A Comment on Entrpreneur or Manager: Who Really Runs the Firm?

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ENTREPRENEUR OR MANAGER: WHO REALLY RUNS THE FIRM?

Abstract

In this journal De Fraja studied the choice problem faced by the owner of a valuable project. He could run the project himself or sell it to an outside investor and work as a manager. The main result was that both cases are possible. Numerical simulation was used for the derivation. Unfortunately the results are flawed. It is possible to prove that it is always optimal for the owner to work as a manager. In the following corrected results will be presented. Moreover, it will be indicated under what additional constraints the results claimed by De Fraja will hold.

1. Introduction

In a recent paper in this journal De Fraja [1996] studied the choice problem faced by the owner of a valuable project. Basically the owner may run the firm as an entrepreneur or sell it to an outside investor and work as a manager.

De Fraja claimed that either structure could be preferred depending on the probability of bad states of nature and/or the amount of capital needed. This result certainly corresponds to empirical findings since in general both structures coexist. The results were derived by means of numerical simulation. Unfortunately, the computations performed are flawed. Under the assumptions made in the paper it is not possible to arrive at the conclusions presented. In fact, it is always optimal to sell the firm to an outside investor. This result is a bit unsatisfactory since it contradicts reality. A generalization that allows to derive the desired results will be presented.

2. Optimal Behavior of Entrepreneur

2.1 The Model

It is assumed that the owner is equipped with a utility function given by

(1) $U(c,l) = (c^{\alpha} l^{1-\alpha})^{r} \qquad \alpha, r \in (0,1).$

In (1) c denotes consumption measured in monetary terms, l is leisure. Note that the owner is risk averse since U is strictly concave. The choice of ownership structure occurs in conditions of uncertainty. The project consists of the sale of a product on a market. The inverse demand is given by

(2) $p = \theta - q, \qquad \theta \in \{\theta_L, \theta_H\}, \quad \theta_H > \theta_L > 0.$

It is assumed that low demand θ_L occurs with probability x. High demand θ_H then has probability 1-x. Total cost is given by

(3) $C(q) = lq.$

Hence, costs may be decreased by the owner or manager if effort is exerted. To finance the project a capital investment of $K > 0$ is needed. In case of entrepreneurship debt will be raised, otherwise, K is equity provided by an outside investor.

The entrepreneur makes two decisions. The first decision is about ownership structure. The state of nature is not known to the owner and potential investors before this decision is made. This assumption allows to rule out problems associated with asymmetric information. The second decision is about the effort exerted as entrepreneur or manager. Before this decision is made the state of nature is revealed. Hence, there is no uncertainty with respect to production decisions.

As was already noted it is assumed that the entrepreneur is able to repay debt regardless of the state of nature. This requires the assumption

(4) $\frac{\theta_L^2}{4} > K.$

In case of running the firm as an entrepreneur the individual chooses l and c according to the optimization problem

(5) $\max_{c,l}\left\{(c^{\alpha} l^{1-\alpha})^{r}\right\}$

under the constraint

(6) $c \le q(\theta - q - l) - K.$

Since debt may be repaid in both states of nature we may assume that the risk free interest rate is zero. Maximization yields the necessary (and sufficient) conditions

(7) $q = \frac{\theta - l}{2},$

(8) $l = \frac{\theta - \sqrt{\alpha^2\theta^2 + 4(1-\alpha^2)K}}{1+\alpha}.$

By condition (4), equation (8) yields optimal leisure in both cases $\theta = \theta_L, \theta_H$. Also, debt K may be repaid. From (6) the optimal c may be easily determined since optimality requires equality in (6).

Total expected utility is now given by

(9) $U^E = x(c_L^{\alpha} l_L^{1-\alpha})^{r} + (1-x)(c_H^{\alpha} l_H^{1-\alpha})^{r},$

where c_L, l_L, c_H and l_H are determined through (7), (8) and (6) (with equality) by substituting θ_L and θ_H for θ, respectively.

It can be shown that

(10) $\frac{dl}{d\theta} > 0$

holds. Hence, the entrepreneur exerts more effort in bad states of nature than in good ones.

