Dynamics in the UK Labour Market: Job Finding Probabilities and Mismatch*

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Abstract

The first part of this paper analyses (i) cyclical properties of job finding probabilities and (ii) presents results, regarding the effect of educational heterogeneity among workers on the average job finding probabilities and on their cyclical properties. In the second part of the paper, we analyse the concept of educational mismatch. In particular, we analyse (ii) how personal characteristics of workers affect the incidences of over- and under-education and (iii) how the proportions of over- and under-educated workers change with the level of GDP (cyclicality). (iv) The last part presents a mathematical framework that analyses and explains the fluctuations in the proportion of educationally mismatched employees, in terms of the labour market flow rates.

Key Words: Labour Market Flows, Job Finding Probability, Over-education, Under-education

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1 Introduction

In this paper, we analyze two important concepts of labour markets: job finding probabilities and educational mismatch *(over- and under-education)* in the UK labour market, for the period of 1996-2015. Although the concepts may seem to be unrelated to each other, the analysis, carried out in this paper, presents an insight into specific issues in each of the topics and, as will be shown, we also attempt to link and analyze the two concepts together. More precisely, the topics and issues that we analyze include the cyclical properties of job finding probabilities and of the proportions of educationally mismatched workers; the effects of personal characteristics of individuals on reemployment probabilities and on the probabilities of becoming educationally mismatched; and, uniquely in this paper, we link the concepts of job finding rates and educational mismatch and develop a framework that explains the dynamics in the proportion of mismatched employees, in terms of the fluctuations in labour market flow rates[1].

Analyzing labour market flow rates and probabilities is crucial for explaining the movements in unemployment rate and for understanding what derives unemployment to rise during recessions and to decrease during expansions. In the literature on worker flows, the cyclicality of flow rates and probabilities is defined as the correlation of the rates/probabilities with the level of economic activity, which can be approximated as the level of GDP or unemployment rate. The ”conventional wisdom” in the field, regarding the cyclical properties of labour market flow rates/probabilities, proposes that the important factor to explain the fluctuations in unemployment rate is the job separation rate, as it follows a clear counter-cyclical pattern, whereas the job finding probabilities/rates follow an acyclic movement. However, the recent findings in the literature, which are mentioned in the next section, seem to contradict to this conventional wisdom, by presenting evidence in favour of procyclicality of job finding rates/probabilities and

[1]As in the following analysis we do not consider the re-entrants or new entrants to the labour force, the terms ”job finding probability” and ”reemployment probability” will be used interchangeably, as representing the same concepts.

their importance in explaining fluctuations in unemployment rate. These recent findings lead to a debate in the field and many papers attempted to analyze cyclical properties of labour market flow rates and probabilities.

Thus, given the controversial evidence, we attempt to contribute to the debate in the field, by analyzing the cyclical properties of job finding probabilities. In addition to this, we, also, analyze how job finding probabilities and their cyclical properties change, depending on educational attainments of workers. The main findings indicate that, in line with the recent literature, job finding probabilities follow a procyclical movement pattern, with the procyclicality being preserved for every analyzed education group. Moreover, the analysis of the average job finding probabilities shows a possible concave relationship between reemployment probabilities and attained education levels of workers: the average job finding probabilities rise with higher levels of schooling, but shows a declining trend at very high levels of education. Using the findings in the literature, we attempt to explain why it may happen. As an additional exercise, we also analyse the behaviour of job finding probabilities, during the period of global financial crisis, and we show that starting from the lowest level of education (GCSE), the extent of the drop in job finding probabilities, during the crisis, declines, as the education level goes up.

Regarding the educational mismatch, a worker is defined as over-educated, if his/her attained education level is higher than the one, required by the job that a worker is assigned to (the converse definition holds for an under-educated worker). The analysis of the UK labour force shows that, the share of working age population, with high education (University graduates and above), increased from 20% in 1996 to 38% in 2013 (Office for National Statistics (2013)). Given this rapid increase in the percentage of individuals with high education, for the same period, the proportion of over-educated individuals faced an increase of, around, 2 percentage points, as well. However, the time series analysis of the proportions of under-educated workers show that, on average, it has been, approximately, constant until the period of global financial crisis and recovery, during which the proportion of under educated individuals rose rapidly. In general, in the literature for different periods and countries, it is found that, around, 10-20 % of workers are over- and under-educated for their jobs.

Given the facts above, for over-educated employees, they imply that, around, 10-20% of workers carry out the jobs that require the schooling levels below than the workers' attained ones. In terms of skill/productivity utilisation, it may imply that over-educated employees under-utilising their skills/productivities, which may indicate an economic inefficiency. Regarding the under-education case, the fact that firms employ workers that have insufficient levels of education for the job proposes an interesting puzzle as well. Various theoretical explanations were developed to explain these phenomenons, which we will mention and analyse below.

Although, since 1970s, the concepts of under- and over-education were extensively analyzed, the main focus of research papers was on the earning outcomes of being educationally mismatched. However, only a few papers attempted to analyze the implications of personal characteristics of workers on the incidences of over- and under-education and to link those implications to theoretical explanations of educational mismatch, in order to attempt to understand the above mentioned phenomenons.

Given this shortcoming in the literature, the first analysis that we will carry out, regarding educational mismatch, is to analyse the relationship between various labour market related personal characteristics of individuals, especially work experience, and the probabilities of becoming over- and under-educated. The findings present an important insight into the theoretical explanations of educational mismatch, especially, into the relationship between the worker productivities and incidences of over- and under-education. As it will be discussed below, the results also have implications on the earnings-experience relation, proposed by the human capital theory.

Moreover, to our knowledge, this paper is the first one in the field that attempts to analyze the business cycle movements of the proportions of over- and under-educated workers, out of all employed ones. The findings indicate an acyclic over-education proportion and counter-cyclical under-education proportion . We present an elegant explanation for a possible reason of counter-cyclicality of under-education proportion, which, also, explains the rapid increase in the proportion of under-educated workers, during the period of global financial crisis and recovery.

Finally, in the last section, we link the concepts of labour market flows and educational mismatch together, and, also for the first time in this paper, we present a mathematical framework that analyses dynamics in the proportion of mismatched employees in terms of the fluctuations in job finding and job separation rates. If applied to the data, the framework shows the contributions of the fluctuations in labour market flow rates on the variation in mismatch proportion and so explains what derives the observed business cycle movements in the proportion of mismatched employees.

The paper is organised as follows: the third section focuses on the concept of job finding probabilities; the fourth section analyses educational mismatch and, finally, the fifth section present the mathematical framework that analyses mismatch proportion, in terms of labour market flow rates. In the sections three and four, we, first, present the review of literature, then, the analytical framework and the results. Conclusion can be found in section six, followed by the appendix of results/proofs and references.

2 Data

For the purposes of the analysis, carried out in the paper, we will make the use of the Quarterly Labour Force Survey (LFS henceworth) data for the United Kingdom for the period of 1996Q2-2015Q3, which can be accessed from the UK Data Service website[2]. It is a random

[2]https://discover.ukdataservice.ac.uk/series/?sn=2000026

sample design that includes, around, 60,000 households, with each household retaining in the sample for five consecutive quarters. The sample is split into five waves, and each quarter, one wave, which is the fifth of the sample (around 12,000 households), is replaced with the new one (Labour Force Survey, 2011). When the interviewees respond to survey questions for the first time, they are interviewed face-to-face; but, the later interviews are carried out on telephone. Quarterly LFS questionaries include various questions about inividuals' personal characteristics, labour market status, job details, earnings and other elements. The latest list of variables, with descriptions, can be found in volume 3 of the LFS user guide for 2015 (Labour Force Survey (2015)).

Labour Force Survey (2011) highlights that the major problem of the dataset is the non-response bias, with the response-error bias being another significant problem of it. Regarding the non-response bias, Gomes (2012) highlights that the non-contact rate in LFS is, around 5% and the rate of refusal to take part in the survey is 10-15%. In general, it should be noted that there are no practically accepted methods of dealing with those shortcomings. The extensive discussion of the methodology and shortcomings of LFS can be found in Clarke and Tate (2000).

In the following analysis, we will use 78 occupation groups. After dropping irrelevant observations (e.g. aged less than 16, not in the labour force etc.), there are, approximately, $20,000$ observations left in each quarter, with $5,000-6,000$ of which are unemployed workers and the rest are the employed ones[3]. After merging all quarters together, there are, around, 1,7 million observations in the dataset.

Practically, we faced several problems with LFS data. First, regardless of having data available since 1993, we could use it only since 1996Q2, for the reason that there were major inconsistencies in education group variables between 1993-95 and all other quarters' datasets. In addition, some of the important variables did not exist in some quarters: for instance, in

[3]We drop individuals that are not in the labour force, as in the following chapters we do not consider the flows out of the labour force.

2004Q1 there was no variable that identified education level of a respondent and in 2001Q1 the occupation variable disappeared. Because of these, we were not able to estimate job finding probabilities, for education groups, for two quarters in 2004 (Q1 and Q2), and we were not able to obtain any estimates, regarding the educational mismatch, in 2001Q1 and 2004Q1.

3 Job Finding Probabilities

In this section of the paper, we will carry out the analysis of probabilities of finding a job for unemployed workers, in the United Kingdom, for the period of 1996-2015. More precisely, as the first step, we will attempt to analyze the movements of job finding probabilities at the business cycle frequencies. In addition, we will address the question of how job finding probabilities and their cyclical properties vary, depending on educational attainments of individuals. In particular, we will examine how reemployment probabilities change with every higher levels of education and whether the cyclical properties remain the same in every analysed schooling group. As an additional exercise, we will also analyse the behaviour of job finding probabilities during the period of global financial crisis.

3.1 Related Literature

Labour Market Flows and Business Cycles:

The literature on the labour market flows, mainly, attempts to study the implications of job finding and separation rates/probabilities on the business cycle. In particular, papers in the literature aim to understand what causes the unemployment rate to rise during recessions and to decline during expansions: do we observe the increase during recessions because more individuals get separated from their jobs, or because less individuals are able to find jobs or both (converse for expansions)? The conventional wisdom on this question proposes that the job separations are more important to explain the unemployment fluctuations than the job findings, because the job separation rates move clearly opposite to the business cycle (counter-cyclical) and they are more fluctuative, whereas the job finding rates are less fluctuative and do not possess clear business cycle movements (acyclic)[4]. However, recently, this conventional wisdom was challenged by the results of Shimer (2012) and Hall (2005). Using US CPS monthly data, both Shimer (2012) and Hall (2005) showed that the job finding probabilities/rates are procyclical, whereas the job separation probabilities/rates are acyclic. Moreover, Shimer (2012) showed that, approximately, 90% of the fluctuations in unemployment rate are due to the movements in job finding probabilities and the results are not driven by the compositional changes in the labour force. The findings of Shimer (2012) and Hall (2005) lead to a debate in the field and, recently, many other researchers attempted to analyse the same issue, using data-sets for different countries and employing different estimation approaches.

As in this paper we do not particularly focus on the explanation of the fluctuations in unemployment rate, in the following discussion we will present the results of the papers, related to the cyclical properties of job finding probabilities/rates only. However, as a summary, it can be said that, in contrast to Shimer (2012), other recent papers found that, both, job finding and job separation rates/probabilities contribute to a similar extent to the fluctuations in unemployment rate (notice that the results do not support the conventional wisdom either). For the discussions and findings about the contributions of labour market flow rates/probabilities to the variation in unemployment, please, see the below cited papers. An additional discussion can, also, be found in Petrongolo and Pissarides (2008).

It must be noted that most of the papers in the literature analyze the labour market flow rates, rather than probabilities. However, as the job finding and job separation procedures are, naturally, assumed to follow a Markov jump process *(e.g. Poisson process)*, it must be the case that the probabilities follow the same pattern as the rates. Thus, the presented results of the

[4]Based on Blanchard and Diamond (1989, 1990), Davis and Haltiwanger (1992) etc.

papers, regarding the behaviours of fob finding rates can be applied for the case of job finding probabilities as well.

To our knowledge, in contrast to the conventional wisdom, in the cases of all recent papers, job finding rates/probabilities are found to be procyclical. For instance, Elsby *et al.* (2013) follows Shimer (2012)'s framework to estimate the labour market flow rates and applies it using the data from 14 OECD countries. In addition to all other findings, in the cases of all countries, Elsby *et al.* (2013) finds procyclical job finding rates.

Additionally, Fujita and Ramey (2009) develops another one of the commonly followed *(two-state labour market)* frameworks for estimating the labour market flow rates and for the analysis of contributions of those rates to fluctuations in unemployment. Using US CPS monthly data, similarly to Shimer (2012) and Elsby *et al.* (2013), it finds an evidence that job finding rates are procyclical. Fujita and Ramey (2009) analyses cyclicality by considering the correlation between the job finding rates and labour productivity (approximated as GDP divided by the number of employed individuals in the sample). Furthermore, another important paper in the filed, Yashiv (2007), uses US CPS data and develops a three-state labour market model. The paper uses the correlation of job finding rates with the level of GDP as an indicator of the cyclicality and finds that job finding rates are pro-cyclical.

Finally, Gomes (2012) develops a three-state labour market model, and for its analysis, it uses the UK LFS data as well (for the period of 1993-2010). In addition, Gomes (2012) employs the same methodology for analysing the cyclical properties of labour market transition probabilities as we do in this paper; namely, it uses the methodology for the cyclicality analysis, proposed in Baker (1992)[5]. As a result, in line with above papers, Gomes (2012) finds an evidence in favour of procyclical job finding probabilities.

Job Finding Probabilities and Educational Attainment:

[5]We will present the analytical details of this method in the next sub-section.

In addition to the general analysis of labour market flow probabilities, Gomes (2012) also presents the analysis of job finding probabilities, conditional on educational attainments of individuals. Having separated workers into three education groups, which are higher education (Bachelor's plus), A-level and GSCE or equivalent, Gomes (2012) shows that job finding probabilities in the highest education group are twice as high as in the lowest education group. Moreover, it shows that reemployment probabilities are procyclical in every analyzed schooling group, and the extents of cyclical fluctuations decrease, as the education level goes up.

Another related paper that presents important results, regarding reemployment probabilities, conditional on educational attainment, is Kettunen (1997). Using 1985 data on Finnish unemployed workers, Kettunen (1997) employs the hazard function approach, in particular Weibull model, to estimate reemployment probabilities. The paper shows that starting from less than nine years up to 13-14 years of schooling, job finding probabilities rise with every additional year of education. However, starting from Bachelor's degree holders, reemployment probabilities begin to decline and unemployed workers with Master's and PhD levels of education have the lowest job finding probabilities. The possible theoretical explanation of this phenomenon can be found in the book of the same author: in Chapter 6 of Kettunen (1993). Using a search theoretical model, Kettunen (1993) shows that the direct effect of higher education on reemployment success, through the increased job arrival rates, is positive. However, as higher education increases individuals' reservation wages (i.e. individuals become more selective), the indirect effect of higher education on reemployment probabilities is negative. Thus, at very high levels of schooling, the individuals' reservation wages may become very high (i.e. individuals may become very selective), such that the indirect effect dominates and job finding probabilities decline. The findings of Kettunen (1997) cannot directly be compared to the ones of Gomes (2012), as the latter paper does not analyze Master's and PhD levels of education, separately; instead, it groups them together.

