A Note on
'Emissions Taxation in Durable Goods Oligopoly'

Marco Runkel*

Department of Economics, University of Siegen, D-57068 Siegen, Germany

email: runkel@wap-server.fb5.uni-siegen.de

Abstract: This note corrects an error in the analysis of Goering/Boyce (1999) and extends their results. In this way, it refutes the claim that the durability of rented products plays a decisive role for the second-best emission taxation under imperfect competition.

1. Introduction

Intuitively, one would expect that the second-best emission tax in an imperfectly competitive industry (the tax which maximizes the social welfare provided it is the only instrument available) falls short of the marginal environmental damage (underinternalization) since it has to account for two distortion simultaneously: the environmental externality and the market power of the firms. However, in an interesting recent article Goering/Boyce (1999) (hereafter referred to as G&B) argue this to be not necessarily true in a durable good oligopoly in which the products are rented. They claim that the optimal emission tax exceeds the marginal damage (overinternalization) if a) the demand and the decay functions are linear, b) the emissions depend only on output and c) the production cost function exhibits increasing returns to durability (subsequently, the conditions a - c will be referred to as the GB-case). They explain this result by a third distortion only inherent in durable good markets, namely '... the misallocation due to producers choosing a durability which does not minimize the social cost of providing a given service level.' (G&B, p. 136) In their view, an increase in the emission tax has the additional benefit of moving the firms closer to the socially optimal durability, and consequently overinternalization may be welfare enhancing.

A closer look at previous results of the durability literature, however, raises doubt whether the reasoning of G&B is correct: Under laissez-faire in a renting durable

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good industry *without* pollution, the product durability is independent of the market structure owing to Swan's independence result (Swan (1970)) and thus it is socially optimal not only under perfect competition but also under oligopoly (Goering (1992)). Hence, the misallocation of durability under laissez-faire in the renting durable good oligopoly *with* pollution can only rest on the environmental externality and doesn't represent a separate distortion. This implies that there are only the two above-mentioned distortions at work and that intuitively the second-best emission tax falls short of the marginal damage: If the emissions are taxed with a rate equal to the marginal damage then the first distortion, namely the environmental externality *together with* the misallocation of durability, is fully corrected whereas the second distortion, namely the market power of the firms, tends to restrict the output and the stock of the durable below their socially optimal levels. According to the usual second-best argument, society can gain from lowering the emission tax since this reduction shifts the output and the stock closer to their efficient levels. Thus the second-best tax underinternalizes the marginal damage.[1]

The present note supports this conclusion by identifying an error in the analysis of G&B. More specificly, their proof of overinternalization is incorrect since in the GB-case the individual firm's profit doesn't attain a maximum owing to a non-concave profit function. As a consequence, in the GB-case there is no industry equilibrium and no second-best emission tax. Furthermore, it is shown that in the case of general demand, cost, emissions and decay functions the second-best tax turns out to be smaller than the marginal damage so long as the objective function of the individual firm is concave and the emission function satisfies a mildly restrictive condition. This result turns out to be in line with previous results on nondurable goods. Hence, the claim of G&B that the durability of rented products plays a decisive role for the second-best emission taxation under imperfect competition is refuted.

2. The Analysis of G&B

For a renting durable good industry with n firms G&B seek to determine the emission tax w which maximizes the long-run social welfare, i.e. which solves the problem

$$\max_{w} V(w) := \int_0^{n\bar{Q}(w)} f(g)dg - nc[\bar{\delta}(w)]\bar{q}(w) - E[n\varepsilon(\bar{\delta}(w), \bar{q}(w))]. \tag{1}$$

[1] For a formal proof of these assertions see Runkel (1999) who employs a two-period model in which the durable causes pollution by the solid waste at the end of its life.

$\bar{\delta}$, $\bar{q}$ and $\bar{Q}$ denote the long-run equilibrium values for the individual firm's product durability, output level and stock of the durable good, respectively. f is the demand function satisfying $f' < 0$ and $f' + \bar{Q}f'' < 0$. The unit production costs are $c(\delta)$ with $c' > 0$. $\varepsilon(\delta, q)$ with $\varepsilon_q, \varepsilon_{qq} > 0$ and $\varepsilon_\delta \geq 0$ represents the firm's emission function and E is the industry emission damage. Without loss of generality the marginal damage is normalized to unity, i.e. $E(n\varepsilon) := n\varepsilon$ with $E' = 1$. The first-order condition for the welfare maximum reads

$$V' = 0 \quad \text{and hence} \quad f\,\frac{d\bar{Q}}{dw} - (\bar{q}c' + \varepsilon_\delta)\,\frac{d\bar{\delta}}{dw} - (c + \varepsilon_q)\,\frac{d\bar{q}}{dw} = 0 \tag{2}$$

where it has been taken into account that $\bar{\delta}$, $\bar{q}$ and $\bar{Q}$ are functions of the emission tax w owing to the industry equilibrium conditions.

