Department of Mathematical Sciences, University of Essex

Research Group: Mathematics

PhD scholarship

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PhD Projects 2019-2022

(27 July 2019)

Supervisor: Dr Murat Akman, Lecturer in Mathematics Topic: Minowski problem and Brunn-Minowski type inequalities for nonlinear capacities Email: <u>murat.akman@essex.ac.uk</u>

Supervisor: Dr Chris Antonopoulos, Lecturer in Applied Mathematics Topic: Development of mathematical approaches to study synchronisation phenomena and functional connectivity in the brain Email: <u>canton@essex.ac.uk</u>

Supervisor: Dr Jessica Claridge, Lecturer in Mathematics Topic: to be announced Email: jessica.claridge@essex.ac.uk

Supervisor: Professor Edd Codling, Professor of Mathematical Biology Topic: to be announced Email: <u>ecodling@essex.ac.uk</u>

Supervisor: Dr Jesus Martinez Garcia, Lecturer in Mathematics Topic: Moduli spaces of geometric objects via computational algebra Email: <u>jm19279@essex.ac.uk</u>

Supervisor: Dr Georgi Grahovski, Senior Lecturer in Applied Mathematics

Topic: to be announced **Email:** grah@essex.ac.uk

Supervisor: Professor Peter Higgins, Professor of Pure Mathematics Topic: Permutation matchings in regular semigroups / Classes of algebraes defined by existence of solutions of equations Email: peteh@essex.ac.uk

Supervisor: Dr Alastair Litterick, Lecturer in Mathematics Topic: Subgroup structure of reductive algebraic groups Email: tba (Dr Litterick joins the department 1 September 2019)

Supervisor: Dr George Papamikos, Lecturer in MathematicsTopic: to be announcedEmail: tba (Dr Papamikos joins the department 1 October 2019)

Supervisor: Dr David Penman, Senior Lecturer in Pure Mathematics Topic: to be announced Email: <u>dbpenman@essex.ac.uk</u>

Supervisor: Professor Chris Saker, Professor of Mathematics Education Topic: to be announced Email: cjsake@essex.ac.uk

Supervisor: Professor Hadi Susanto, Professor of Applied Mathematics Topic: to be announced Email: <u>hsusanto@essex.ac.uk</u>

Supervisor: Dr Alexei Vernitski, Senior Lecturer in Mathematics Topic: to be announced Email: <u>asvern@essex.ac.uk</u>

Supervisor: Professor Gerald Williams, Professor of Algebra Topic: Exploring knots, manifolds, and geometric structures using cyclically presented groups Email: gerald.williams@essex.ac.uk Minowski problem and Brunn-Minowski type inequalities for nonlinear capacities

Supervisor: Dr Murat Akman, Lecturer in Mathematics Email: <u>murat.akman@essex.ac.uk</u>

The problems focuses on two classical potential-theoretic problems in convex geometry; inequalities of Brunn-Monkowski type and Minkowski type problems associated to quasi-linear elliptic PDEs modelled on p-Laplace equation.

Classical Brunn-Minkowski inequality relates the volume or more generally Lebesgue measures of compact subsets of Euclidean space. Although this inequality is known for more than a century, its stability has been studies very recently by M Christ, A. Figalli, and D. Jerison. One fundamental problem in this area is the potential theoretic version of this inequality and its stability.

The classical Minkowski problem asks for necessary and sufficient conditions on a non negative Borel measure on the unit sphere to be the surface area measure of a convex body. One key goal is to study this problem which consists of existence, uniqueness and regularity from the potential- theoretic point of view. The regularity of this problem which requires further work on the regularity of solutions to a system of differential equations involving Monge-Ampere equation.

Development of mathematical approaches to study synchronisation phenomena and functional connectivity in the brain

Supervisor: Dr Chris Antonopoulos, Lecturer in Applied Mathematics Email: <u>canton@essex.ac.uk</u>

Many studies have tried so far to decode neurophysiological data using information from temporal representations. Neural synchronisation plays a crucial role in cognitive functions and in performing tasks as it facilitates the transmission of information among brain regions, and thus their communication. This allows regions to coordinate their actions to perform cognitive functions and tasks. In this project, we will develop further mathematical approaches used in the study of artificial data to analyse the emergence of synchronisation patterns in neurophysiological signals from the brain (e.g. EEG-fMRI data from healthy individuals and patients with brain disorders). The PhD student will study a) emergent synchronisation phenomena in artificial and recorded data from the brain and b) develop mathematical methods to infer functional connectivity, using statistical and information theory tools. The benefit of the studentship originates from the need of integrating the work in a) and b) into a solid piece of work. The impact of the project stems from the combined use of mathematical, network theory and connectivity analysis techniques to study electrophysiological data. The collaboration with neurologist, MD D. San-Juan Orta at INNN (Mexico City) is instrumental in improving health quality internationally and will provide us with expertise knowledge.