One of the last two papers that presents relevant findings for this section is Riddell and Song (2011); the paper uses the US CPS data and estimates the effect of high school graduation and the further education on reemployment probabilities. In contrast to Kettunen (1997), the findings of Riddell and Song (2011) show that job finding probabilities increase for every level of education: the high school graduation increases reemployment probabilities by, around, 20 percentage points and every further year of schooling rises job finding probabilities by, around, 3 percentage points. Finally, Russell and O'Connell (2001) makes the use of the discrete time hazard function approach to estimate job finding probabilities of individuals, aged under 25, in nine European countries. Similar to Riddell and Song (2011), Russell and O'Connell (2001) shows that every higher year of schooling affects reemployment probabilities positively.

3.2 Framework

The paper will follow the methodology, popularised in Shimer (2012), for the purposes of estimation of job finding probabilities. There are three main reasons for choosing Shimer (2012)'s approach. First, for its correction of time aggregation bias, which will we discuss below, in details; second, differently from many other papers, it estimates job finding probabilities, not rates (rates can also be estimated, separately, as shown below); finally, in practise, it is, relatively, straightforward to implement.

We will focus on the two-state labour market, where the flows out of the state of unemployment can be made only into the state of employment and vice-versa (we will not consider the flows in/out of the labour force). Following Shimer (2012)'s notation, let $t \in \{0, 1, 2, \cdots\}$ be the time index set and for every t, let $[t, t+1)$ be defined as a period t. Practically, $t \in \{0, 1, 2, \cdots\}$ denotes a data measurement date and a period t is a period between the two data measurement points; as the Quarterly LFS data is measured every three months, for our purposes, a period t is a 3 months period. Define $F_t \in [0, 1]$ as a job finding probability, during a period t; in particular, F_t is the probability that a worker, starting period t unemployed, finds at least one job during $[t, t+1)$. In addition, define $X_t \in [0,1]$ as an employment exit probability.

Further, for some time $\tau \in [0,1]$, elapsed after a data measurement date t, let $e_{t+\tau}$ and $u_{t+\tau}$ denote the numbers of employed and unemployed individuals at time $t+\tau$, respectively (note that if $\tau = 1$, $t+\tau$ implies the following data measurement date). Finally, let $u^s_{t+\tau}$ be the number of workers, who are unemployed at time $t+\tau$, but were employed at some time $t' \in [t, t+\tau]$ (Shimer (2012) calls $u^s_{t+\tau}$ as a "short term unemployment") . More precisely, if $\tau = 1$, u^s_{t+1} denotes the number of workers, who are unemployed at data measurement date $t+1$, but were employed at some time $t' \in [t, t+1]$; i.e. it captures the number of workers, who became unemployed between the two data measurement dates. In addition, as, in our case, t is a three months period, u^s_{t+1} can also be said to denote the number of workers, who are unemployed for less than three months[6].

Having defined the above notation, Shimer (2012) assumes that, during a period t, the job finding process of unemployed individuals follows a Poisson process with arrival rate $f_t \equiv -\log(1-F_t)$ (job finding rate) and the job separation process of employed workers follows Poisson with arrival rate $\chi_t \equiv -\log(1-X_t)$ (job separation rate) (note that as $F_t \in [0,1]$ and $X_t \in [0,1]$, $f_t \geq 0$ and $\chi_t \geq 0$). In addition to these, Shimer (2012) proposes that unemployment and short term unemployment evolve over time, according to the following differential equations:

$$\dot{u}_{t+\tau} = e_{t+\tau}\chi_t - u_{t+\tau}f_t \tag{1}$$

$$\dot{u}^s_{t+\tau} = e_{t+\tau}\chi_t - u^s_{t+\tau}f_t \tag{2}$$

Eliminating $e_{t+\tau}\chi_t$ from both equations and solving them for $\tau = 1$ yields to the following

[6] In Quarterly LFS, unemployed workers are asked about their durations of unemployment; thus, we will identify u^s_{t+1} as the number of all unemployed workers, who report the duration of less than three months, in a given quarter.

expression for the job finding probability, during a period t, according to which we will estimate it:

$$F_t = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t} \tag{3}$$

At this point it is important to mention the correction for time aggregation bias, implemented in Shimer (2012). The time aggregation bias arises due to the fact that, usually, data on worker flows are available at some discrete time frequencies, but, the flows in and out of the labour market states happen at continuous time scale. Thus, the flows that take place between the data measurement dates are not observed, which may impose bias on the estimated flow probabilities/rates. On the example of Quarterly LFS, as data is collected every three months, we do not observe the transitions that workers may make between the states of unemployment and employment, during the three months that data is not collected. Nordmeier (2014) presents the analysis of the problem of time aggregation, using German administrative data; according to that paper, if not corrected, the time aggregation problem imposes a downward bias on the estimated flow rates/probabilities, because, the flows between the states are not fully captured[7]. From the above definitions, recall that u^s_{t+1} denotes the number of workers, who became unemployed between the two data measurement dates; that is it captures the flow from the state of employment into unemployment, during the period that data is not collected. Thus, by introducing u^s_{t+1}, Shimer (2012) attempts to correct for the time aggregation bias.

However, note that in the case of Quarterly LFS data, u^s_{t+1} is able to capture only one unobserved transition for every observation. The reason is the following. Recall that we use the reported duration of unemployment, in order to identify workers that are in u^s_{t+1}. Given this, note that even if a worker has a single unobserved transition or multiple ones (i.e. gets out of employment then in and out again), he/she would report the duration less than three months, so

[7]Some further discussion regarding the effects of time aggregation bias on the cyclical properties of employment exit probability can be found in Shimer (2012).

that we would not be able to observe how many transitions a worker made and would count it as a single transition. Therefore if, during the period that data is not collected, an individual makes multiple transitions between the states, in our Quarterly LFS data, only one of those unobserved transitions will be captured. Moreover, as during the three months period such multiple unobserved transitions are more likely to exist, following Nordmeier (2014), we may expect our results in the next sub-section to be downward biased.

Note that the definition of job finding probabilities, defined as in (3), does not capture any dimensions of individual heterogeneity among unemployed indiviuals. In fact, Shimer (2012) proves that the definition in (3) is the average job finding probability among individuals, during a period t. In order to capture educational heterogeneity, we introduce the education group index set $\mathbb{K}$; in addition, we modify the definition in (3) and define the job finding probability of an individual in an education group $\kappa \in \mathbb{K}$, during a period t as:

$$F_{t,\kappa} = 1 - \frac{u_{t+1,\kappa} - u^s_{t+1,\kappa}}{u_{t,\kappa}} \tag{4}$$

where $u_{t,\kappa}$ is the number of unemployed individuals in an education group $\kappa \in \mathbb{K}$, at time t; similar interpretations hold for other components.

Practically, we segment individuals in our data-set into education groups and estimate job finding probabilities for every education group, separately. However, as can be noted, other than the educational heterogeneity, expression (4) does not capture any other dimensions of individual heterogeneity among unemployed workers. Moreover, as in the case of expression (3), it can be proved that $F_{t,\kappa}$ is the average job finding probability in an education group $\kappa \in \mathbb{K}$, during a period t.

Finally, for the analysis of cyclical properties of job finding probabilities, similarly to Gomes (2012), we will follow the methodology, popularised in Baker (1992). Following Baker (1992)'s approach, in order to analyse the cyclical movements of job finding probabilities, the logs of the estimated F_t (and $F_{t,\kappa}$) will be regressed, using Ordinary Least Squares, on seasonal dummy variables (ω_j), a linear time trend (φ) and quarterly unemployment rate (ϱ_t)[8]:

$$\log(F_t) = \alpha_0 + \sum_{j=1}^{3} \alpha_j \omega_j + \rho\varphi + \phi\varrho_t + \varepsilon_t \tag{5}$$

Additionally, the coefficient on unemployment rate ($\frac{\partial \log(F_t)}{\partial \varrho_t} = \phi$) will indicate the cyclicality of job finding probabilities: if $\phi > 0$ and statistically significant, job finding probabilities move together with unemployment rate, which implies that F_t is counter-cyclical (the converse holds for procyclicality). In addition, if ϕ turns out to be not statistically significant, we will identify the series as being acyclic.

3.3 Results

Job Finding Probabilities, without Educational Heterogeneity:

Given the definition of F_t, as it is in (3), we estimate job finding probabilities, first, without considering educational heterogeneity. The time series plot of the obtained F_t can be found in Appendix A. As expected, the sharp decline in job finding probabilities during the period of global financial crisis (during 2007,8-2012,13) can be noticed from the plot. In addition, F_t shows a rising trend, during the post-crisis period (2014-2015). Regarding the period of 1996-2004, the rising trend in job finding probabilities can be noticed as well. The increasing trend for the period of 1996-2004 and the following decline, until the end of the crisis period, is in line with the findings of Gomes (2012). Although, for the 1996-2004 period, for the US case, Shimer (2012) shows the opposite, declining, trend in job finding probabilities. Additionally, the average job finding probability, over the sample period, is found to be, around, 0.3632 with standard deviation 0.008.

Regarding the cyclical properties of job finding probabilities, implementing the regression equation, as it is defined in (5), yields to the results, presented in Appendix B.

[8] Baker (1992) applies this method to analyse cyclicality in unemployment durations.

As can be noted from the regression results, the coefficient on unemployment rate is, around, -0.1 and significant, which implies that, in line with the recent literature, job finding probabilities show a pro-cyclical behaviour. Recall that Gomes (2012) uses the same methodology to analyse cyclical properties of job finding probabilities, in the UK. The coefficient on unemployment rate, found in Gomes (2012), is, around, -0.09, which is very close to the one that we obtained.

As a final exercise, following the analysis of time series plots, we attempt to analyse the approximate extent of the decline in job finding probabilities, during the global financial crisis. In order to do so, we divide our sample period into three sub-periods (pre-crisis (1996Q2-2007Q4), crisis & recovery (2008Q1-2013Q4) and post-crisis (2014Q1-2015Q2) periods), estimate the averages of F_t, for each period, and compare them. The findings show that, during the pre-crisis period, the job finding probabilities averaged, around, 0.4, but, during the crisis period, it dropped by 11 percentage points to 0.29, later rising to 0.34, during the post-crisis period.

Job Finding Probabilities, with Educational Heterogeneity:

In order to analyse the behaviour of job finding probabilities, conditional on educational attainments of workers, we divide all unemployed individuals in our sample into five education groups: Master's, Bachelor's, A-level holders, GSCE or equivalent holders and individuals with City and Guilds qualifications[9]. The time series plots of job finding probabilities, for every education group, can be found in Appendix C[10].

As a first step of the analysis, we consider the averages of job finding probabilities, for each education group, over the entire sample period:

[9] City and Guilds of London Institute (City and Guilds qualifications) offers professional trainings and vocational educational qualifications, in the United Kingdom.

[10] The reason of focusing on these education groups, only, is the presence of large numbers of *unemployed* workers in those groups, in our dataset, for every quarter. Unfortunately, in many quarters, the numbers of *unemployed* workers in other education groups were insufficient to calculate job finding probabilities for them.

	Job Finding Probabilities	
	Mean	Standard Deviation
Master's	0.4333***	0.0149***
Bachelor's	0.4776***	0.0102***
A-level	0.4533***	0.0129***
GCSE	0.3835 ***	0.0117***
City and Guilds	0.3151***	0.0120***

The found results can be said to be similar to the ones, presented in Kettunen (1997), although not exactly the same. From the table it can be seen that the average job finding probabilities increase with every level of education, starting from the City and Guilds level up to the Bachelor's degree one. However, for Master's degree holders, the average reemployment probabilities decline by 4 percentage points. Although we do not find evidence that the Master's degree holders have the lowest job finding probabilities, as Kettunen (1997) did, we do find evidence in favour of declining reemployment probabilities at very high levels of schooling. In addition, this finding may indicate a possible concavity of job finding probabilities, in terms of the levels of schooling.

Although our analysis does not present any information about the possible cause of the decline in job finding probabilities, we may attempt to explain this phenomenon, by using a model, presented in Kettunen (1993), which was mentioned in the literature review section. The model implies that the direct effect of higher levels of schooling on reemployment probabilities is positive, as a result of the higher arrival rates of job offers. However, as the reservation wages of individuals also rise with the level of schooling (i.e. individuals become more selective in accepting job offers), the indirect effect of higher education is negative. Therefore, the total effect of educational attainment on reemployment probabilities is ambiguous. Thus, at very high levels of education (e.g. Master's and PhD), due to high reservation wages of workers, the, negative, indirect effect may dominate the, positive, direct one and so reemployment probabilities may decline.

From the results it can also be seen that, in contrast to Gomes (2012), we do not find evidence that individuals in the Bachelor's and above education groups have *twice* as high job finding probabilities as the ones in the GCSE schooling group.

In addition to the above discussion, as in the previous sub-section, we analyze the extent of the decline in job finding probabilities, during the period of global financial crisis; as before, we divide our sample into three sub-periods (pre-crisis, crisis/recovery and post-crisis) and estimate the average job finding probabilities, for each education group, in those periods:

	Average Job Finding Probabilities		
	Pre-crisis	Crisis & Recovery	Post-crisis
Master's	0.4602***	0.3943***	0.3917***
Bachelor's	0.5049***	0.4260***	0.4833***
A-level	0.5012***	0.3731***	0.4228***
GCSE	0.4398***	0.2951***	0.3246***
City and Guilds	0.3356***	0.2730***	0.3295***

As can be seen from the above table, during the crisis&recovery period, the average reemployment probabilities for Master's degree holders declined by 7 percentage points and, in contrast to all other education groups, it continued to decline during the post-crisis period as well. Nevertheless, the extent of the decline in job finding probabilities, during the crisis&recovery period, for Master's degree holders was the least one, relatively to other education groups. Furthermore, an intuitive and expected feature that can be noticed from the above table is that, starting from the GCSE level of education, the higher is the education level, the less is the extent of decline in job finding probabilities, when crisis hits the economy. Thus, it can be claimed that individuals with lower education levels suffer the most, in terms of reemployment success, during deep recessions, relatively to individuals with higher education levels. In particular, note that although the average job finding probability, over the whole sample period, for A-level holders was just 2 percentage points lower than for Bachelor's degree holders, the decline in the average reemployment probabilities, during the crisis, for the A-level education group was, nearly, twice as high as the decline for the Bachelor's schooling group. The results for City and Guilds education group are not in line with the above discussion: although workers in this education group have the lowest average job finding probabilities, the decline in reemployment probabilities for them was just 0.06 percentage points. This feature may be explained by the fact that, in contrast to GCSE and A-levels, City and Guilds qualifications are the specific professional qualifications, which can include not only low skilled qualifications. In addition, it could, also, be claimed that the decline was just 6 percentage points, because the average job finding probability for the City and Guilds group was already low, even before the crisis.

As a final step of this section, we analyze whether job finding probabilities preserve the pro-cyclical behaviour in each education group or not. The results of the OLS cyclicality regression equation (as defined in (5)), for every education group, are presented in Appendix D.

As can be seen from the results in Appendix D, in the cases of every education group, the coefficients on unemployment rate are negative and significant. Thus, for each analyzed education group, job finding probabilities show pro-cyclcial behaviours. In addition, in line with Gomes (2012), the extent of cyclical fluctuations, which can be approximated by the absolute value of the coefficient on unemployment rate, tends to increase at lower levels of education. Furthermore, the coefficients on unemployment rate for the first four groups of schooling are very similar to the ones, found in Gomes (2012)[11].

[11] Gomes (2012) does not consider City and Guilds qualifications. In addition, the estimated coefficients, found in that paper, are as follows; for Higher Education group *(Bachelor's and Master's together)*: -0.077*; A-levels: -0.106*; GCSE: -0.097*.