To derive these equilibrium conditions, a noncooperative integral game is used: Firm i chooses the time path for durability $\delta_i(t)$ and output $q_i(t)$, $t \,\epsilon\, [\tau, \infty[$, in order to maximize the present value of its rental profit

$$\Pi_i = \int_\tau^\infty \Big\{ f[Q_i(t) + Q_{-i}(t)]Q_i(t) - c[\delta_i(t)]q_i(t) - w\varepsilon[\delta_i(t), q_i(t)] \Big\} e^{-r(t-\tau)} dt$$

$$\text{s.t.} \qquad Q_j(t) = \int_{-\infty}^t \phi[t-s, \delta_j(s)]q_j(s)ds, \qquad j = 1, \ldots, n \tag{3}$$

with $Q_{-i} := \sum_{j \neq i} Q_j$. ϕ with $\phi_{t-s} < 0$ and $\phi_\delta > 0$ represents the decay function of the durable good. Under an open-loop information structure, firm i takes as given Q_j, $j \neq i$, and hence the associated Hamiltonian is defined as[2]

$$H[q_i(t), \delta_i(t), Q_i(t), \mu_i(s \geq t)] := \int_t^\infty \phi[s-t, \delta_i(t)]q_i(t)\mu_i(s)ds$$
$$+ \Big\{ f[Q_i(t) + Q_{-i}(t)]Q_i(t) - c[\delta_i(t)]q_i(t) - w\varepsilon[\delta_i(t), q_i(t)] \Big\} e^{-r(t-\tau)} \tag{4}$$

where the co-state $\mu_i > 0$ may be interpreted as shadow price of firm i's stock Q_i (see Kamien/Muller (1976)). Marking the profit-maximizing values by a superscript 'o' and applying the maximum principle of Bakke (1974), the necessary conditions for a profit maximum of firm i reads

$$H_{Q_i(t)} = \Big\{ f[Q_i^o(t) + Q_{-i}(t)] + Q_i^o(t)f'[Q_i^o(t) + Q_{-i}(t)] \Big\} e^{-r(t-\tau)} = \mu_i(t), \tag{5}$$

$$[q_i^o(t), \delta_i^o(t)] = \arg \max_{q_i(t), \delta_i(t)} H[q_i(t), \delta_i(t), Q_i^o(t), \mu_i(s \geq t)], \tag{6}$$

[2] The Hamiltonian in the anlysis of G&B also contains the restrictions for Q_j, $j \neq i$, but under open-loop strategies the Hamiltonian (4) yields exactly the same results.

for all $t \epsilon [\tau, \infty[$. In order to characterize the solution to (6), G&B use the first-order conditions (see also Kamien/Muller (1976))

$$H_{q_i(t)} = H_{\delta_i(t)} = 0, \qquad \forall\, t \,\epsilon\, [\tau, \infty[. \tag{6a}$$

In the symmetric long-run equilibrium, i.e. $x_i^o(t) = x^o(t) = \bar{x}$ for all i, t and $x \,\epsilon\, \{Q, q, \delta\}$, eqs. (5) and (6a) become

$$[f(n\bar{Q}) + \bar{Q}f'(n\bar{Q})]\rho(\bar{\delta}) - c(\bar{\delta}) - w\varepsilon_q(\bar{\delta}, \bar{q}) = 0, \tag{7}$$

$$[f(n\bar{Q}) + \bar{Q}f'(n\bar{Q})]\rho'(\bar{\delta})\bar{q} - c'(\bar{\delta})\bar{q} - w\varepsilon_\delta(\bar{\delta}, \bar{q}) = 0, \tag{8}$$

with $\bar{Q} = \rho(\bar{\delta})\bar{q}$ from (3) and $\rho(\bar{\delta}) := \int_t^\infty \phi(s-t, \bar{\delta})e^{-r(s-t)}ds$.[3] (7) and (8) together with $\bar{Q} = \rho(\bar{\delta})\bar{q}$ determine a long-run industry equilibrium which depends on the level of the emission tax w.

