minkowski theory and has far-reaching applications across geometry and analysis. The $L_p$-Brunn-Minkowski inequality, a cornerstone of the $L_p$-Brunn-Minkowski theory, provides a profound extension of the classical result. The case $p \geq 1$ has been fully established, while the validity of this inequality for $p < 1$ is a major open question. In this context, Böröczky-Lutwak-Yang-Zhang proposed the $L_p$ Brunn-Minkowski inequality conjecture in 2012. To this day, this conjecture remains largely unresolved and stands as one of the most significant open problems.

Parallel to developments in geometric inequalities for volume, analogous $L_p$-Brunn-Minkowski-type inequalities have emerged in the *Calculus of Variations*. In this setting, the central objects are variational functionals, such as the $q$-capacity, the torsional rigidity, and the first eigenvalue of the Laplace operator, which play crucial roles in physics and engineering. In this talk, we will review the background of the $L_p$-Brunn-Minkowski inequalities for variational functionals and present some recent progress in the case $0 \leq p < 1$.

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10 June
3pm

## Steady States of the 2D Incompressible Euler Equation and Semilinear Elliptic Equations

Yupei Huang
Imperial College London

The two-dimensional incompressible Euler equations exhibit complicated global behavior, and a natural approach to understanding the long-time asymptotics of the solutions is to study the structure of steady states. This leads to the question of whether the stream function of a steady state must satisfy a semilinear elliptic equation. We show that, in the analytic category and on non-radial domains, the stream function of every steady state indeed satisfies such an equation. In contrast, for a broad class of simply connected domains, we construct smooth Morse-type steady states whose stream functions fail to satisfy any semilinear elliptic equation.

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## A measure-theoretic generalisation of Danskin's theorem and semi-differentiability of supremal functionals

9 June
3:30pm

Nikos Katzourakis

University of Reading

Let $\Omega \subseteq \mathbb{R}^n$ be open. J. Danskin proved in the 1960s that the envelope $f$ of a family of continuous functions $F : \Omega \times \mathrm{K} \longrightarrow \mathbb{R}$, parameterised by the points of a compact $\mathrm{K} \subseteq \mathbb{R}^m$, given by

$$f(u) := \max_{k \in \mathrm{K}} F(u,k),$$

is semi-differentiable on $\Omega$, under the assumption that $F(\cdot,k)$ is differentiable for all $k \in \mathrm{K}$. This result is of utmost importance in numerous applications, including game theory, economics, finance, stochastic optimisation, and relates to the envelope theorem in comparative statics. Various significant extensions have been established since, but none allows to replace "maximum over K" with "essential supremum over K", for measurable $F(u,\cdot)$. This is not a technicality, as examples show that no direct generalisation can exist. By introducing some new measure-theoretic apparatus, we extend Daskin's theorem to $F$'s defined on the product of a Banach space with a measure space. As a corollary, we obtain a previously unknown *foundational regularity result in the Calculus of Variations in* $L^\infty$, asserting that $L^\infty$ *functionals are semi-differentiable everywhere*, with an explicit formula for the semi-derivative, despite generally being nowhere Gateaux differentiable. This yields a *variational characterisation* of (absolute) minimisers in $L^\infty$ via partial differential inequalities. In the absence of differentiability, the latter serve as *intrinsic* $L^\infty$ *counterparts of the Euler-Lagrange equations* in the case $p = \infty$, bypassing the need for extrinsic $L^p$-approximations as $p \to \infty$. Aronsson-type equations, which ordinarily are only necessary for (absolute) minimality in $L^\infty$, are deducible from the semi-differentials.

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## Regularity for variational problems with PDE constraints

8 June
9:30am

Zhuolin Li

Max Planck Institute

We consider variational problems with PDE constraints, which lie in the $\mathscr{A}$-free framework initiated by Tartar and Murat. While the lower-semicontinuity in this setting has been well studied under the constant rank condition, much less is known regarding regularity. I will first give a short introduction to this type of problems, and then talk about how to establish regularity, including higher integrability and partial continuity, despite the degeneracy inherent to the constant rank condition. This talk is based on joint work with Christopher Irving and Bogdan Raita.