4 Mismatch

In this section, of the paper we will carry out several analysis, related to the phenomenon of educational mismatch. An employee is considered to be educationally mismatched, if his/her level of education is below (in the case of under-education) or higher (in the case of over-education) than the schooling level, required to carry out the job that a worker is assigned to.

One of the first economists, who analyzed the concept of educational mismatch, particularly over-education, was Richard Freeman (1976), in his book, called "The Overeducated American", where he studies the US graduate labour market. Since then, the concept of educational mismatch, especially over-education, has been paid particular attention, due to its implications on economic efficiency. The reason of it is that over-educated workers carry out the jobs that require the schooling levels below than what they have attained, which may be regarded as a clear waste of resources. In addition to this, under-educated individuals are assigned to jobs that require the schooling level higher than what they possess, which raises the question of why would employers hire workers that do not meet the educational requirements to carry out the job.

In this section, the first question that we will address is how several personal characteristics, especially work experience, affect the probabilities of becoming over- and under-educated. The possible answer to this question will help us to understand the implications of over- and under-education on worker productivities and to explain the concept of educational mismatch, especially the above mentioned phenomenons[12]. Secondly, we will attempt to analyze how the proportions of over- and under-educated workers, out of all employed ones, move at the business cycle frequencies: do we observe the proportions of over- and under-educated workers to rise during recessions or the converse is true; or is it the case that the movements are acyclic?

[12]We will discuss the importance of this issue more, in details, in the literature review section. Also, as it will be shown in "Results" section, this analysis will present an insight into the wage-experience relationship, proposed in the human capital theory.

4.1 Related Literature

Mainly, the literature on educational mismatch focuses on the analysis of implications of the incidences of over- and under-education on the earnings of workers. As this paper does not attempt to estimate any results, regarding the earnings of educationally mismatched individuals, we do not discuss those findings in details below. However, as a summary of the below presented papers, it can be said that the findings on the relationship of earnings and educational mismatch indicate that, typically, over-educated individuals earn less than the *correctly allocated workers, in other jobs, with the same years of education*. In addition, under-educated employees earn more than the *correctly allocated workers, in other jobs, with the same levels of schooling*. Moreover, it is also found that the under-educated employees earn less than the individuals *in the same job, but who are correctly allocated*; because, by definition, individuals, who are in the same jobs with under-educated workers and correctly allocated, have *higher levels of education* than the under-educated employees, and so they are paid higher wages (Allen and van der Velden (2001), McGuinness (2006), Hartog (2000) and others).

Empirical Approaches:

In the literature on educational mismatch, there are two main approaches that are applied to measure the required level/years of education in an occupation/job and so to measure the incidences of over- and under-education: subjective and objective measures[13].

A measure of educational mismatch is considered to be subjective, if the information about the mismatch is collected by asking directly a worker. That is, either workers are asked whether they consider themselves as being mismatched or not in their current job; or the workers are asked to define the minimum education level, which is sufficient to carry out their jobs. In the latter case, the employees' own schooling attainments are compared with the worker-defined minimum required education level to identify whether they are mismatched or not. The papers

[13] According to Groot and Maassen van den Brink (2000), Hartog (2000) and McGuinness (2006)

that use the subjective measure of mismatch include Dolton and Vignoles (2000), Allen and van der Velden (2001) and Sicherman (1991).

In the case of the objective measure, the realised matches from a given data are analysed. In particular, the required level of education is either defined as the average or modal years of schooling, in a given occupation; and the deviations from that required level are defined as being educationally mismatched. As will be shown below, in this paper, we will also follow the objective approach to measure mismatch. The papers that follow the objective measure include Verdugo and Verdugo (1989), Mendes de Oliviera *et al.* (2000), Goot (1996) and Rubb (2003).

All of the above mentioned approaches have own negative and positive aspects. For instance, the average years of schooling definition may be very sensitive to outliers in a given data. Therefore, the measure of the incidences/proportions of over- and under-educated workers may be biased, where the direction of the bias is ambiguous (depends on the values of outliers). In addition, practically, the modal years of schooling definition may not be straightforward to implement, if the distribution of education years of workers, in a given occupation, is bi-modal (i.e. it is not clear which mode should be chosen as a required education level)[14]; however, this definition is less sensitive to outliers. More discussion about the drawbacks and benefits of the approaches can be found in Hartog (2000), McGuinness (2006) and Dolton and Vignoles (2000).

The Estimated Proportion of Educational Mismatch:

In the literature, the estimated proportions/percentages of over- and under-educated employees, out of all employed workers, may vary, depending on which measurement approach is implemented[15]. For instance, McGoldrick and Robst (1996) analyses the percentage of over-

[14]In fact, because of this practical difficulty, we were not able to implement the modal years of education definition; we discuss this issue, in details, below.

[15]Although, as it is mentioned in McGuinness (2006), the qualitative results of the analyses, related to educational mismatch (e.g. the effect of mismatch on earnings), are not sensitive to the differences in measurement approaches.

education and finds that, if the subjective measure approach is employed, the percentage of over-education is 30%; however, if the objective approach is used, the estimated percentage of over-educated individuals is, around, 16%. Hartog (2000) presents the review of papers in the field and concludes that, in the case of the objective measure, the proportions of over- and under-education are found to be, around, 10-15% each. In addition, Groot and van den Brink (2000) analyses various related papers and reports that applying the objective measure yields the percentage of over-education to be, around, 13% and for the subjective measure case, it is, approximately, 28%. Another important paper, Groot (1996), uses the objective measure, in particular, the average schooling years approach, for the UK (BHPS(1991) data), and finds that, around, 10% of females and 13% of males are over-educated and 10% of males and 8% of females are under-eduated in their jobs. Finally, as was mentioned, Dolton and Vignoles (2000) employs the subjective measure approach for the UK, using the 1980 National Survey of Graduates and Diplomates; the paper's findings indicate that, in line with the above presented papers that use the same measure, around, 30% of workers are over-educated.

Theoretical Explanations:

Based on the papers cited above, it can be said that there are two main theoretical approaches that are implemented to explain the phenomenon of educational mismatch: the Human Capital Theory (HCT henceforth) and the Assignment Theory approaches.

As it is well-known, one of the key results of the classical HCT is that workers are paid the value of their marginal productivities, which implies that workers' productivities in their jobs must be fully utilized. Thus, based on this fully utilized productivity approach, the HCT explanation proposes that despite of having high education levels, over-educated individuals suffer from the lack of other labour market related human capital (e.g. work experience or job training etc.), which diminishes their productivities, relative to the correctly allocated workers with the same levels of education. Thus, because of the low levels of productivity, those (over-educated) workers are not employed in the jobs, at which other individuals, with the same amount of education and sufficient levels of other labour market related human capital, work at. As a result, despite of high levels of schooling attainments, those individuals have to accept the offers from the jobs, which require lower education levels than theirs, face a wage penalty (see above for the effect on earnings), and so become over-educated. Based on a similar explanation, for the case of under-educated workers, HCT claims that because of the surplus of some labour market related characteristics, those workers have higher levels of productivity, than the individuals with the same levels of education. Therefore, those individuals are able to work at jobs that require higher levels of schooling, than their attained ones, and so they become under-educated and receive a wage surplus. In addition, due to the high levels of productivity, firms hire those workers, despite of not having sufficient levels of education (otherwise why would a firm hire a worker with the education level below the required one?). Based on this explanation, if the HCT approach is valid, it must be the case that the surplus of labour market related characteristics decreases the chances of becoming over-educated and affects the probability of becoming under-educated positively.

Thus, based on the HCT approach, in order to understand better and attempt to explain existence of over- and under-educated individuals, it is important to analyze how some work related personal characteristics affect the incidences of educational mismatch. Recall that this is one of the questions that we will analyze in this section. Furthermore, as it will be shown in ”Results” sub-section, the analysis of the implications of work experience on the incidence of over-education also presents an insight into the *(concave)* relationship between earnings and the levels of work experience of individuals, provided by the Human Capital Theory.

In the literature, Groot (1996) uses a multinomial logit model and, along with other findings, it estimates that, in line with HCT explanation, over-education decreases with work experience, but under-education increases with it. In addition, Battu and Sloane (2004) estimates a multinomial logit model for the UK (using the objective measure definition of mismatch and Fourth National Survey of Ethnic Minorities data for 1993-94) and, similar to Groot (1996), it finds that the incidence of over-education decreases at high levels of work experience. However, in contrast to Groot (1996)'s findings and to HCT explanation, it finds that higher work experience is associated with lower incidences of under-education as well.

Although does not present any clear explanation for the existence of under-education, assignment models are used as another approach to explain the case of over-education. Assignment model approach does not propose that over-educated workers have low productivity levels; instead, it offers the following explanation. The fact that higher educated workers earn higher wages serves as a guidance for income maximizing agents to invest into higher education. However, if the rate, at which the number of higher educated individuals grow, is higher than the rate, at which the number of jobs for those higher educated workers increase, more of the individuals with high levels of schooling will get assigned to jobs that have lower educational requirements, which will result in over-education.

Mismatch and Business Cycles:

Finally, we will present some results, regarding the cyclical properties of the incidences of over- and under-education, from the literature. To our knowledge, there are no papers that analyze cyclical properties of the proportions of educationally mismatched workers, which points out another important contribution of this paper. We highlight the importance of analyzing the proportions of over- and under-educated employees, out of all employed ones, and not the incidences (i.e. numbers) of over- and under-education, because, the cyclicality analysis of the incidences alone may be misleading. The reason of it is that movements in the incidences may be driven by some random changes in the labour force, which may not represent the true business cycle fluctuations.

There are two papers that present results, which can be claimed to, approximately, represent the cyclical properties of the *incidences* of over- and under-education. Although, it must be mentioned that, in the papers, the results are not explicitly interpreted as the cyclical properties of incidences. One of the papers is Groot and van den Brink (2000). In that paper, authors use the OLS approach and regress the incidences of over- and under-education on unemployment rate and on some other parameters (i.e. if the incidences are identified by a binary variable, authors estimated an LPM model), which is very similar to the cyclicality regression that we discussed and implemented in above section (from Baker (1992)), but not the same. The results of that paper indicate that the incidence of under-education moves together with unemployment rate, so shows a counter-cyclical behaviour. In contrast, over-education shows a pro-cyclical behaviour. Additionally, Rubb (2003) analyzes the incidence of over-education only (using US CPS data) and finds that the incidence of over-education is higher during the expansion period (1995-99), than during the recession one (1991-92). This finding may be interpreted as an evidence in favour of procyclicality of the incidence of over-education.

4.2 Framework

As was mentioned in the previous sub-section, in order to identify over- and under-educated workers, we will follow the objective measure approach. In particular, following Verdugo and Verdugo (1989), Groot (1996) and Rubb (2003), we will define the required years of schooling in a given occupation as the average years of attained education, among the employees of that occupation. In addition, a worker in an occupation will be defined as over-educated if his/her attained schooling years exceeds the defined required years of education by one standard deviation (the converse definition holds for under-educated workers).

Before proceeding into analytical definitions, it is important to mention the reason of implementing the average years of schooling definition. First, we are not able to implement the subjective measure approach for the reason that we do not have data on worker responses to the questions, regarding educational mismatch (Quarterly LFS does not provide such information). Second, regarding the modal years of schooling definition, as was also briefly mentioned above, if the distribution of the years of attained education of workers, in a given occupation, is bi-modal (i.e. large proportions of workers are concentrated around two different years of schooling), it is not clear which mode should be chosen to define the required years of education. This was a problem that we faced in many occupation groups in our sample. One could claim that if the discrete mode definition is used, then the required level of education can clearly be chosen, as it is very unlikely to obtain exactly the same number of workers having two different years of education, in a given occupation. However, we claim that this approach can be very misleading and the generated results may be biased, if the two modes are not very close to each other.

We will present the reason of it with a practical example from our data-set. Tabulating education years of workers in a ”Marketing and Sales Directors” occupation shows that, over the whole sample period, in every quarter, big proportions of individuals in this occupation are concentrated around 11-12 (GSCE; AS level) and 16 (Bachelor's) years of education; thus, the distribution is bi-modal, in every quarter. Moreover, with the discrete mode definition, the modes tend to vary among GCSE and Bachelor's levels of education, depending on the considered year and quarter. This implies that, if the modal years of education definition was used, we would have to define the required level of education as being GCSE in some quarters and Bachelor's in others, which is a clear inconsistency that would result our measures of over- and under-education to be biased. Similar problems are also observed in other occupations (e.g. ”Transport Managers” occupation)[16]. Thus, because of this practical inconsistency, we employ

[16]One could claim that the bi-modality of the distribution may reflect the fact that individuals in the same occupation may be employed in jobs that require different skill and education levels. For the future research, in order to overcome this issue, one possible solution would be to analyze the modal years of education in a given occupation, conditional on industries. Moreover, another possible solution for solving this problem could be to use Bayesian Mixture Model approach (see Xu *et al.* (2014) for an explanation), treat the distribution of the years

the average schooling years approach, instead of the modal years of schooling one. Although, our approach might be sensitive to outliers, which may slightly bias our results (mentioned in literature review section).

For the analytical framework, define $\mathbb{Z}$ as an occupation group index set, $z \in \mathbb{Z}$ as an occupation, and for every t, let $\mathbb{T}$ be the time index set. Additionally, define $\mathcal{N}_{z,t}$ as a number of employees, in an occupation $z \in \mathbb{Z}$, at time $t \in \mathbb{T}$. Let $\epsilon_{i_z,t}$ be the attained years of education of an employee i, in an occupation $z \in \mathbb{Z}$, at time $t \in \mathbb{T}$. In addition, we define $\epsilon^r_{z,t}$ as being the required years of education, in an occupation $z \in \mathbb{Z}$, at time $t \in \mathbb{T}$. Following the discussion above, we define $\epsilon^r_{z,t}$ as follows.

Definition The required years of education, in an occupation $z \in \mathbb{Z}$, at time $t \in \mathbb{T}$, is defined as:

$$\epsilon^r_{z,t} = \frac{\sum_{i_z} \epsilon_{i_z,t}}{\mathcal{N}_{z,t}} \tag{6}$$

In addition to this, let $\sigma_{z,t}$ be the standard deviation of education years, in an occupation $z \in \mathbb{Z}$, at time $t \in \mathbb{T}$, and define $\mathfrak{M} \in \{0, 1\}$ as a binary variable that identifies a mismatched worker. Then, an employee is defined as being mismatched, in particular over- and under-educated, in an occupation $z \in \mathbb{Z}$, at time $t \in \mathbb{T}$, according to the following definition.