Now, G&B proceed as follows. They insert $d\bar{Q}/dw = \rho'\bar{q}\,d\bar{\delta}/dw + \rho\, d\bar{q}/dw$, (7) and (8) into (2). Solving the resulting expression with respect to w yields

$$w^* = 1 + \bar{Q}f' \cdot \left(\rho'\bar{q}\,\frac{d\bar{\delta}}{dw} + \rho\,\frac{d\bar{q}}{dw}\right) \Big/ \left(\varepsilon_\delta\,\frac{d\bar{\delta}}{dw} + \varepsilon_q\,\frac{d\bar{q}}{dw}\right). \tag{9}$$

as second-best emission tax. Totally differentiating (7), (8) and $\bar{Q} = \rho(\bar{\delta})\bar{q}$ yields $d\bar{q}/dw$ and $d\bar{\delta}/dw$. By inserting these derivatives into (9) G&B obtain the overinternalization result for the GB-case, i.e. the second-best tax exceeds the marginal damage ($w^* > 1$) if the demand and the decay functions are linear ($f'' = \phi_{\delta\delta} = \rho'' = 0$), if the emissions depend only on output ($\varepsilon_\delta = 0$) and if the production costs exhibits increasing returns to durability ($c'' < 0$) (see their proposition 5). This conclusion will now be shown to be incorrect.

3. Erratum and Further Results

The error made by G&B is that they don't realize the Hamiltonian (4) to be not concave in $[q_i(t), \delta_i(t)]$ if $f'' = \phi_{\delta\delta} = \varepsilon_\delta = 0$ and $c'' < 0$: (4) is globally concave in $[q_i(t), \delta_i(t)]$ if its Jacobian is negative definite, i.e. if $H_{q_i(t)q_i(t)}, H_{\delta_i(t)\delta_i(t)} < 0$ and $H_{q_i(t)q_i(t)}H_{\delta_i(t)\delta_i(t)} - H^2_{q_i(t)\delta_i(t)} > 0$ or, respectively,

$$w\varepsilon_{qq} > 0, \tag{10}$$

[3] Actually, the correct expression is $\bar{Q} = \bar{q}\int_t^\infty \phi(s-t, \bar{\delta})ds$. But to obtain the correct derivatives of $\bar{Q}$, the expression $\bar{Q} = \rho(\bar{\delta})\bar{q}$ must be used. See fn. 13 in G&B.

$$c''q_i(t) + w\varepsilon_{\delta\delta} - q_i(t)\int_t^\infty \phi_{\delta\delta}[s-t,\delta_i(t)]e^{r(t-\tau)}\mu_i(s)ds > 0, \tag{11}$$

$$w\varepsilon_{qq}\Big\{c''q_i(t) + w\varepsilon_{\delta\delta} - q_i(t)\int_t^\infty \phi_{\delta\delta}[s-t,\delta_i(t)]e^{r(t-\tau)}\mu_i(s)ds\Big\} - \Big\{c' + w\varepsilon_{\delta q} - \int_t^\infty \phi_{\delta}[s-t,\delta_i(t)]e^{r(t-\tau)}\mu_i(s)ds\Big\}^2 > 0. \tag{12}$$

For $\varepsilon_\delta = 0$ we obtain $\varepsilon_{\delta\delta} = 0$ which together with $\phi_{\delta\delta} = 0$ simplifies the condition (11) to $c''q_i(t) > 0$. Obviously, for $c'' < 0$ this condition is violated with the consequence that in the GB-case the Hamiltonian is globally concave in $q_i(t)$ but is globally *convex* in $\delta_i(t)$. These properties of H in turn imply that in the GB-case the second-order conditions for the maximization problem (6) are not satisfied. Hence, in the GB-case the first-order conditions (6a) don't characterize the solution to (6) and therefore G&B are not entitled to use (7) and (8) as long-run equilibrium conditions in order to determine $d\bar{Q}/dw$, $d\bar{q}/dw$ and $d\bar{\delta}/dw$. Thus, (9) doesn't represent the second-best tax in the GB-case and G&B's proof of overinternalization fails to be correct.