Suppose now, that the firm is sold to an outside investor. Then the owner has to determine the sales price V, effort l_L and l_H, as well as her salary t_L and t_H. The optimization problem is given by

$$(11)\ \max_{V,l_L,l_H,t_L,t_H}\left\{x((t_L+V)^{\alpha}l_L^{1-\alpha})^r+(1-x)((t_H+V)^{\alpha}l_H^{1-\alpha})^r\right\}$$

under the constraints

$$(12)\ ((t_L+V)^{\alpha}l_L^{1-\alpha})^r \geq (V+u)^{\alpha r},$$

$$(13)\ ((t_H+V)^{\alpha}l_H^{1-\alpha})^r \geq ((t_L+V)^{\alpha}(l_L+\theta_H-\theta_L)^{1-\alpha})^r,$$

$$(14)\ x(\tfrac{1}{4}(\theta_L-l_L)^2-(t_L+V))+(1-x)(\tfrac{1}{4}(\theta_H-l_H)^2-(t_H+V))-K\geq 0$$

and the usual non-negativity assumptions. Note, that the investor is assumed to be risk neutral. Also, note, that an optimal solution requires equality in (14). Otherwise, t_H may be increased without violating any constraints. However, this would increase the objective function contradicting optimality. Next, we may assume $V = 0$ without loss of generality. This follows by substituting t_L for $t_L + V$ and t_H for $t_H + V$. (12) will be less restrictive then, allowing higher values for the objective function. Setting $V = 0$ implies that the firm's sales price is paid as part of the salary independent of the state of nature realized.

2.2 Optimality

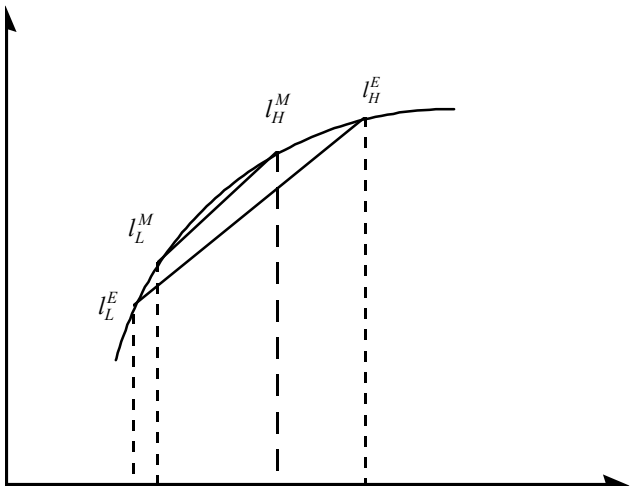

FIG. 1

It is now shown that under the assumptions presented selling off the firm is always a better strategy. Clearly, the utility function of the entrepreneur/manager is strictly concave. In case she acts as an entrepreneur she has to repay K in any state of nature. In the other case the investor receives more than the amount invested in the good state but less than invested in the bad state of nature. This in turn implies that all other parameters being equal the entrepreneur receives higher payments in the good state than the manager whereas in the bad state the entrepreneur receives lower payments. The same is true for utility in the good or bad state, respectively, since leisure is optimally adjusted. This implies that in the bad state utility of the manager is higher than that of the entrepreneur, while in the good state it is lower. Due to the

strict concavity of the utility function expected utility of the entrepreneur is always less than expected utility of the manager. Fig. 1 once more illustrates the argument. An 'E' in the upper index refers to an entrepreneurial, a 'M' to a managerial structure, respectively.

3. Correction of Numerical Results

The optimization problem may be solved by means of Lagrange multipliers (see e.g. Bertsekas [1982], pp. 71). Table 1 reproduces the results of the original paper, and shows the corrected results. Clearly variations in the parameters x and K have no effect on the optimality of selling off the firm. Numerically the problem is not very well conditioned. Rounding errors heavily affect the final result. This may be an explanation for the erroneous numbers reported in the original paper.

TABLE 1

Change in the probability of low demand

		Orig. pap.	Corrected		Orig. pap.	Corrected		Orig. pap.	Corrected
	IL	IL	IL	IH	IH	IH	Exp. Utility	Exp. Utility	Exp. Utility
K	Entr.	Manag.	Manag.	Entr.	Manag.	Manag.	Entr.	Manag.	Manag.
0.00	1.790	20.54	16.49	12.35	22.00	12.03	12.36	6.21	12.48
56.25	1.790	20.59	11.01	12.35	18.15	11.21	10.89	8.54	11.30
112.50	1.790	15.74	6.81	12.35	7.71	9.60	7.95	8.23	8.69
168.70	1.790	4.70	3.74	12.35	7.09	7.79	5.01	5.55	5.61