Definition An employee i, in an occupation $z \in \mathbb{Z}$, at time $t \in \mathbb{T}$, is defined as mismatched (over- and under-educated) by the following condition:

$$\mathfrak{M}_{i_z,t} = \begin{cases} 1 \quad \text{if} \quad \epsilon_{i_z,t} > \epsilon^r_{z,t} + \sigma_{z,t} & \text{(over-educated)} \\ 1 \quad \text{if} \quad \epsilon_{i_z,t} < \epsilon^r_{z,t} - \sigma_{z,t} & \text{(under-educated)} \\ 0 \quad \text{if} \quad \epsilon_{i_z,t} \in [\epsilon^r_{z,t} - \sigma_{z,t}, \epsilon^r_{z,t} + \sigma_{z,t}] & \text{(correctly matched)} \end{cases} \tag{7}$$

Following this definition, for further notational purposes, we define $\Theta \in \{0, 1\}$ as a binary variable, representing an over-educated worker, and $\Upsilon \in \{0, 1\}$ as the one that identifies an

of schooling as if it consists of two different sub-populations and estimate under- and over-education for each sub-population, separately.

under-educated employee. Then following (7), we have that:

$$\Theta_{i_z,t} = \begin{cases} 1 & \text{if} \quad \epsilon_{i_z,t} > \epsilon^r_{z,t} + \sigma_{z,t} \\ 0 & \quad \text{otherwise} \end{cases} \quad (8)$$

$$\Upsilon_{i_z,t} = \begin{cases} 1 & \text{if} \quad \epsilon_{i_z,t} < \epsilon^r_{z,t} - \sigma_{z,t} \\ 0 & \quad \text{otherwise} \end{cases} \quad (9)$$

Given the above definitions, the aggregate proportions of mismatched (μ) over- (θ) and under-educated (γ) individuals at time $t \in \mathbb{T}$ can, clearly, be represented as follows:

$$\mu_t = \frac{\sum_{z\in\mathbb{Z}} \sum_{i_z} \mathfrak{M}_{i_z,t}}{\sum_{z\in\mathbb{Z}} \mathcal{N}_{z,t}}; \qquad \theta_t = \frac{\sum_{z\in\mathbb{Z}} \sum_{i_z} \Theta_{i_z,t}}{\sum_{z\in\mathbb{Z}} \mathcal{N}_{z,t}}; \qquad \gamma_t = \frac{\sum_{z\in\mathbb{Z}} \sum_{i_z} \Upsilon_{i_z,t}}{\sum_{z\in\mathbb{Z}} \mathcal{N}_{z,t}}$$

In addition, as in the previous section, in order to analyze the cyclical properties of proportions, we will follow the methodology, proposed in Baker (1992). Namely, for some λ_t (in this case $\lambda_t \in \{\mu_t, \theta_t, \gamma_t\}$), we will estimate the coefficients of the following regression equation, using Ordinary Least Squares:

$$\log(\lambda_t) = \alpha_0 + \sum_{j=1}^{3} \alpha_j \omega_j + \rho\varphi + \phi\varrho_t + \varepsilon_t \quad (10)$$

where ω_j represent seasonal dummy variables, φ is a linear time trend and ϱ_t is the quarterly unemployment rate. As in the previous case, $\frac{\partial \log(\lambda_t)}{\partial \varrho_t} = \phi$ represents the cyclical properties of $\lambda_t(\in \{\mu_t, \theta_t, \gamma_t\})$.

Finally, to estimate the relationship between the various personal characteristics of workers and the probabilities of becoming over- and under-educated, we will implement the logit model (Cox (1958)) . Formally, let ψ be the binary response variable (i.e. $\psi \in \{\mathfrak{M}, \Theta, \Upsilon\}$) and let $\vec{\boldsymbol{x}}$ to be the vector of personal characteristics. Then, the logit model is specified as:

$$P(\psi = 1 \mid \vec{\boldsymbol{x}}) = G(\beta_0 + \vec{\boldsymbol{x}}\vec{\boldsymbol{\beta}}) \quad (11)$$

where $G(\cdot)$ is a logistic function, defined as:

$$G(\beta_0 + \vec{\boldsymbol{x}}\vec{\boldsymbol{\beta}}) = \frac{\exp(\beta_0 + \vec{\boldsymbol{x}}\vec{\boldsymbol{\beta}})}{1 + \exp(\beta_0 + \vec{\boldsymbol{x}}\vec{\boldsymbol{\beta}})}$$

4.3 Results

The Proportion of Educational Mismatch:

Following the definitions of over- and under-education, defined as above, and applying them to the Quarterly LFS data shows that 25.73% of employed individuals are mismatched in their jobs, 14.5% out of which are over-educated and 11.23% are under-educated. By gender differences, among males, 26.75% are mismatched: 14.78% are over-educated and 11.96% are under-educated. Whereas, among females, 24.17% are mismatched, 14.07% of which are over-educated and 10,11% of which are under-educated. Note that the estimated percentages of over- and under-educated workers can be said to be in line with the papers that use the average schooling years definition of mismatch (Groot (1996), Hartog (2000) and others).

Personal Characteristics and Probability of Becoming Mismatched:

Before proceeding to the analysis of cyclical properties, we analyze how some personal characteristics of individuals are associated with the probabilities of becoming over- and under-educated. As was mentioned above, we estimate the logit model, with several labour market related personal characteristics as explanatory variables. Mainly, our purpose is to analyze how work experience is associated with probabilities (odds ratio in the case of logit) of becoming over- and under-educated[17]. Consider the table of the regression results, presented in Appendix E.

As can be seen from the table in Appendix E, one more year of work experience decreases the odds of becoming over-educated by, around, 3%($\approx (1 - \exp(-0.0284)) \times 100$) and increases the odds of becoming under-educated by 1.7%($\approx (\exp(0.0173) - 1) \times 100$), with all the coefficients being significant. From the results on the odds ratios it can be claimed that, *ceteris paribus*, the incidence of under-education is higher among workers with high work-experience and the incidence of over-education is higher among workers with low work-experience. Thus,

[17] Experience is approximated as $age - education(years) - 5$

workers with higher work experience are more likely to become under-educated and individuals with low work-experience are more likely to become over-educated. Note that this result is in line with the findings of Groot (1996), for the UK. Moreover, the findings are also in line with HCT explanation of over- and under-education. As lower work experience affects worker productivities negatively, *ceteris paribus*, individuals with low levels of work experience have lower levels of productivity, relatively to individuals with the same levels of schooling, but with sufficient amounts of work experience. Therefore, individuals with low levels of work experience are more likely to work in jobs, for which they are over-qualified, in terms of educational attainment. The converse holds for under-educated workers. Moreover, as can be noticed from the table in Appendix E, the odds ratio of becoming correctly matched increases with work experience as well. In addition to these, another result of the logit regression indicates that individuals, who received any type of a job training are more likely to become under-educated and less likely to become over-educated. As the job training can be considered to affect worker productivities positively, this finding is in line with the HCT approach as well.

The above presented results can also be used to provide an evidence for the mechanism that is presented in Burdett *et al.* (2011), which attempts to explain the life-cycle wage growth of workers, which is proposed by the human capital theory, since the pioneering works of Becker (1964) and Mincer (1974). As it is well known, HCT proposes that earnings of individuals rise with experience, but at a decreasing rate (concavity). However, as it is claimed in Burdett *et al.* (2011), the human capital theory fails to explain the job-to-job movements of individuals (due to the assumed competitive environment), which is a drawback of the theory, as a big proportion of the wage rises with experience occurs, when individuals move from one job to another. Given this shortcoming of HCT, Burdett *et al.* (2011) attempts to model the wage-experience relationship as a result of the job-to-job movements. It develops a search-theoretical model and proposes that, at the beginning of his/her career, a young, inexperienced, labour market entrant accepts a low wage job and eventually, while accumulating higher work experience, finds another job with higher wages and moves on to that one. However, after a while, new higher wage job opportunities get exhausted, which explains why the wages increase with experience at a decreasing rate. We claim that, based on our results, Burdett *et al.* (2011) model can be explained on the example of the relationship of over-education and experience.

Recall that, as it is found in the literature, over-educated individuals earn less than the *correctly allocated workers, in other jobs, with the same years of education*. Additionally, as we found above, workers with low levels of work experience are more likely to become over-educated, and the converse implies that as experience rises, the incidence of over-education decreases. Moreover, it was also shown that the incidence of correct match among workers rises with experience. Thus, based on our results, we can claim that young, inexperienced, labour market entrants, with high education, are more likely to get assigned to jobs that require lower levels of education than theirs, so become over-educated and face a wage penalty. However, as workers accumulate higher levels of work experience, they are less likely to be over-educated and more likely to become correctly matched, which implies that as experience rises, under-educated employees are more likely to move on to jobs that matches their education levels better (become correctly matched) and so provides higher payment. In addition, as after finding a job that matches an employee's education level and skills correctly, the possibilities of finding a job that matches the skill and education levels even better and provides even higher earnings get exhausted, so the return to an additional year of experience starts to diminish, which implies a concave relationship between experience and earnings.

Moreover, if we estimate another logit model (Appendix F), where we attempt to obtain the relationship between the incidences of being over- and under-educated and the odds of searching for a different job, while being employed (on-the-job search), we would obtain that being over-educated increases the odds ratio of looking for a different job by 50% (being under-educated decreases it)[18]. This result may, also, be used as supporting the above provided explanation: although the finding does not imply that over-educated individuals are more likely to *move* on to different jobs, it implies that over-educated workers are more likely to carry out on-the-job search and so attempt to move on to other jobs.

Mismatch Proportions and the Business Cycles:

The time series plots of the estimated proportions of mismatched, over- and under-educated employees, out of all employed workers, can be found in Appendixes G, H and I, respectively. As can be noticed from the plots, the proportion of over-educated individuals has a rising trend over the sample period (it rises by, around, 2 percentage points from 1996 to 2015). In addition, the proportion of under-educated individuals shows a, approximately, constant trend up to 2010s, then there is a sharp increase in that proportion.

As was explained above and implemented in the previous section, in order to analyse the cyclical properties of the proportions of over-/under-education, mismatch and of the *correct* match (which can be obtained as $(1 - \mu_t)$ for every $t \in \mathbb{T}$) we will implement the methodology, popularised in Baker (1992). The regression results are reported in the table in Appendix J.

As can be seen from the table in Appendix J, the coefficient on unemployment rate for the over-education proportion is not statistically significant ($p - value = 0.611$). Thus, it can be claimed that the over-education proportion follows an acyclic movement pattern: it does not move at business cycle frequencies. In addition, the results indicate that the movements in the mismatch proportion follow a counter-cyclical pattern and the proportion of correctly matched individuals shows a pro-cyclical movement pattern, with the significant coefficients on unemployment rate. Thus, in terms of the co-movement with job finding probabilities, the results indicate that when job finding probabilities are high, the proportion of correctly matched individuals rise as well. This fact may, simply, indicate that, during expansions, there are more

[18] LFS provides a binary variable that identifies whether an employed individual is looking for a different job.

jobs around and, as most of the workers get correctly matched in their jobs, the proportion of correctly matched workers increases (the converse explanation holds for recessions).

The counter-cyclicality of mismatch proportion rises more interesting questions. Recall that the over-education proportion follows an acyclic movement; thus, it must be the case that the counter-cyclicality of mismatch proportion has been driven by the movements in the proportion of under-educated workers. Indeed, from the table in appendix J it can be seen that the movements in the proportion of under-educated employees follow a counter-cyclical pattern, with a significant coefficient on unemployment rate. That is, during recessions, firms adjust their employee numbers, such that the under-educated individuals' proportion increase. Using our findings above and the properties of the earnings of under-educated individuals, we can explain this phenomenon as follows.

Recall that the HCT explanation of educational mismatch proposes that due to the surplus of labour market related human capital, under-educated individuals possess higher productivity levels, than individuals with the same levels of education. It is the high productivity levels that allow firms to employ workers with insufficient levels of education. In addition, our results of the logit model, presented in Appendix E, provided evidence in support of this claim, on the examples of work experience and job training. Moreover, recall that, as was mentioned in the literature review part of this section, *the under-educated workers are paid less* than the employees, who are *in the same job* with them, but, *who are correctly allocated* and by definition, *have higher levels of education*. Thus, from a firm's point of view, under-educated workers have sufficient levels of productivities, as the other employees, to carry out the work and it is cheaper to employ them. Therefore, as firms, typically, decrease output and so employment during recessions, it can be claimed that, given the desired output level, it is cheaper for them to adjust its employees, such that, the proportions of under-educated workers increase[19]. This

[19] It could also be the case that the under-educated individuals possess high work-specific experiences, which

explanation presents an insight of why the proportion of under-educated individuals rises during recessions. The decrease during expansions can, simply, be justified with the same explanation as the increase in the *correct* match proportion, given above[20].

5 Analyzing Dynamics: Decomposition of the Mismatch Proportion

In the previous section we analyzed fluctuations in the mismatch proportion and its movement over the business cycle. Recall that we found the business cycle movements in mismatch proportion to be counter-cyclical. In addition, we presented some explanations, regarding what may derive this counter-cyclicality.

In this section, we link the concepts of labour market flow rates and educational mismatch together and develop a mathematical framework that attempts to analyse what derives the mismatch proportion to increase during recessions and to decrease during expansions. In particular, when applied to the data, the framework will answer the question of what amount of the variation in mismatch proportion can be explained by the fluctuations in job finding and job separation rates of mismatched and correctly matched employees, separately.

The framework will be constructed by using a methodology, similar to the one that was popularised in Fujita and Ramey (2009) for capturing the contributions of labour market flow rates to the variation in unemployment. In particular, we will modify Fujita and Ramey (2009)'s approach for a two-state labour market and apply it for the case of mismatch proportion. As it will be shown below, we will log-linearize the mismatch proportion, in terms of job finding

yields firms to value those workers more.

[20]Note that the explanation of the increase during recession may *(mistakenly)* indicate that if it is true, then the firm would hire only under-educated employees or keep the large proportions of its employees as under-educated ones, in every state of the economy, as it is cheaper to do so for reaching the same level of output. The statement would be correct, if there was a large supply of those under-educated individuals in the economy, which is not the case (see proportions of under-educated workers above or in the presented papers).

and job separation rates of mismatched and correctly matched individuals, and will modify the resulted expression, in order to capture the contributions of those rates to the fluctuations in the proportion of mismatched employees. At this point it is important to mention that we will not develop separate frameworks for the proportions of under- and over-educated individuals; instead, we will focus the overall proportion of educationally mismatched workers: on the mismatch proportion

However, in contrast to Fujita and Ramey (2009), we will not attempt to use the developed framework to *estimate* the amounts of the contributions of separation and job finding rates to the variation in mismatch proportion. The main reason of it is that it is essential to make the use of a panel data to estimate the parameters of the framework, presented below. More precisely, it is not possible to identify and estimate job separation and job finding rates of mismatched and correctly matched individuals, separately, from the data that we used in above sections, namely from the Quarterly LFS. However, although we do not estimate the amounts of contributions, the below presented framework can be regarded as being important for the research in the field, as it presents a ready framework to measure the contributions of labour market flow rates to the variation mismatch proportion. In addition, as will be discussed below, it presents an intuitive inside into the relationship of mismatch proportion and labour market flows. As was mentioned, to our knowledge, there are no other papers in the field that attempt to analyse the fluctuations in mismatch proportion, in terms of labour market flow rates.

5.1 Definitions and Assumptions

As before, we will focus on the two-state labour market model. Let T be the time index set. Then, for every $t \in T$, define M_t as being the number of mismatched workers, C_t as the number of correctly matched employees, E_t as the number of all employees and, finally, define L_t as representing the labour force. Note that, as the number of all employees consists of the numbers of mismatched and correctly matched ones, it must be true that:

$$E_t = M_t + C_t \tag{12}$$

Thus, as can be seen, one of the crucial factors of our framework is the separation of the state of employment into two sub-states: state of mismatched employment and the state of *(correctly)* matched employment.