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10 June
2pm

## Lie symmetry reductions of the p-Laplace equation and invariant solutions

George Papamikos
University of Essex

I will present the construction of the complete symmetry group of Lie point transformations for the p-Laplace equation for all real p, including $p = \infty$ (Aronsson equation), and classify the corresponding symmetry generators. This classification leads to a classification of symmetry reductions to ODEs. I will also present exact invariant solutions (p-harmonic functions) which we construct as solutions of the obtained reduced ODEs. I will also explain how to derive a known duality of the p-Laplace equation directly from Noether's theorem. More general symmetries of contact type, and their use, will also be presented. This is joint work with M. Akman.

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9 June
12pm

## Stabilisation of damped and viscoelastic linear wave equations exposed to external Neumann manipulations

idem Susuzlu
izmir Institute of Technology

In this talk, we consider the stabilization of damped and viscoelastic linear wave equations on a bounded domain, subject to an inhomogeneous Neumann manipulation on a portion of the domains boundary. Compared with the homogeneous case, these models present additional challenges, as the rate at which the energy of the solutions changes depends directly on the boundary trace of the temporal derivative. Due to regularity issues, it is not clear a priori how this quantity should be controlled in terms of the given data. However, we prove uniform stabilization of solutions, with decay rates determined by the Neumann input. We supplement these results with numerical simulations under the stated assumptions, and also present additional simulations for data that do not necessarily satisfy the required assumptions for decay; the latter offer essential physical insights into how energy might change in the presence of improper boundary data.

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9 June
4:30pm

## On vectorial PDE constrained optimisation problems in $L^\infty$ arising from aquifer modelling

Frederick Temple
University of Reading

We study the minimisation of a cost functional in $L^p$ and $L^\infty$ where the admissible class is constrained by a second-order linear elliptic PDE. We demonstrate the existence of constrained $L^p$ minimisers for all $p > n$. Furthermore, as $p \to \infty$, these $L^p$ minimisers tend to $L^\infty$ minimisers. We also utilise a non-smooth Karush-Kuhn-Tucker theory to obtain variational inequalities in $L^p$ and $L^\infty$. Our aim is to use these methods to provide a regularisation strategy to the ill-posed inverse problem of identifying the elasticity tensor, given a source and some measurements/observations.

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## Discrete-To-Continuum Limits in Graph-Based Semi-Supervised Learning

8 June
11am

Matthew Thorpe
University of Warwick

Semi-supervised learning (SSL) is the problem of finding missing labels from a partially labelled data set. The heuristic one uses is that similar feature vectors should have similar labels. The notion of similarity between feature vectors explored in this talk comes from a graph-based geometry where an edge is placed between feature vectors that are closer than some connectivity radius. A natural variational solution to the SSL is to minimise a Dirichlet energy built from the graph topology. And a natural question is to ask what happens as the number of feature vectors goes to infinity? In this talk I will give results on the asymptotics of graph-based SSL using an optimal transport topology. The results will include a lower bound on the number of labels needed for consistency and, time permitting, some recent extensions to infinite dimensional settings.

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## Characterising 1-rectifiable metric spaces via connected tangent spaces

9 June
2pm

Phoebe Valentine
University of Warwick

Characterising rectifiability using the existence of tangents is a very classical technique in GMT and positive results can be found even when using very weak notions of a tangent. This work builds on geometric ideas from Besicovitch in the plane to prove that 1-rectifiability in complete metric spaces is implied by the existence of connected tangents almost everywhere. This requires exploitation of the inherent gappiness of purely 1-unrectifiable sets which must first be quantified. The result is stated for metric spaces but is in fact also new in Euclidean space.

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## Some Analytic and Geometric Features of Solutions to Nonlinear Diffusion Equations on Static and Evolving Weighted Manifolds

8 June
3:30pm

Vahideh Vahidifar
University of Sussex

In this talk, I will present several analytic and geometric properties of solutions to nonlinear diffusion equations on weighted manifolds. I establish two types of gradient estimates, HamiltonSoupletZhang and LiYau estimates, for positive solutions. The diffusion operator considered here is the Witten (weighted) Laplacian, and the analysis is carried out in the general setting where both the Riemannian metric and the weighted measure (potential function) evolve in time. These estimates are obtained under various curvature assumptions on the BakryÉmery Ricci tensor and are subsequently applied to derive several important consequences, including Harnack inequalities, spectral bounds, sharp logarithmic Sobolev inequalities (LSI), and Liouville-type global constancy results. I will also discuss applications to entropy dissipation inequalities, before moving to the setting of discrete spaces and graphs, and finally to applications in functional inequalities and geometric deep learning.

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# Author Index