Moduli spaces of geometric objects via computational algebra

Supervisor: Dr Jesus Martinez Garcia, Lecturer in Mathematics Email: jm19279@essex.ac.uk

Most objects in geometry (including algebraic varieties, metrics, vector bundles, or maps) can be deformed continuously. In order to study how perturbations of the object affect its properties, it is customary to construct a moduli space. This is a classifying space whose points represent each of the objects in the deformation. Moduli spaces were first considered by Hilbert in the 1890s and they became a driving force in geometry in recent years, facilitating solutions to many conjectures. While many moduli spaces are constructed abstractly, lack of computational techniques make it difficult to describe them. The main tool for constructing moduli spaces is Geometric Invariant Theory (GIT). Developed in the 1960s, GIT translates the construction of a moduli space into computational algebra. However, not much work has been done into automatizing the problem.

The student will construct moduli spaces via GIT in many different situations by developing algorithms in computational algebra and implementing them in software. As a result, it will be possible to describe many different moduli spaces and their deformations. The student will learn not only a great deal of algebraic geometry, but also transferable skills including symbolic computing, parallel programming, mathematical writing and presenting their work in public.

Permutation matchings in regular semigroups - Classes of algebraes defined by existence of solutions of equations

Supervisor: Professor Peter Higgins, Professor of Pure Mathematics Email: <u>peteh@essex.ac.uk</u>

Both topics require an undergraduate background in abstract algebra and a first course in algebraic semigroups. The second requires knowledge of universal algebra and model theory. The student might consider first completing a 12 months Masters by Research with the same supervisor.

(1) The background is contained in three papers of the supervisor:

https://arxiv.org/abs/1901.05198

https://arxiv.org/abs/1812.03743

https://arxiv.org/abs/1811.04472

However, a fundamental question remains, which is does every regular semigroup with a permutation matching have an involution matching? This topic also has scope for computational exploration using GAP or some other suitable software.

(2) This is a new and exciting topic as shown in the paper by Higgins and Jackson <u>https://arxiv.org/abs/1810.13012</u>

We have proved that elementary classes of algebras closed under the taking of homomorphic images and direct products are exactly those which have a basis of equations over the algebra. This is particularly fruitful in semigroup theory where so many of these classes are not varieties. In the regular case in particular we have an alternative to the theory of E-varieties as our approach uses only the natural binary operation of the semigroup class. This topic is still in its infancy and fundamental questions remain unanswered. For example, at the time of writing there is no example of an equation class as we call them known to require more than three alternations of existential quantifier in its definition whereas the theory suggests that any number is possible.

Subgroup structure of reductive algebraic groups

Supervisor: Dr Alastair Litterick, Lecturer in Mathematics Email: tba (Dr Litterick joins the department 1 September 2019)

Reductive algebraic groups are a family of algebraic objects which occur throughout mathematics and theoretical physics. They include important matrix groups such as general linear groups, orthogonal groups and symplectic groups over algebraically closed fields. Because of their fundamental importance in group theory, there has been an ongoing effort for many decades to understand their subgroup structure. If the underlying field is the complex numbers then this is essentially well-understood, but the theory is much more subtle over fields of positive characteristic.

This project encompasses several open problems in group theory, including both classification problems and more conceptual problems such as transferring results from algebraic groups to the so-called finite groups of Lie type. Depending on the student, a combination of group theory, algebraic geometry, group cohomology, representation theory and computational algebra could be brought to bear on these problems.

A researcher working on this project would need good knowledge of group theory, and preferably some basic knowledge of commutative algebra (rings and algebras over fields). Familiarity with algebraic geometry or computational algebra (e.g. GAP, MAGMA, SAGE) would also be helpful, although this is not essential.

Exploring knots, manifolds, and geometric structures using cyclically presented groups

Supervisor: Professor Gerald Williams, Professor of Algebra Email: gerald.williams@essex.ac.uk

Knots (and knotted structures) can be distinguished from one another by number theoretic invariants such as the Jones or Alexander polynomials, or by algebraic invariants such as the fundamental group (or knot group). It is known that torus knot groups have descriptions (specifically, group presentations) that admit a cyclic symmetry, in the sense that they are examples of cyclically presented groups. Certain 3-dimensional manifolds that are branched covers of knots also have such a description. Not all cyclically presented groups correspond to knots in these ways, however.

This project is to investigate which cyclically presented groups can be knot groups (or correspond to related knotted structures) and to provide classification results within certain families of cyclically presented groups. The mathematics involved in this research will centre around combinatorial group theory and use techniques from topology, matrix theory, and number theory, and is suitable for applicants with background in group theory and one or more of the other areas.