In addition, let m_t be the rate of employed individuals that are mismatched (mismatch rate henceforth) and let c_t be the rate of workers that are correctly matched (match rate henceforth); given these, we define m_t and c_t as:

$$m_t = \frac{M_t}{L_t}; \tag{13}$$

$$c_t = \frac{C_t}{L_t} \tag{14}$$

By defining e_t as the employment rate at time t, from (12), (13) and (14) we obtain that:

$$e_t = \frac{E_t}{L_t} = \frac{M_t}{L_t} + \frac{C_t}{L_t} = m_t + c_t \tag{15}$$

In addition to the definitions above, let f_t^m and f_t^c be rates of finding a job, at which individuals become mismatched and correctly matched, respectively. More precisely, f_t^m is the rate, at which an unemployed/employed individual makes a transition to a job, where he/she is mismatched; and f_t^c is the rate, at which an unemployed/employed individual makes a transition to a job, where he/she is correctly matched[21]. Likewise, let s_t^m and s_t^c be the job separation rates of mismatched and correctly matched individuals, correspondingly *(i.e. the rates at which mismatched and matched individuals lose their jobs)*[22]. Given the above definitions, now, it can clearly be seen that for the estimation of job finding rates of mismatched and matched in-

[21]For simplicity we will refer to f_t^m and f_t^c as job finding rates of mismatched and correctly matched individuals, respectively.

[22]Note that job separation does not necessarily imply the movement into the state of unemployment; individuals may, indeed, move to other jobs (where they may become mismatched or correctly matched)

dividuals, separately, it is essential to have a panel data; so with our Quarterly LFS data, it is impossible to obtain those parameters[23].

As can be noted from the above definitions, the crucial factor in our framework is that job finding and job separation rates differ, depending on whether an individual becomes/is mismatched or correctly matched in the job. An intuition behind this idea can be motivated by several examples. For instance, as noted in the previous section, it could be the case that educational mismatch is a transitionary stage for young workers, in order to accumulate some labour market related human capital (e.g. work experience); thus, we can expect that the jobs, formed as a result of a mismatch, may not last as long as the ones, formed as a result of a correct match, so $s_t^m > s_t^c$. Similar examples can be presented for the cases of job finding rates (f_t^m; f_t^c).

Following the discussion above, for the development of our framework, we make the following crucial assumption:

Assumption 1 *Given, f_t^m, f_t^c, s_t^m and s_t^c, the rates of mismatched and matched employees (mismatch and match rates) can be approximated by the following expressions:*

$$m_t \simeq \frac{f_t^m}{f_t^m + s_t^m} \tag{16}$$

$$c_t \simeq \frac{f_t^c}{f_t^c + s_t^c} \tag{17}$$

A justification for this assumption originates from Shimer (2012). As it is shown in that paper, the unemployment rate (u_t) can be approximated as:

$$u_t \simeq \frac{s_t}{s_t + f_t} \tag{18}$$

where, s_t and f_t are job separation and finding rates, respectively, without any heterogeneity. Given this finding of Shimer (2012), the employment rate (e_t) can be approximated as:

$$e_t \simeq 1 - \frac{s_t}{s_t + f_t} = \frac{f_t}{f_t + s_t} \tag{19}$$

[23] One may obtain an access and use the panel version of LFS. For more information, please, see https://discover.ukdataservice.ac.uk/series/?sn=2000026

In addition, if we segment employed workers, depending on whether they are mismatched or correctly matched, we obtain the approximations, given in Assumption 1.

The approximations of m_t and c_t, presented in Assumption 1, can be applied to define the trend components ($\overline{m}, \overline{c}, \overline{f}^m, \overline{f}^c, \overline{s}^m$ and $\overline{s}^c$). The trend values are crucial for the log-linearisation procedure that we apply below; they can be regarded as being the steady state values of each parameter. Empirically, as it is also noted in Fujita and Ramey (2009), the trend values may be regarded as being the trend components of our six series, obtained via the Hodrick-Prescott filter (HP filter henceworth), or the trends can be assumed to be equal to a one period lagged value of each component of the framework *(e.g.* $\overline{m} = m_{t-1}$*)*. We define trend components as follows:

$$\overline{m} \equiv \frac{\overline{f}^m}{\overline{f}^m + \overline{s}^m} \tag{20}$$

$$\overline{c} \equiv \frac{\overline{f}^c}{\overline{f}^c + \overline{s}^c} \tag{21}$$

5.2 Framework

Following the notation in the fourth section, we define the mismatch proportion as:

$$\mu_t = \frac{M_t}{E_t} \tag{22}$$

Before we can analyze the contributions of labour market flow rates to the fluctuations in mismatch proportion, we need to show that μ_t can be expressed as a function of the rates. The following proposition presents such result.

Proposition 5.1 *The mismatch proportion can be represented as:*

$$\mu_t = \frac{f_t^m(f_t^c + s_t^c)}{f_t^m(f_t^c + s_t^c) + f_t^c(f_t^m + s_t^m)}$$

Proof See Appendix K

Thus, as (by the result of Proposition 5.1) the mismatch proportion can be expressed as a function of labour market flow rates, it can be claimed that, in order to understand the observed business cycle dynamics of mismatch proportion, it is essential to analyze the contributions of job finding and separation rates to those fluctuations. Furthermore, it is important to examine which components are more important for explaining the dynamics and so to understand what derives the μ_t to rise during recessions and to decline during expansions[24].

Following Proposition 5.1, the trend value of the mismatch proportion ($\overline{\mu}$) can be represented in terms of the other trend components as:

$$\overline{\mu} = \frac{\overline{f}^m(\overline{f}^c + \overline{s}^c)}{\overline{f}^m(\overline{f}^c + \overline{s}^c) + \overline{f}^c(\overline{f}^m + \overline{s}^m)}$$

Given Proposition 5.1, we prove the following lemma, which presents one of the central results of this chapter:

Lemma 5.2 *A one-period deviation of mismatch proportion from its trend, can be expressed as:*

$$\Delta \ln \mu_t = (1-\overline{\mu})(1-\overline{m})\Delta \ln f_t^m - (1-\overline{\mu})(1-\overline{c})\Delta \ln f_t^c - (1-\overline{\mu})(1-\overline{m})\Delta \ln s_t^m + (1-\overline{\mu})(1-\overline{c})\Delta \ln s_t^c + \varepsilon_t$$

where ε_t denotes the error term.

Proof See Appendix L

The result of Lemma 5.2 shows the deviation of mismatch proportion from its trend value as a sum of the parameters, which are related to the deviations of job finding and job separation rates from their trends, plus a disturbance term. More precisely, as at time $t \in T$, $\overline{m}$ and $\overline{\mu}$ are some given parameters, the term $(1-\overline{\mu})(1-\overline{m})\Delta \ln f_t^m$ can be interpreted as a factor that represents the part of the deviation of μ_t from its trend that is originated by the deviation of f_t^m

[24]The main idea is similar to the analysis of fluctuations in unemployment.

from $\overline{f}^m$. Likewise, as $\overline{\mu}$ and $\overline{c}$ are some fixed parameters, at some time $t \in T$, $(1-\overline{\mu})(1-\overline{c})\Delta \ln f_t^m$ can be regarded as representing the part of the deviation in μ_t from $\overline{\mu}$ that depends on the deviation of f_t^c from its trend. Similar interpretations hold for the other components.

Note that the result of Lemma 5.2 presents an intuitive insight into the relationship between the mismatch proportion and labour market flow rates. In particular, the increase in the rate, at which individuals become mismatched (f_t^m) increases the mismatch proportion and the increase in the rate at which mismatched individuals lose jobs decreases μ_t; which are the expected and intuitive outcomes. Moreover, again, as expected, the increase in the rate at which individuals become correctly matched (f_t^c) decreases the mismatch proportion and the increase in the separation rate of matched workers increases it. The found negative effect of f_t^c may reflect the fact that one of the factors, which may derive a decrease in the proportion of mismatched workers, is the movement of mismatched employees to the jobs that matches their education levels better. Similarly, the positive effect of s_t^c may be explained using the fact that some proportion of the correctly matched workers that get separated from their jobs may become mismatched.

Moreover, from this discussion note that, although we cannot estimate and analyse job finding and job separation rates of mismatched and matched employees, we can comment on their cyclical properties. As the mismatch proportion follows a counter-cyclical trend and as f_t^m and s_t^c affect the deviations in μ_t positively, it implies that job finding rate of mismatched employees and job separation rates of correctly matched workers must follow a counter-cyclical or an acyclic pattern. In addition, as f_t^c and s_t^m affect the deviations in μ_t negatively, the job finding rate of matched workers and the job separation rate of mismatched workers must follow a procyclical or an acyclic pattern.

In addition, similarly to Fujita and Ramey (2009), for notational simplicity, we define the following expressions:

$$\Delta \ln \mu_t \equiv d\mu_t \tag{23}$$

$$(1-\overline{\mu})(1-\overline{m})\Delta \ln f_t^m \equiv df_t^m \tag{24}$$

$$-(1-\overline{\mu})(1-\overline{c})\Delta \ln f_t^c \equiv df_t^c \tag{25}$$

$$-(1-\overline{\mu})(1-\overline{m})\Delta \ln c_t^m \equiv ds_t^m \tag{26}$$

$$(1-\overline{\mu})(1-\overline{c})\Delta \ln s_t^c \equiv ds_t^c \tag{27}$$

Then, using the above definitions, the resulted expression of Lemma 5.2 can be rewritten as:

$$d\mu_t = df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t \tag{28}$$

As was mentioned above and as it is implemented in Fujita and Ramey (2009), the factors $\{df_t^m, df_t^c, ds_t^m, ds_t^c\}$ can be estimated, for every $t \in T$, by applying the HP filtering of data or by first differencing it. In the case of first differencing the given data, for the trend components we have $\overline{f}^j = f_{t-1}^j$ and $\overline{s}^j = s_{t-1}^j$, where $j \in \{m, c\}$.

Note that the result of Lemma 5.2 does not represent the total variation in mismatch proportion, rather it presents an expression for a *one-period* deviation of mismatch proportion from its trend. Given this, in order to obtain the expression that shows the *total* variation in mismatch proportion, we prove the following lemma:

Lemma 5.3 *The total variation in the mismatch proportion, relative to its trend, can be expressed as:*

$$\begin{aligned}
\mathrm{Var}(\mu_t) &= \mathrm{Var}(df_t^m) + \mathrm{Var}(df_t^c) + \mathrm{Var}(ds_t^m) + \mathrm{Var}(ds_t^c) + \mathrm{Var}(\varepsilon_t) + \\
&+ 2\mathrm{Cov}(df_t^m, df_t^c) + 2\mathrm{Cov}(df_t^m, ds_t^m) + 2\mathrm{Cov}(df_t^m, ds_t^c) + \\
&+ 2\mathrm{Cov}(df_t^m, \varepsilon_t) + 2\mathrm{Cov}(df_t^c, ds_t^m) + 2\mathrm{Cov}(df_t^c, ds_t^c) + \\
&+ 2\mathrm{Cov}(df_t^c, \varepsilon_t) + 2\mathrm{Cov}(ds_t^m, ds_t^c) + 2\mathrm{Cov}(ds_t^m, \varepsilon_t) + 2\mathrm{Cov}(ds_t^c, \varepsilon_t) \\
&= \mathrm{Cov}(d\mu_t, df_t^m) + \mathrm{Cov}(d\mu_t, df_t^c) + \mathrm{Cov}(d\mu_t, ds_t^m) + \mathrm{Cov}(d\mu_t, ds_t^c) + \mathrm{Cov}(d\mu_t, \varepsilon_t)
\end{aligned}$$

Proof See Appendix M

From the result of Lemma 5.3, note that $\mathrm{Var}(d\mu_t)$ is the total variation in mismatch proportion, relative to its trend value, over all $t \in T$. In addition, the parameter $\mathrm{Cov}(d\mu_t, df_t^m)$ shows the amount of total variation in $d\mu_t$ that is derived from the fluctuations in $df_t^m (= (1-\overline{\mu})(1-\overline{m})\Delta \ln f_t^m)$. More precisely, recall that the term $(1-\overline{\mu})(1-\overline{m})\Delta \ln f_t^m$ can be interpreted as a factor that represents part of the deviation of μ_t from its trend that is originated by the deviation of f_t^m from $\overline{f}^m$. Therefore, $\mathrm{Cov}(d\mu_t, df_t^m)$ can be interpreted as a component that shows the amount of total variation in mismatch proportion that is derived from the fluctuations in the rate, at which workers become mismatched[25]. Similar interpretations hold for $\mathrm{Cov}(d\mu_t, df_t^c), \mathrm{Cov}(d\mu_t, ds_t^m), \mathrm{Cov}(d\mu_t, ds_t^c)$ and $\mathrm{Cov}(d\mu_t, \varepsilon_t)$. Given the estimated df_t^j and ds_t^j for $j \in \{m, c\}$ and $d\mu_t$, estimating $\mathrm{Cov}(\cdot)$ parameters are, relatively, trivial.

For obtaining the proportions of the total variation in μ_t that are derived from the fluctuations in labour market flow rates, we modify the result of Lemma 5.3 by dividing it by $\mathrm{Var}(d\mu_t)$:

$$\frac{\mathrm{Cov}(d\mu_t, f_t^m)}{\mathrm{Var}(d\mu_t)} + \frac{\mathrm{Cov}(d\mu_t, f_t^c)}{\mathrm{Var}(d\mu_t)} + \frac{\mathrm{Cov}(d\mu_t, s_t^m)}{\mathrm{Var}(d\mu_t)} + \frac{\mathrm{Cov}(d\mu_t, s_t^c)}{\mathrm{Var}(d\mu_t)} + \frac{\mathrm{Cov}(d\mu_t, \varepsilon_t)}{\mathrm{Var}(d\mu_t)} = 1 \quad (29)$$

In fact, in their paper, Fujita and Ramey use the proportions of total variations to analyze the contributions of fluctuations in labour market flow rates on the variation in unemployment. Thus, the proportions of the variation in $d\mu_t$, which we derived above, can be interpreted as the contributions of labour market flow rates on the variation *(in our case)* of mismatch proportion.

Following Fujita and Ramey (2009)'s notation, we further denote the proportions of total variations as:

$$\frac{\mathrm{Cov}(d\mu_t, f_t^m)}{\mathrm{Var}(d\mu_t)} = \beta_m^f \qquad \qquad \frac{\mathrm{Cov}(d\mu_t, f_t^c)}{\mathrm{Var}(d\mu_t)} = \beta_c^f$$

[25]Note that, the effect of the variation in df_t^m may be the direct effect or the indirect effect through its correlations with other parameters *(e.g. the effect may be driven by* $\mathrm{Cov}(df_t^m, df_t^c)$ *or by the correlation of* df_t^m *with other parameters).*

$$\frac{\mathrm{Cov}(d\mu_t, s_t^m)}{\mathrm{Var}(d\mu_t)} = \beta_m^s \qquad \frac{\mathrm{Cov}(d\mu_t, s_t^c)}{\mathrm{Var}(d\mu_t)} = \beta_c^s$$

$$\frac{\mathrm{Cov}(d\mu_t, \varepsilon)}{\mathrm{Var}(d\mu_t)} = \beta^\varepsilon$$

Given the above notations, the expression (18) can be shown as:

$$\beta_m^f + \beta_c^f + \beta_m^s + \beta_c^s + \beta^\varepsilon = 1 \tag{30}$$

The interpretation of β_m^f is similar to the one of $\mathrm{Cov}(d\mu_t, df_t^m)$; β_m^f shows the proportion of the total variation in μ_t, relative to its trend, that is derived by the fluctuations in job finding rate of mismatched employees. As noted above, β_m^f can, also, be interptered as a contribution of f_t^m to the variation in the mismatch proportion; in other words, β_m^f shows what proportion of the variation in mismatch proportion can be explained by the fluctuations in f_t^m. Similar interpretations hold for other parameters. Moreover, Fujita and Ramer (2009) claims that β_j^i for $j \in \{m, s\}$ and $i \in \{f, s\}$, by construction, are equivalent to the concept of beta in finance. In addition to the estimation of $\mathrm{Cov}(\cdot)$, by estimating $\mathrm{Var}(d\mu_t)$, the values of β_j^i can easily be found.