This conclusion gives rise to the question how to determine the second-best tax in the GB-case. If there is no upper bound to durability[4] then we obtain

PROPOSITION 1. *Suppose there is no upper bound to durability, i.e.* $\delta_i(t)\,\epsilon\,[\delta^{min},\infty[$ *with* $\delta^{min} > -\infty$ *for all* $t\,\epsilon\,[\tau,\infty[$. *Then in the GB-case there doesn't exist a second-best emission tax at all.*

PROOF: In the GB-case the condition $H_{\delta_i(t)} = 0$ simplifies to $c'[\delta_i(t)] = \int_t^\infty \bar{\phi}_\delta[s-t]e^{r(t-\tau)}\mu_i(s)ds > 0$ with $\bar{\phi}_\delta[s-t] := \phi_\delta[s-t,\delta_i(t)]$. The RHS of this equation is independent of $\delta_i(t)$ and hence the equation has exactly one solution for $\delta_i(t)$ due to the monotony of c'. Consequently, this together with the global convexity of H with respect to $\delta_i(t)$ implies that for any given $q_i(t)$ the Hamiltonian is U-shaped with respect to $\delta_i(t)$. Hence, in the GB-case there doesn't exist a $[q_i(t),\delta_i(t)]$ which solves (6) since H can always be increased by increasing $\delta_i(t)$ owing to $\delta_i(t)\,\epsilon\,[\delta^{min},\infty[$. Because (6) is a necessary condition for the solution of (3) there doesn't exist a profit maximum of the individual firm, neither a short-run nor a long-run industry equilibrium, no welfare function such as (1) and consequently no second-best tax. (Q.E.D.)

Owing to the nonexistence of the second-best emission tax proven in proposition 1 it is impossible to say anything about the size of this tax rate, and hence G&B are wrong in claiming overinternalization to be second-best optimal in the GB-case.

[4] For example, the exponential decay function $\phi[a,\delta_i] = e^{-a/\delta_i}$ typically requires $1/\delta_i \epsilon [0,1]$ or, equivalently, $\delta_i \epsilon [1,\infty[$. If δ_i represents the life-time of the product then $\delta_i \epsilon [0,\infty[$.

Of course, the nonexistence of the industry equilibrium may be avoided by assuming the set of admissible values for durability to be compact, i.e. $\delta_i(t) \,\epsilon\, [\delta^{min}, \delta^{max}]$ with $-\infty < \delta^{min} < \delta^{max} < \infty$: For any given w the profit maximum of firm i in the GB-case is then described by (5), $H_{q_i(t)} = 0$ and a corner solution for $\delta_i(t)$, namely $\delta_i(t) = \delta^{min}$ or $\delta_i(t) = \delta^{max}$ depending on the relative size of H at these boundaries. The symmetric long-run industry equilibrium is then determined by (7), $\bar{Q} = \rho(\bar{\delta})\bar{q}$ and $\bar{\delta}$ equal to δ^{min} or δ^{max}. However, beside the somewhat ad hoc nature of the assumption $\delta_i(t) \,\epsilon\, [\delta^{min}, \delta^{max}]$ there may arise an other technical problem: A marginal change in w may alter the relative size of H at the boundaries $\delta_i(t) = \delta^{min}$ and $\delta_i(t) = \delta^{max}$ and hence it may switch the equilibrium durability from δ^{min} to δ^{max} or vice versa. Consequently, $\bar{\delta}(w)$, $\bar{q}(w)$, $\bar{Q}(w)$ and $V(w)$ in (1) are discontinuous functions and the marginal analysis used in (2) may fail.[5]

To obtain more specific results, let us return to the question which G&B actually intended to answer: Their major aim is to compare the second-best emission tax in a renting durable good industry with that in a nondurable good industry previously investigated by e.g. Barnett (1980) or Ebert (1992). The latter authors, however, consider the case of general demand, cost and emission functions and assume a concave objective function of the individual firm as well as an interior solution for all variables. A suitable comparison thus requires to suppose analogous conditions for the durable good industry. Unfortunately, G&B only consider the GB-case and don't explictly investigate second-best taxation in the case of general functions in which the Hamiltonian is concave and in which there is an interior solution. This gap is closed by

PROPOSITION 2. *Suppose H is globally concave in $[q_i(t), \delta_i(t)]$, the maximization problem (6) has an interior solution for all $t \,\epsilon\, [\tau, \infty[$ and the emission function satisfies $\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q} \geq 0$. Then $w^* < 1$.*