Change in the level of capital investment

		Orig. pap.	Corrected		Orig. pap.	Corrected		Orig. pap.	Corrected
	IL	IL	IL	IH	IH	IH	Exp. Utility	Exp. Utility	Exp. Utility
K	Entr.	Manag.	Manag.	Entr.	Manag.	Manag.	Entr.	Manag.	Manag.
0.00	12.00	20.81	19.82	20.00	32.03	16.38	13.46	11.99	13.85
56.25	7.92	20.75	16.07	17.25	21.50	14.52	11.80	11.13	12.19
112.50	4.84	20.68	10.41	14.87	13.89	12.31	10.12	10.02	10.59
168.70	2.27	18.30	7.33	12.75	7.90	10.05	8.34	8.48	9.01
225.00	0.00	11.15	4.99	10.81	6.32	7.85	5.93	6.99	7.38

4. Generalization of De Fraja s Model

It was assumed in the original paper that investors are risk neutral. From constraint (14) it is clear that the expected payment for the investor is zero. Consequently, investor's return is equal to the risk free interest rate. This assumption is not a realistic one. In modern capital market theory investors are generally assumed to be risk averse (see e.g. Copeland, Weston [1992], pp. 77). Then they require a higher return than zero in case of risky payment streams with an expected value of zero. We use the data $\theta_H = 50$, $\theta_L = 30$, $\alpha = 3/7$, $r = 0.7$, $u = 1$, $x = 0.5$, $K = 180$ but now we assume that the investor requires an expected return of $R = 0.15$ for the risky investment ($x = 0.5$). Constraint (14) has to be modified by substituting $K(1+R)$ instead of K. This yields an expected utility of 7.91 for the managerial structure which is lower than expected utility for the entrepreneurial one. This shows that the introduction of risk aversion for investors may indeed produce the results predicted by De Fraja in his original paper.

The effects of a change of x on the optimal firm structure are not obvious. If x decreases and the variance of investor's earnings decreases he requires a lower expected rate of return. Generally, this leads to an increase in expected utility of the manager since constraint (14) is less binding. If x decreases and the variance of investor's earnings increases the investor requires a higher expected rate of return. This in turn, will decrease expected utility of the manager.

manager. Then the monotony result claimed in the original paper would hold. Table 2 shows results of another numerical simulation. Using the data of the example that was used throughout the paper the table shows results for various combinations of x and R. The first column contains the results for the entrepreneurial structure. Since the variance is not monotonic in x, the optimal structure may depend on the risk-return-tradeoff of the economy.

TABLE 2

		Manager Expected Utility			Investor Variance		
x	Entr.	R = 0.15	R = 0.10	R = 0.05	R = 0.15	R = 0.10	R = 0.05
0.1	11.88	11.47	11.68	11.89	2734	2677	2637
0.2	10.90	10.64	10.86	11.08	4783	4623	4478
0.3	9.92	9.78	10.01	10.24	6332	6070	5827
0.4	8.93	8,87	9.11	9.36	7424	7067	6734
0.5	7.95	7.91	8.17	8.43	8056	7614	7203
0.6	6.97	6.89	7.17	7.45	8192	7679	7204
0.7	5.99	5.81	6.11	6.41	7751	7188	6676
0.8	5.01	4.62	4.95	5.28	6577	6006	5501
0.9	4.03	3.27	3.67	4.04	4334	3855	3452

5. Concluding Remarks

The original paper by De Fraja does not allow to conclude that entrepreneurial as well as managerial structures coexist. Based on the model assumptions it is always beneficial for a project owner to sell the firm to an outside investor and work as a manager. By doing so the risk associated with uncertain states of nature may be reduced while monetary earnings do not drop too sharply. The results obtained by De Fraja for the managerial structure are flawed which is probably due to rounding errors during the numerical simulation.

It was shown in the paper that the results hypothesized by De Fraja may hold, if the assumption of risk neutrality is dropped. Risk aversion of both managers and investors leads to coexistence of various firm structures.

The optimal structure usually is not monotonic in the probability x of the bad state of nature. The results may change again if bankruptcy is possible. Then debt is risky too. As a consequence lenders may require a rate of return that depends on the variance of earnings. We hypothesize that the optimal firm structure depends on the degree of riskiness of debt in comparison to equity. Detailed results, however, are not yet available.

References

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