Thus, by estimating the parameters of the above developed framework, one can analyze what derives the mismatch proportion to decrease, during expansions, and to increase, during recessions. In order to understand the implications of the framework better, consider the following example.

As we mentioned, due to its positive effect on the *one-period* deviation in mismatch proportion, relative to its trend, f_t^m must follow the same pattern as μ_t, so a counter-cyclical pattern or it must be acyclic. Suppose it is counter-cyclical, then, if, say, β_m^f was large, relative to other beta parameters, it would indicate that the big proportion of the decrease in μ_t, during expansions, is derived from the decrease in f_t^m (and the converse for expansions). Thus, if β_m^f is large, relative to other betas, one can claim that job finding rates of mismatched workers are more important in explaining the business cycle fluctuations in μ_t, as a big proportion of the variation in μ_t is derived by the fluctuations in f_t^m. Similar examples can be constructed for other beta parameters and labour market flow rates.

6 Conclusion

This paper analyzed the concepts of job finding probabilities and educational mismatch in the UK labour market, for the period of 1996-2015.

Regarding job finding probabilities, we presented results that analyzed the cyclical properties of probabilities and showed how the educational heterogeneity among workers affects the average reemployment probabilities and their cyclical properties. Our results indicate job finding probabilities are procyclical, with the procyclical pattern being preserved in each analysed education group. In addition, we found that, starting from the City and Guilds level of qualifications, the average job finding probabilities increase up to Bachelor's level of education, but declines for Master's degree holders, which may indicate a possible concave relation between job finding probabilities and educational attainments.

On the educational mismatch side, we analysed the implications of various personal characteristics of workers on the incidences of over- and under-education. In addition to other findings, we showed that higher work experience decreases the incidence of over-education, but increases the incidence of under-education, which was claimed to be in line with the human capital theory explanation of educational mismatch. Regarding the cyclical properties, we found that the proportion of over-education is acyclic, the proportions of under-education and mismatch follow counter-cyclical movements and the business cycle movement of the proportion of correct match is procyclical.

Finally, in the last section, we linked the concepts of labour market flows and educational mismatch together, and presented a mathematical framework that analyzed the dynamics in the mismatch proportion, in terms of the fluctuations in labour market flow rates.

A Appendix. The Plot of the Job Finding Probabilities, without Educational Heterogeneity

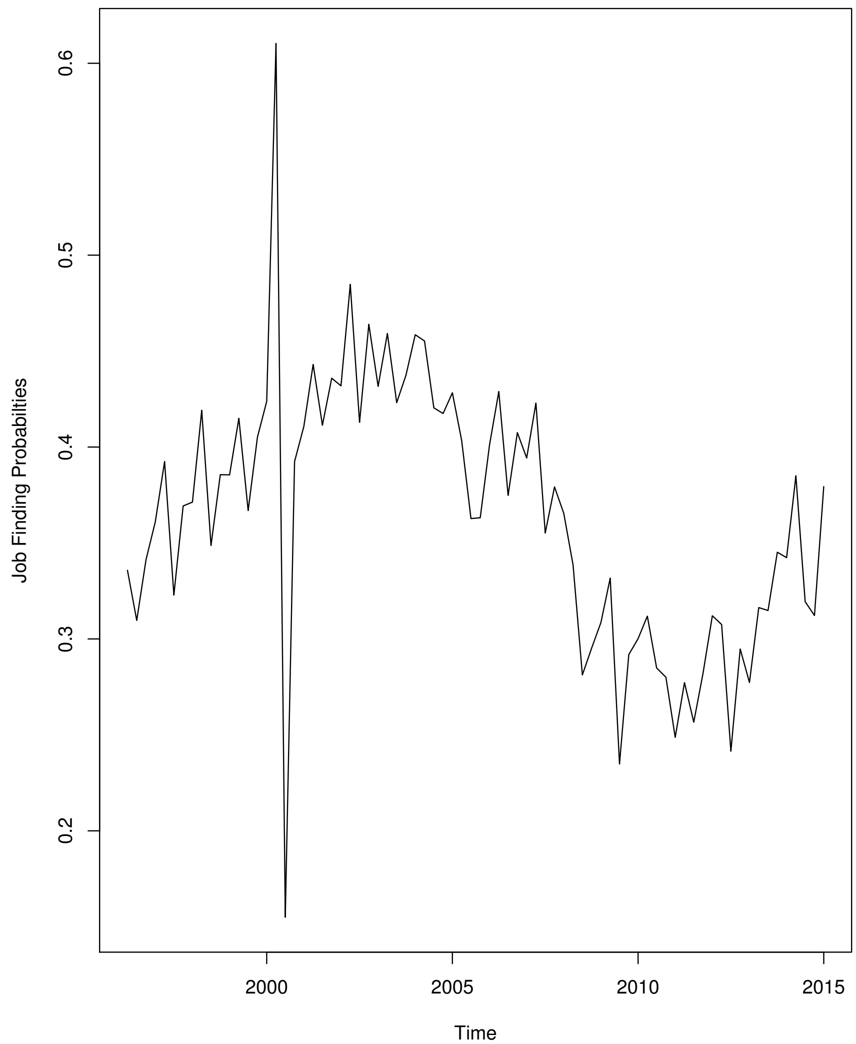

Figure 1: Job Finding Probabilities (No Educational Heterogeneity)

B Appendix. OLS Cyclicality Regression Results: Job Finding Probabilities, without Educational Heterogeneity (numbers in brackets indicate standard errors)

	Log of Job Finding Probability ($\log(F_t)$)
Unemployment Rate	-0.1067***
	(0.0128)
Linear Time Trend	-0.0018***
	(0.0006)
Winter	0.1214***
	(0.0417)
Spring	0.1367***
	(0.0411)
Summer	0.2083***
	(0.0417)
Constant	-0.4090***
	0.0815
R^2	0.6381
Adjusted R^2	0.6126
F statistic	25.03

C Appendix. The Plot of the Job Finding Probabilities, with Educational Heterogeneity

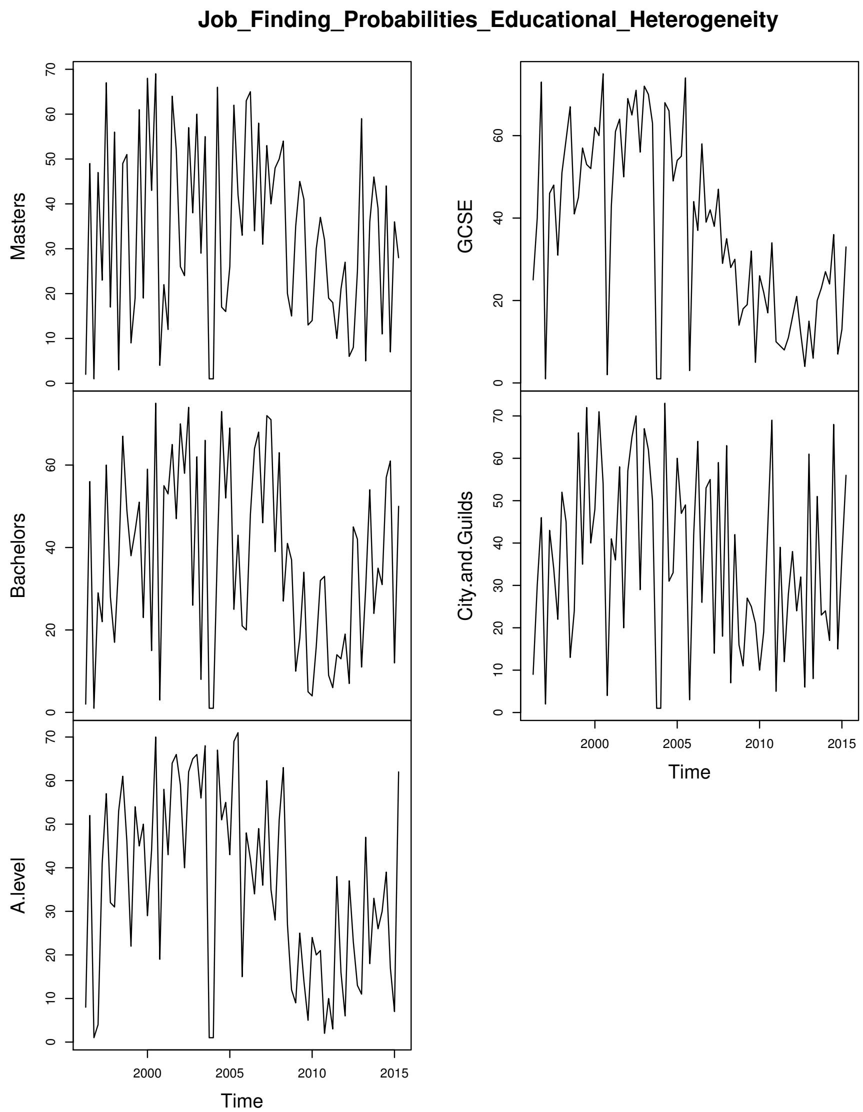

Figure 2: Job Finding Probabilities (with Educational Heterogeneity)

D Appendix. OLS Cyclicality Regression Results: Job Finding Probabilities, with Educational Heterogeneity (numbers in brackets indicate standard errors)

	Master's	*Bachelor's*	*A-level*	*GCSE*	*City and Guilds*
Unemployment Rate	-0.0620***	-0.0885***	-0.1368***	-0.0910***	-0.1000***
	(0.0305**)	(0.0147**)	(0.0241**)	(0.0206**)	(0.0482**)
Linear Time Trend	-0.0009***	0.0004***	-0.0017***	-0.0055***	0.0004***
	(0.0016**)	(0.0007**)	(0.0012**)	(0.0011**)	(0.0025**)
Winter	0.1618***	0.0224***	0.0604***	0.1646***	0.1557***
	(0.0952**)	(0.0481**)	(0.0768**)	(0.0681**)	(0.1589**)
Spring	0.0087***	-0.0408***	0.1962***	0.1602***	0.2320***
	(0.0939**)	(0.0469**)	(0.0767**)	(0.0653**)	(0.1549**)
Summer	0.2012***	0.1855***	0.2403***	0.2518***	0.3611***
	(0.0941**)	(0.0475**)	(0.0777**)	(0.0661**)	(0.1568**)
Constant	-0.5334***	-0.2628***	-0.0314***	-0.3496***	-0.8240
	(0.1873**)	(0.0929**)	(0.1518**)	(0.1318**)	(0.3112**)
R^2	0.1682	0.4915	0.4568	0.5471	0.1278
Adjusted R^2	0.1062	0.4541	0.4168	0.5138	0.0646
F statistic	2.71	13.14	11.43	16.43	2.02

E Appendix. Logit Results: Over-/under-education and personal characteristics (numbers in brackets indicate standard errors)

	Over-education	*Under-education*	*Correct Match*
Experience	-0.0284***	0.0173***	0.0070***
	(0.0003**)	(0.0003**)	(0.00021**)
Education Years	0.6072***	-1.477***	-0.1210***
	(0.0016**)	(0.00626**)	(0.00093**)
Male	0.4766***	0.1087***	-0.217***
	(0.00692**)	(0.00698**)	(0.00475**)
Married	-0.2150***	0.2764***	-0.0335***
	(0.00741**)	(0.00782**)	(0.00520**)
Tenure	-0.0622***	-0.3199***	0.2023***
	(0.001480**)	(0.01735**)	(0.01065**)
Job Training*(=1, if received job training)*	-0.2960***	0.3092***	0.0341***
	(0.00851**)	(0.0091**)	(0.00603**)
Regions of residence	...	...	...
...	...	...	...
Log-Likelyhood	-297775.77	-289047.64	-586629.52
Pseudo R^2	0.2863	0.2449	0.0200

F Appendix. Logit Results: Over-/under-education and the On-the-job search (numbers in brackets indicate standard errors)

	Whether Looking for a Different Job
Over-educated	0.4410***
	(0.0123**)
Under-educated	-0.103***
	(0.0143**)
Experience	-0.0249***
	(0.00034**)
Education years	-0.0346***
	(0.00204**)
Male	0.00719***
	(0.00846**)
Married	-0.1284***
	(0.0090**)
Tenured	-1.257***
	(0.01296**)
Job Training*(=1, if received job training)*	0.0594***
	(0.01048**)
Regions of residence	...
...	...
Log-Likelyhood	-237733.41
Pseudo R^2	0.0355