PROOF: If H in (4) is globally concave in $[q_i(t), \delta_i(t)]$ and if there is an interior solution to the maximization of H then the first-order conditions (6a) are suitable to characterize the solution to (6). Consequently, (7), (8) and $\bar{Q} = \rho(\bar{\delta})\bar{q}$ describe the long-run industry equilibrium and (9) represents the correct expression for the second-best emission tax. Hence, it remains to determine $d\bar{q}/dw$ and $d\bar{\delta}/dw$. First, note that in the long-run

[5] It is interesting to note that if a variation in w doesn't change the relative size of H at its boundaries then $d\bar{\delta}/dw = 0$. Using (7) and $\bar{Q} = \rho(\bar{\delta})\bar{q}$ in (2) then yields $w^* = 1 + \bar{Q}f'\rho/\varepsilon_q < 1$. This would *refute* the overinternalization result of G&B in the GB-case. As mentioned in the text, however, a jump in equilibrium durability can't be ruled out, in general.

equilibrium the concavity conditions (10) to (12) become

$$w\varepsilon_{qq} > 0, \qquad c''\bar{q} + w\varepsilon_{\delta\delta} - \rho''\bar{q}(f + \bar{Q}f') > 0, \tag{13}$$

$$w\varepsilon_{qq}[c''\bar{q} + w\varepsilon_{\delta\delta} - \rho''\bar{q}(f + \bar{Q}f')] - w^2[\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q}]^2 > 0 \ , \tag{14}$$

where (5) and (8) are employed. Totally differentiating (7), (8) and $\bar{Q} = \rho(\bar{\delta})\bar{q}$ and applying Cramer's rule yields

$$\frac{d\bar{q}}{dw} = \frac{1}{|J|}\Big\{[(n+1)f' + n\bar{Q}f''][\rho'^2\bar{q}^2\varepsilon_q - \rho\rho'\bar{q}\varepsilon_\delta] - \varepsilon_q[c''\bar{q} + w\varepsilon_{\delta\delta} - \rho''\bar{q}(f + \bar{Q}f')] - w\varepsilon_\delta(\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q})\Big\}, \tag{15}$$

$$\frac{d\bar{\delta}}{dw} = \frac{[(n+1)f' + n\bar{Q}f''][\rho^2\varepsilon_\delta - \rho\rho'\bar{q}\varepsilon_q] - w\varepsilon_\delta\varepsilon_{qq} - w\varepsilon_q(\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q})}{|J|}, \tag{16}$$

where the Jacobian determinant $|J|$ is positive as is easily shown by employing $\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q} \geq 0$, (13) and (14). The size of the second-best emission tax is determined by inserting (15) and (16) in (9). After some tedious computations the two bracket terms in (9) become, respectively,

$$\rho'\bar{q}\,\frac{d\bar{\delta}}{dw} + \rho\,\frac{d\bar{q}}{dw} = -\frac{1}{|J|}\Big\{\rho\varepsilon_q[c''\bar{q} + w\varepsilon_{\delta\delta} - \rho''\bar{q}(f + \bar{Q}f')] + w\rho'\bar{q}\varepsilon_\delta\varepsilon_{qq} + w(\rho'\bar{q}\varepsilon_q + \rho\varepsilon_\delta)(\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q})\Big\} < 0, \tag{17}$$

$$\varepsilon_\delta\,\frac{d\bar{\delta}}{dw} + \varepsilon_q\,\frac{d\bar{q}}{dw} = \frac{1}{|J|}\Big\{[(n+1)f' + n\bar{Q}f''][\rho\varepsilon_\delta - \rho'\bar{q}\varepsilon_q]^2 - w\varepsilon_\delta^2\varepsilon_{qq} - 2w\varepsilon_\delta\varepsilon_q(\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q}) - \varepsilon_q^2[c''\bar{q} + w\varepsilon_{\delta\delta} - \rho''\bar{q}(f + \bar{Q}f')]\Big\} < 0. \tag{18}$$

The signs of these expressions are obtained from $|J| > 0$, (13) and $\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q} \geq 0$. Using (17), (18) and $f' < 0$ in (9) proves $w^* < 1$. (Q.E.D.)