G Appendix. The Plot of the Proportion of Mismatched Employees

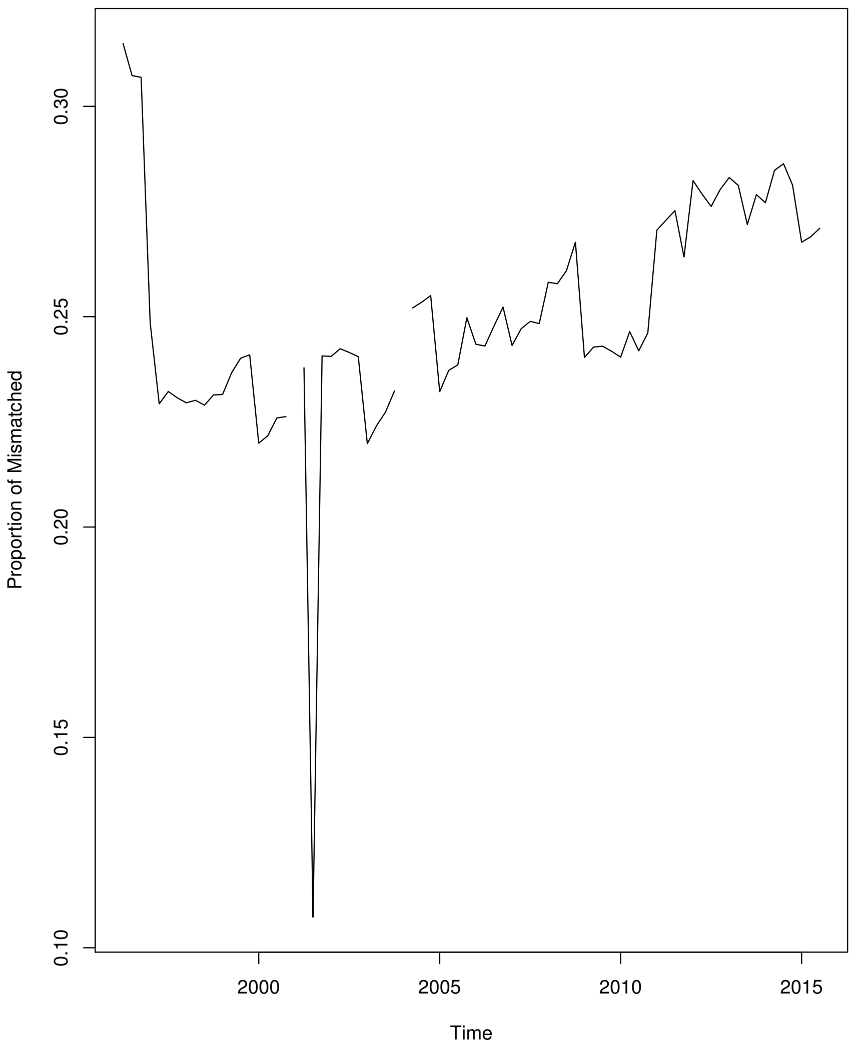

Figure 3: Proportion of Mismatch Employees

H Appendix. The Plot of the Proportion of Over-educated Employees

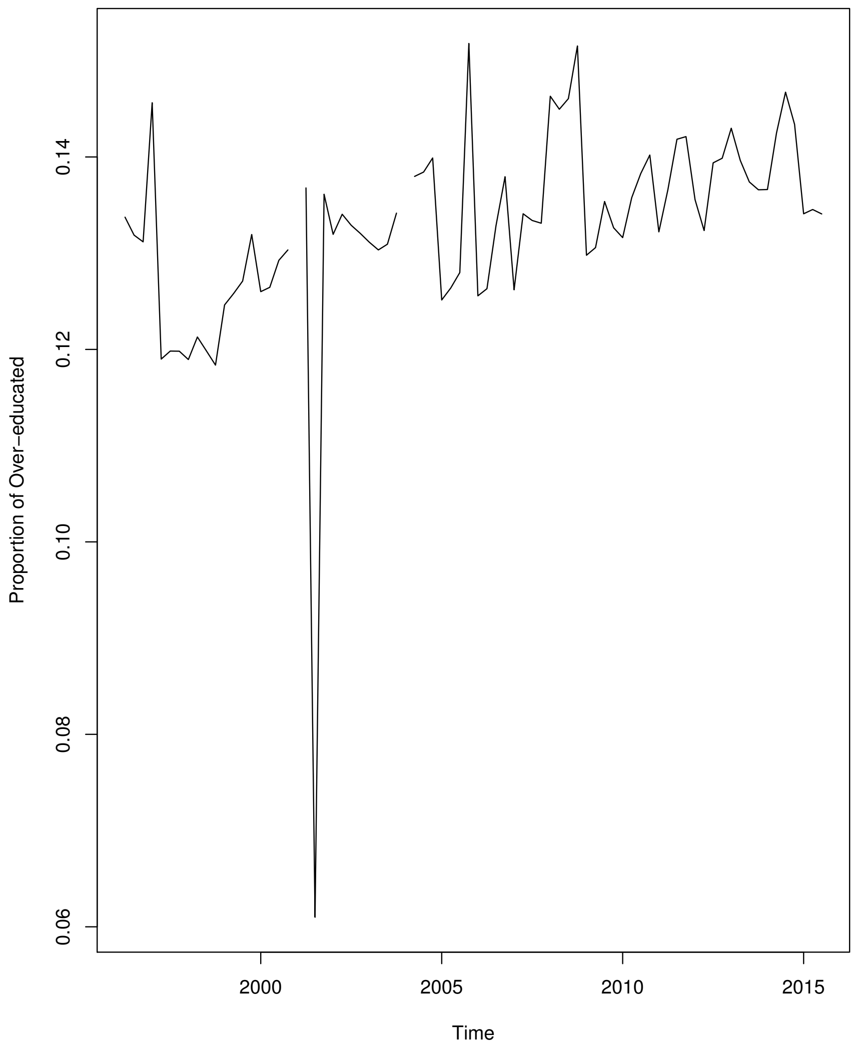

Figure 4: Proportion of Over-educated Employees

I Appendix. The Plot of the Proportion of Under-educated Employees

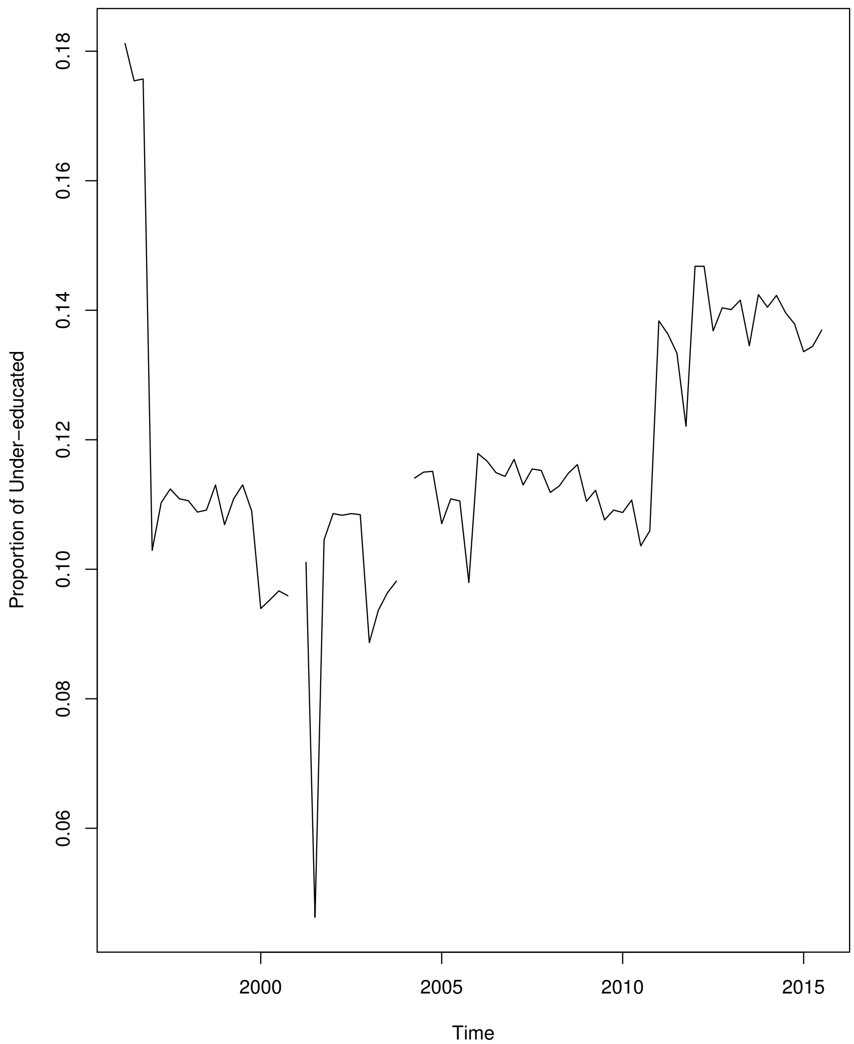

Figure 5: Proportion of Under-educated Employees

J Appendix. OLS Cyclicality Regression Results: Mismatch (numbers in brackets indicate standard errors)

	Over-education	*Under-education*	*Mismatch*	*Match*
Unemployment Rate	0.0051***	0.0684***	0.0362***	-0.0120***
	(0.01005**)	(0.01578**)	(0.01149**)	(0.00304**)
Linear Time Trend	0.0017***	0.0016***	0.0016***	-0.00048***
	(0.00053**)	(0.0.00084**)	(0.0061**)	(0.00016**)
Winter	-0.0342***	-0.01801***	-0.0275***	0.0090***
	(0.03331**)	(0.05230**)	(0.03808**)	(0.1007**)
Spring	-0.0256***	0.0167***	-0.0062***	0.0017***
	(0.03194**)	(0.05013**)	(0.03650**)	(0.00965**)
Summer	-0.0549***	-0.039***	0.2518***	0.0104***
	(0.03235**)	(0.05078**)	(0.0661)	(0.00978**)
Constant	-2.0975***	-2.645***	-1.665***	-0.1988**
	(0.0641**)	(0.10064**)	(0.07328**)	(0.01938**)
R^2	0.1875	0.3162	0.2753	0.3424
Adjusted R^2	0.1287	0.2228	0.5138	0.2947
F statistic	3.19	6.38	5.24	7.18

K Appendix. Proof of Proposition 5.1

Proposition 5.1 *The mismatch proportion can be represented as:*

$$\mu_t = \frac{f_t^m(f_t^c + s_t^c)}{f_t^m(f_t^c + s_t^c) + f_t^c(f_t^m + s_t^m)}$$

Proof Recall that:

$$\mu_t = \frac{M_t}{E_t}$$

Given that $E_t = M_t + C_t$, we have the following:

$$\mu_t = \frac{M_t}{M_t + C_t}$$

$$\mu_t = \frac{\frac{M_t}{L_t}}{\frac{M_t}{L_t} + \frac{C_t}{L_t}}$$

$$\mu_t = \frac{m_t}{m_t + c_t}$$

From the definitions of m_t and c_t, defined as in (16) and (17), we have that:

$$\mu_t = \frac{\frac{f_t^m}{f_t^m + s_t^m}}{\frac{f_t^m}{f_t^m + s_t^m} + \frac{f_t^c}{f_t^c + s_t^c}}$$

Modifying further the above expression completes the proof and yields to:

$$\mu_t = \frac{f_t^m(f_t^c + s_t^c)}{f_t^m(f_t^c + s_t^c) + f_t^c(f_t^m + s_t^m)} \quad \blacksquare$$

L Appendix. Proof of Lemma 5.2

Lemma 5.2 *A one-period deviation of mismatch proportion from its trend, can be expressed as:*

$$\Delta \ln \mu_t = (1-\overline{\mu})(1-\overline{m})\Delta \ln f_t^m - (1-\overline{\mu})(1-\overline{c})\Delta \ln f_t^c - (1-\overline{\mu})(1-\overline{m})\Delta \ln s_t^m + (1-\overline{\mu})(1-\overline{c})\Delta \ln s_t^c + \varepsilon_t$$

where ε_t denotes the error term.

Proof Using the result of Proposition 5.1 and applying the Taylor Series expansion of μ_t, around the defined trend values, yields to:

$$\mu_t = \overline{\mu} + \frac{\partial \overline{\mu}}{\partial \overline{f}^m}(f_t^m - \overline{f}^m) + \frac{\partial \overline{\mu}}{\partial \overline{f}^c}(f_t^c - \overline{f}^c) + \frac{\partial \overline{\mu}}{\partial \overline{s}^m}(s_t^m - \overline{s}^m) + \frac{\partial \overline{\mu}}{\partial \overline{s}^c}(s_t^c - \overline{s}^c) + \varepsilon_t$$

where ε_t is the error term that captures the reminder term.

We modify the above expression further as follows:

$$\frac{\mu_t - \overline{\mu}}{\overline{\mu}} = \frac{\partial \overline{\mu}}{\partial \overline{f}^m}\frac{\overline{f}^m}{\overline{\mu}}\left[\frac{f_t^m - \overline{f}^m}{\overline{f}^m}\right] + \frac{\partial \overline{\mu}}{\partial \overline{f}^c}\frac{\overline{f}^c}{\overline{\mu}}\left[\frac{f_t^c - \overline{f}^c}{\overline{f}^c}\right] + \frac{\partial \overline{\mu}}{\partial \overline{s}^m}\frac{\overline{s}^m}{\overline{\mu}}\left[\frac{s_t^m - \overline{s}^m}{\overline{s}^m}\right] + \frac{\partial \overline{\mu}}{\partial \overline{s}^c}\frac{\overline{s}^c}{\overline{\mu}}\left[\frac{s_t^c - \overline{s}^c}{\overline{s}^c}\right] + \varepsilon_t \tag{31}$$

Before modifying the expression (31) further, we will present a definition of the log-linearisation procedure. Log-linearisation of some variabe ϕ_t (in our case $\phi \in \{\mu, f^m, f^c, s^m, s^c\}$) implies taking the log-deviation of that variable around its trend value ($\overline{\phi}$), where log-deviation of ϕ_t around $\overline{\phi}$ is defined as:

$$\Delta \ln \phi_t \equiv \ln \phi_t - \ln \overline{\phi} \tag{32}$$

However, note that:

$$\ln \phi_t - \ln \overline{\phi} = \ln\left(\frac{\phi_t}{\overline{\phi}}\right)$$

Then, using the first-order Taylor expansion of ϕ_t, around $\overline{\phi}$, we obtain that:

$$\ln\left(\frac{\phi_t}{\overline{\phi}}\right) \approx \ln 1 + \frac{1}{\overline{\phi}}(\phi_t - \overline{\phi}) = \frac{\phi_t - \overline{\phi}}{\overline{\phi}}$$

Thus, we have that:

$$\Delta \ln \phi_t \approx \frac{\phi_t - \overline{\phi}}{\overline{\phi}} \tag{33}$$

Using (33) we can modify (31) further as follows:

$$\Delta \ln \mu_t = \frac{\partial \overline{\mu}}{\partial \overline{f}^m} \frac{\overline{f}^m}{\overline{\mu}} \Delta \ln f_t^m + \frac{\partial \overline{\mu}}{\partial \overline{f}^c} \frac{\overline{f}^c}{\overline{\mu}} \Delta \ln f_t^c + \frac{\partial \overline{\mu}}{\partial \overline{s}^m} \frac{\overline{s}^m}{\overline{\mu}} \Delta \ln s_t^m + \frac{\partial \overline{\mu}}{\partial \overline{s}^c} \frac{\overline{s}^c}{\overline{\mu}} \Delta \ln s_t^c + \varepsilon_t \tag{34}$$

Recall that:

$$\overline{\mu} = \frac{\overline{f}^m(\overline{f}^c + \overline{s}^c)}{\overline{f}^m(\overline{f}^c + \overline{s}^c) + \overline{f}^c(\overline{f}^m + \overline{s}^m)}$$

Given the above expression, the partial derivatives of $\overline{\mu}$ w.r.t. other variables are:

$$\frac{\partial \overline{\mu}}{\partial \overline{f}^m} = \frac{\overline{f}^{c2}\overline{s}^m + \overline{s}^c\overline{f}^c\overline{s}^m}{(\overline{f}^m(\overline{f}^c + \overline{s}^c) + \overline{f}^c(\overline{f}^m + \overline{s}^m))^2} \tag{35}$$

$$\frac{\partial \overline{\mu}}{\partial \overline{f}^c} = -\Big[\frac{\overline{f}^{m2}\overline{s}^c - \overline{s}^c\overline{f}^m\overline{s}^m}{(\overline{f}^m(\overline{f}^c + \overline{s}^c) + \overline{f}^c(\overline{f}^m + \overline{s}^m))^2}\Big] \tag{36}$$

$$\frac{\partial \overline{\mu}}{\partial \overline{s}^m} = -\frac{\overline{f}^c\overline{f}^m(\overline{f}^c + \overline{s}^c)}{\overline{f}^m(\overline{f}^c + \overline{s}^c) + \overline{f}^c(\overline{f}^m + \overline{s}^m))^2} \tag{37}$$

$$\frac{\partial \overline{\mu}}{\partial \overline{s}^c} = \frac{\overline{f}^{m2}\overline{f}^c + \overline{s}^m\overline{f}^c\overline{f}^m}{(\overline{f}^m(\overline{f}^c + \overline{s}^c) + \overline{f}^c(\overline{f}^m + \overline{s}^m))^2} \tag{38}$$

For notational simplicity define:

$$\Omega = \overline{f}^m(\overline{f}^c + \overline{s}^c) + \overline{f}^c(\overline{f}^m + \overline{s}^m)$$

Then, substituting the expressions for partial derivatives into (34) implies:

$$\begin{aligned}\Delta \ln \mu_t = \Big[\frac{\overline{f}^{c2}\overline{s}^m + \overline{s}^c\overline{f}^c\overline{s}^m}{\Omega^2}\Big]\frac{\overline{f}^m}{\overline{\mu}}\Delta \ln f_t^m - \Big[\frac{\overline{f}^{m2}\overline{s}^c - \overline{s}^c\overline{f}^m\overline{s}^m}{\Omega^2}\Big]\frac{\overline{f}^c}{\overline{\mu}}\Delta \ln f_t^c - \\ - \Big[\frac{\overline{f}^c\overline{f}^m(\overline{f}^c + \overline{s}^c)}{\Omega^2}\Big]\frac{\overline{s}^m}{\overline{\mu}}\Delta \ln s_t^m + \Big[\frac{\overline{f}^{m2}\overline{f}^c + \overline{s}^m\overline{f}^c\overline{f}^m}{\Omega^2}\Big]\frac{\overline{s}^c}{\overline{\mu}}\Delta \ln s_t^c + \varepsilon_t\end{aligned}$$

Modifying further and cancelling some terms out yields to:

$$\begin{aligned}\Delta \ln \mu_t = \left[\frac{\overline{f}^c \overline{s}^m}{\Omega}\right] \Delta \ln f_t^m - \left[\frac{\overline{f}^c (\overline{f}^m + \overline{s}^m)}{\Omega}\right] \left[\frac{\overline{s}^c}{\overline{f}^c + \overline{s}^c}\right] \Delta \ln f_t^c - \\ - \left[\frac{\overline{f}^c \overline{s}^m}{\Omega}\right] \Delta \ln s_t^m + \left[\frac{\overline{f}^c (\overline{f}^m + \overline{s}^m)}{\Omega}\right] \left[\frac{\overline{s}^c}{\overline{f}^c + \overline{s}^c}\right] \Delta \ln s_t^c + \varepsilon_t\end{aligned} \tag{39}$$

Note that by definition $\overline{c} = \frac{\overline{f}^c}{\overline{f}^c + \overline{s}^c}$, then:

$$\frac{\overline{s}^c}{\overline{f}^c + \overline{s}^c} = 1 - \overline{c} \tag{40}$$

In addition, by definition of $\overline{\mu}$ we have that:

$$\frac{\overline{f}^c (\overline{f}^m + \overline{s}^m)}{\Omega} = 1 - \overline{\mu} \tag{41}$$

For the factor $\frac{\overline{f}^c \overline{s}^m}{\Omega}$ we have the following:

$$\begin{aligned}\frac{\overline{f}^c \overline{s}^m}{\Omega} &= \frac{\overline{f}^c \overline{s}^m + \overline{f}^m \overline{f}^c - \overline{f}^m \overline{f}^c}{\Omega} \\ &= \frac{\overline{f}^c (\overline{f}^m + \overline{s}^m)}{\Omega} - \frac{\overline{f}^m \overline{f}^c}{\Omega} \\ &= (1 - \overline{\mu}) - \left[\frac{\overline{f}^m \overline{f}^c}{\overline{f}^m (\overline{f}^c + \overline{s}^c) + \overline{f}^c (\overline{f}^m + \overline{s}^m)}\right] \\ &= (1 - \overline{\mu}) - \left[\frac{\frac{\overline{f}^m}{(\overline{f}^m + \overline{s}^m)} \frac{\overline{f}^c}{(\overline{f}^c + \overline{s}^c)}}{\frac{\overline{f}^m}{(\overline{f}^m + \overline{s}^m)} + \frac{\overline{f}^c}{(\overline{f}^c + \overline{s}^c)}}\right] \\ &= (1 - \overline{\mu}) - \left[\frac{\overline{mc}}{\overline{m} + \overline{c}}\right] \\ &= (1 - \overline{\mu}) - \frac{\overline{c}}{\overline{e}} \overline{m} \\ &= (1 - \overline{\mu}) - \left[\frac{\overline{e} - \overline{m}}{\overline{e}}\right] \overline{m}\end{aligned}$$

However, note that:

$$\begin{aligned}
\frac{\overline{e}-\overline{m}}{\overline{e}} &= 1-\frac{\overline{m}}{\overline{e}} \\
&= 1-\frac{\frac{\overline{f}^m}{\overline{f}^m+\overline{s}^m}}{\frac{\overline{f}^m}{\overline{f}^m+\overline{s}^m}+\frac{\overline{f}^c}{\overline{f}^c+\overline{s}^c}} \\
&= \frac{\overline{f}^c(\overline{f}^m+\overline{s}^m)}{\Omega} \\
&= (1-\overline{\mu})
\end{aligned}$$

By substituting the above finding into the expression for $\frac{\overline{f}^c\overline{s}^m}{\Omega}$, we obtain that:

$$\frac{\overline{f}^c\overline{s}^m}{\Omega} = (1-\overline{\mu})(1-\overline{m}) \tag{42}$$

Finally, substituting (40), (41) and (42) into (39) for the corresponding factors completes the proof, by yielding to:

$$\Delta \ln \mu_t = (1-\overline{\mu})(1-\overline{m})\Delta \ln f_t^m - (1-\overline{\mu})(1-\overline{c})\Delta \ln f_t^c - (1-\overline{\mu})(1-\overline{m})\Delta \ln s_t^m + (1-\overline{\mu})(1-\overline{c})\Delta \ln s_t^c + \varepsilon_t \quad \blacksquare \tag{43}$$

M Appendix. Proof of Lemma 5.3

Lemma 5.3 *The total variation in the mismatch proportion, relative to its trend, can be expressed as:*

$$
\begin{aligned}
\mathrm{Var}(\mu_t) &= \mathrm{Var}(df_t^m) + \mathrm{Var}(df_t^c) + \mathrm{Var}(ds_t^m) + \mathrm{Var}(ds_t^c) + \mathrm{Var}(\varepsilon_t) + \\
&+ 2\mathrm{Cov}(df_t^m, df_t^c) + 2\mathrm{Cov}(df_t^m, ds_t^m) + 2\mathrm{Cov}(df_t^m, ds_t^c) + \\
&+ 2\mathrm{Cov}(df_t^m, \varepsilon_t) + 2\mathrm{Cov}(df_t^c, ds_t^m) + 2\mathrm{Cov}(df_t^c, ds_t^c) + \\
&+ 2\mathrm{Cov}(df_t^c, \varepsilon_t) + 2\mathrm{Cov}(ds_t^m, ds_t^c) + 2\mathrm{Cov}(ds_t^m, \varepsilon_t) + 2\mathrm{Cov}(ds_t^c, \varepsilon_t) \\
&= \mathrm{Cov}(d\mu_t, df_t^m) + \mathrm{Cov}(d\mu_t, df_t^c) + \mathrm{Cov}(d\mu_t, ds_t^m) + \mathrm{Cov}(d\mu_t, ds_t^c) + \mathrm{Cov}(d\mu_t, \varepsilon_t)
\end{aligned}
$$

Proof Using the defitions of $\mathrm{Var}(\cdot)$ and $\mathrm{Cov}(\cdot)$, we modify the first part of the above expression as follows:

$$
\begin{aligned}
\mathrm{Var}(\mu_t) = {} & \mathbb{E}[df_t^{m2}] - \left(\mathbb{E}[df_t^m]\right)^2 + \mathbb{E}[df_t^{c2}] - \left(\mathbb{E}[df_t^c]\right)^2 + \mathbb{E}[ds_t^{m2}] - \left(\mathbb{E}[ds_t^m]\right)^2 + \\
& + \mathbb{E}[ds_t^{c2}] - \left(\mathbb{E}[ds_t^c]\right)^2 + \mathbb{E}[{\varepsilon_t}^2] - \left(\mathbb{E}[\varepsilon_t]\right)^2 + \mathbb{E}[df_t^m df_t^c] + \mathbb{E}[df_t^m df_t^c] - \\
& - \mathbb{E}[df_t^m]\mathbb{E}[df_t^c] - \mathbb{E}[df_t^m]\mathbb{E}[df_t^c] + \mathbb{E}[df_t^m ds_t^m] + \mathbb{E}[df_t^m ds_t^m] - \mathbb{E}[df_t^m]\mathbb{E}[ds_t^m] - \\
& - \mathbb{E}[df_t^m]\mathbb{E}[ds_t^m] + \mathbb{E}[df_t^m ds_t^c] + \mathbb{E}[df_t^m ds_t^c] - \mathbb{E}[df_t^m]\mathbb{E}[ds_t^c] - \mathbb{E}[df_t^m]\mathbb{E}[ds_t^c] + \\
& + \mathbb{E}[df_t^m \varepsilon_t] + \mathbb{E}[df_t^m \varepsilon_t] - \mathbb{E}[df_t^m]\mathbb{E}[\varepsilon_t] - \mathbb{E}[df_t^m]\mathbb{E}[\varepsilon_t] + \mathbb{E}[df_t^c ds_t^m] + \\
& + \mathbb{E}[df_t^c ds_t^m] - \mathbb{E}[df_t^c]\mathbb{E}[ds_t^m] - \mathbb{E}[df_t^c]\mathbb{E}[ds_t^m] + \mathbb{E}[df_t^c ds_t^c] + \mathbb{E}[df_t^c ds_t^c] - \\
& - \mathbb{E}[df_t^c]\mathbb{E}[ds_t^c] - \mathbb{E}[df_t^c]\mathbb{E}[ds_t^c] + \mathbb{E}[df_t^c \varepsilon_t] + \mathbb{E}[df_t^c \varepsilon_t] - \mathbb{E}[df_t^c]\mathbb{E}[\varepsilon_t] - \\
& - \mathbb{E}[df_t^c]\mathbb{E}[\varepsilon_t] + \mathbb{E}[ds_t^m ds_t^c] + \mathbb{E}[ds_t^m ds_t^c] - \mathbb{E}[ds_t^m]\mathbb{E}[ds_t^c] - \mathbb{E}[ds_t^m]\mathbb{E}[ds_t^c] + \\
& + \mathbb{E}[ds_t^m \varepsilon_t] + \mathbb{E}[ds_t^m \varepsilon_t] - \mathbb{E}[ds_t^m]\mathbb{E}[\varepsilon_t] - \mathbb{E}[ds_t^m]\mathbb{E}[\varepsilon_t] + \mathbb{E}[ds_t^c \varepsilon_t] + \\
& + \mathbb{E}[ds_t^c \varepsilon_t] - \mathbb{E}[ds_t^c]\mathbb{E}[\varepsilon_t] - \mathbb{E}[ds_t^c]\mathbb{E}[\varepsilon_t]
\end{aligned}
$$

Rearranging the relevant terms together yields to:

$$\begin{aligned}\mathrm{Var}(\mu_t) = &\Big\langle \mathbb{E}[df_t^{m2}] + \mathbb{E}[df_t^m df_t^c] + \mathbb{E}[df_t^m ds_t^m] + \mathbb{E}[df_t^m ds_t^c] + \mathbb{E}[df_t^m \varepsilon_t] \Big\rangle - \\
&- \Big\langle \big(\mathbb{E}[df_t^m]\big)^2 + \mathbb{E}[df_t^m]\mathbb{E}[df_t^c] + \mathbb{E}[df_t^m]\mathbb{E}[ds_t^m] + \mathbb{E}[df_t^m]\mathbb{E}[ds_t^c] + \mathbb{E}[df_t^m]\mathbb{E}[\varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[df_t^{c2}] + \mathbb{E}[df_t^m df_t^c] + \mathbb{E}[df_t^c ds_t^m] + \mathbb{E}[df_t^c ds_t^c] + \mathbb{E}[df_t^c \varepsilon_t] \Big\rangle - \\
&- \Big\langle \big(\mathbb{E}[df_t^c]\big)^2 + \mathbb{E}[df_t^m]\mathbb{E}[df_t^c] + \mathbb{E}[df_t^c]\mathbb{E}[ds_t^m] + \mathbb{E}[df_t^c]\mathbb{E}[ds_t^c] + \mathbb{E}[df_t^c]\mathbb{E}[\varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[ds_t^{m2}] + \mathbb{E}[ds_t^m df_t^c] + \mathbb{E}[ds_t^m df_t^m] + \mathbb{E}[ds_t^m ds_t^c] + \mathbb{E}[ds_t^m \varepsilon_t] \Big\rangle - \\
&- \Big\langle \big(\mathbb{E}[ds_t^m]\big)^2 + \mathbb{E}[ds_t^m]\mathbb{E}[df_t^c] + \mathbb{E}[df_t^m]\mathbb{E}[ds_t^m] + \mathbb{E}[ds_t^m]\mathbb{E}[ds_t^c] + \mathbb{E}[ds_t^m]\mathbb{E}[\varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[ds_t^{c2}] + \mathbb{E}[df_t^m ds_t^c] + \mathbb{E}[ds_t^c ds_t^m] + \mathbb{E}[df_t^c ds_t^c] + \mathbb{E}[ds_t^c \varepsilon_t] \Big\rangle - \\
&- \Big\langle \big(\mathbb{E}[ds_t^c]\big)^2 + \mathbb{E}[df_t^m]\mathbb{E}[ds_t^c] + \mathbb{E}[ds_t^c]\mathbb{E}[ds_t^m] + \mathbb{E}[df_t^c]\mathbb{E}[ds_t^c] + \mathbb{E}[ds_t^c]\mathbb{E}[\varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[{\varepsilon_t}^2] + \mathbb{E}[df_t^m \varepsilon_t] + \mathbb{E}[\varepsilon_t ds_t^m] + \mathbb{E}[df_t^c \varepsilon_t] + \mathbb{E}[ds_t^c \varepsilon_t] \Big\rangle - \\
&- \Big\langle \big(\mathbb{E}[\varepsilon_t]\big)^2 + \mathbb{E}[df_t^m]\mathbb{E}[\varepsilon_t] + \mathbb{E}[\varepsilon_t]\mathbb{E}[ds_t^m] + \mathbb{E}[df_t^c]\mathbb{E}[\varepsilon_t] + \mathbb{E}[ds_t^c]\mathbb{E}[\varepsilon_t] \Big\rangle\end{aligned}$$

$$\begin{aligned}\mathrm{Var}(\mu_t) = &\Big\langle \mathbb{E}[df_t^m(df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t)] \Big\rangle - \Big\langle \mathbb{E}[df_t^m]\mathbb{E}[df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[df_t^c(df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t)] \Big\rangle - \Big\langle \mathbb{E}[df_t^c]\mathbb{E}[df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[ds_t^m(df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t)] \Big\rangle - \Big\langle \mathbb{E}[ds_t^m]\mathbb{E}[df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[ds_t^c(df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t)] \Big\rangle - \Big\langle \mathbb{E}[ds_t^c]\mathbb{E}[df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t] \Big\rangle + \\
&+ \Big\langle \mathbb{E}[\varepsilon_t(df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t)] \Big\rangle - \Big\langle \mathbb{E}[\varepsilon_t]\mathbb{E}[df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t] \Big\rangle\end{aligned}$$

Using the definition of $\mathrm{Cov}(\cdot)$ and noting that $df_t^m + df_t^c + ds_t^m + ds_t^c + \varepsilon_t = d\mu_t$ and substituting it in the above expression completes the proof:

$$\mathrm{Var}(\mu_t) = \mathrm{Cov}(d\mu_t, df_t^m) + \mathrm{Cov}(d\mu_t, df_t^c) + \mathrm{Cov}(d\mu_t, ds_t^m) + \mathrm{Cov}(d\mu_t, ds_t^c) + \mathrm{Cov}(d\mu_t, \varepsilon_t) \quad \blacksquare \tag{44}$$

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