Proposition 2 shows that the conditions which render the second-best emission tax in the renting durable good industry smaller than the marginal damage are not very restrictive: Firstly, a concave objective function and an interior solution are typically assumed in all economic models which don't explicitly focus on corner solutions or existence problems. Especially, they are used in the second-best taxation models suggested by Barnett (1980) or Ebert (1992) which serve as a standard of comparision for the present durable good model. Secondly, the condition $\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q} \geq 0$ is satisfied by all separable emission functions ($\varepsilon_{\delta q} = 0$) and by all those non-separable emission functions for which an increase in the output reduces the marginal damage of durability ($\varepsilon_{\delta q} < 0$) or increases the marginal damage of durability only relative slightly

($\varepsilon_{\delta q} \leq \varepsilon_\delta/\bar{q}$).[6] Furthermore, note that the conditions for $w^* < 1$ listed in proposition 2 are sufficient. Hence, the second-best tax may be smaller than the marginal damage even if $\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q} < 0$.

Certainly, in case of $\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q} < 0$ it can't be ruled out that the second-best emission tax is greater than the marginal damage ($w^* > 1$). However, this case of overinternalization, if it exists, has counterintuitive implications: $w^* > 1$ implies that one and only one of the two bracket terms in (9) has to be *positive*. However, the second bracket term equals the change of the firm's equilibrium emission due to a marginal change in the emission tax, $d\bar{\varepsilon}/dw$, and obviously there is a strong plausibility for this change to be *negative*. If the change in emission is negative then $w^* > 1$ requires the first bracket term in (9) to be *positive*. However, intuitively this term is also expected to be *negative* since it equals the change in the firm's equilibrium stock of the durable due to a marginal change in the emission tax, $d\bar{Q}/dw$. Furthermore, the first bracket term to be positive implies that at least one of the equilibrium values, either that of durability or that of output, *increases* as the emission tax increases. This implication is counterintuitive since both variables are positively correlated with the firm's emission which in this scenario *decreases*.

Of course, plausibility arguments of this type are no good substitute for hard empirical evidence. Hence, although G&B are mistaken in the GB-case (which is their exclusive focus) they actually are correct in pointing out that overinternalization can't be ruled out to be second-best optimal in a renting durable good industry, in general. Owing to proposition 2 a necessary condition for $w^* > 1$ is $\varepsilon_{\delta q} > 0$. However, this observation is *not* really a new insight since in a similar way it is already derived by Barnett (1980) and Ebert (1992) for nondurable goods: In a nondurable good model these authors assume cost and emission functions similar to those used in our model, namely $C(\delta, q)$ and $\varepsilon(\delta, q)$ with $C_\delta > 0$ but $\varepsilon_\delta < 0$ since δ is interpreted not as product durability but as pollution abatement effort of the individual firm. In their model a necessary condition for overinternalization is $C_{\delta q} + w\varepsilon_{\delta q} < 0$ (see Barnett (1980), fn. 6, Ebert (1992), eq. (15)) which in turn implies $\varepsilon_{\delta q} < 0$ if constant returns to scale (i.e. $C(\delta, q) = c(\delta)q$ and $C_{\delta q} = c'(\delta) > 0$) are assumed, as in the present durable good model. Since the sign of ε_δ is reversed this necessary condition is analogous to $\varepsilon_{\delta q} > 0$ in the durable good model.

[6] For example, $\varepsilon(\delta, q) = G(q) + (\alpha q + \beta)\delta$ with $\alpha, \beta, G', G'' > 0$ seems to be a reasonable emission function since a marginal increase in durability causes pollution by additional resources needed to make one unit of the good more durable (α) and by additional resources needed for R&D (β). For this function we obtain $\varepsilon_{\delta q} = \alpha > 0$ but $\varepsilon_\delta/\bar{q} - \varepsilon_{\delta q} = \beta/\bar{q} > 0$.

To sum up, the possibility of overinternalization to be second-best optimal in a renting durable good industry cannot be excluded, in general. If it occurs at all, however, overinternalization a) has counterintuitive implications, b) is not really a new insight but already derived in similar form by Barnett (1980) and Ebert (1992) for nondurable goods and thus c) is *not* due to a 'third distortion' inherent in rental markets for durable goods but is due only to the special form of the emission function.

4. Concluding Remark

In contrast to the argumentation of G&B, the results of the present note deny a decisive role of the durability of rented products in the second-best emission taxation under imperfect competition. It should be also noted, however, that this finding does not carry over to product durability in sales markets: As Bulow's (1986) analysis of planned obsolescence shows there really is a misallocation of durability in sales markets even in the absence of environmental externalities. This distortion may render the second-best emission tax greater than the marginal environmental damage (see Runkel (1999)).

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