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We examine the effects of price discrimination in the Stackelberg competition model for the linear demand case. We show that the leader does not use any price discrimination at all. Rather, the follower does all price discrimination. The leader directs all of its first mover preemptive advantage to attract the highest value consumers who pay a uniformly high price. We observe that profits and total welfare are larger and consumer surplus is smaller than those of the standard Stackelberg competition model.

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Price Discrimination in Stackelberg Competition*

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Abstract

We examine the effects of price discrimination in the Stackelberg competition model for the linear demand case. We show that the leader does not use any price discrimination at all. Rather, the follower does all price discrimination. The leader directs all of its first mover preemptive advantage to attract the highest value consumers who pay a uniformly high price. We observe that profits and total welfare are larger and consumer surplus is smaller than those of the standard Stackelberg competition model.

I. Introduction

There is a large recent literature on price discrimination in oligopolistic contexts.[1] The literature varies in oligopoly models and types of discrimination considered. For example, Borenstein [1985] and Holmes [1989] examine third degree price discrimination in Bertrand models. Hazledine [2006] examines second degree price discrimination in a Cournot model. In the second degree formulation, the firm is able to segment consumer demand by ranges of reservation price. Consumers with reservation price between r_1 and r_2 pay one price, those between r_2 and r_3 pay another, and so on. He shows that the output sold at a particular price is a multiple N of the output sold at the next lowest price, where N is the number of firms in the market.

In this paper, we examine second degree price discrimination as modeled by Hazledine [2006] and apply it to the Stackelberg formulation instead of Cournot. We find that, in contrast to Hazledine's results, under the Stackelberg formulation, only the follower price discriminates. The leader directs all of its first mover preemptive advantage to attract the highest value customers who pay a uniformly high price. The follower price discriminates over the residual demand. Another result of price discrimination is that profits and total welfare are larger and consumer surplus is smaller than those of the standard Stackelberg competition model.

II. The Model and Results

We consider a leader-follower duopoly with no cost of entry. Let c be the constant marginal cost of the leader and the follower. We assume that each consumer buys at most one unit of the good. The firms know valuations of the consumers and can prevent resale of the good. They divide the consumers into *bins* according to their reservation prices. The price of the good for the k^{th} bin is given by:

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[1] See Stole (2007) for a survey.

$$P^k = a - Q^k \tag{1}$$

where $Q^k \equiv \sum_{i=1}^{k}(q_L^i + q_F^i)$ is the total quantity sold in all bins from 1 to k.

A good example for such a setting is airline seats offered for a specific route. Airline tickets are purchased in unit quantity and it is a common practice for airlines to price discriminate. For the airline case, one can think that consumers come to the market at different times and their valuations are inversely related with the length of time between purchase and flight. For example, business people learn of their travel plans at the last moment, and are willing to pay much more than tourists, who are at the other extreme and know their travel plans a year ahead of time. The linear demand assumes a continuum of people between these extremes. From now on we will call the product 'an airline seat' and a seller 'an airline.'

Before stating our results, we would like to remind the reader of some standard results from theories of monopoly and oligopolistic competition: A monopolist produces $\frac{a-c}{2}$ and each of the symmetric Cournot duopolist produces $\frac{a-c}{3}$. Finally, under the standard Stackelberg competition model, the leader produces $\frac{a-c}{2}$, and the follower produces $\frac{a-c}{4}$. Note that the leader produces the monopoly output which is more than the Cournot output. This is because of the strategic benefit of inducing the second mover to reduce output. In what follows, we state our propositions for Stackelberg model with price discrimination.

Proposition 1 *Suppose that price discrimination is permitted in a symmetric linear Stackelberg competition duopoly model with K bins. Then, in equilibrium , only the follower price discriminates. The equilibrium quantities and prices are:*

$$\begin{aligned}
q_L^1 &= \frac{a-c}{2} \text{ and } q_L^j = 0 \text{ for } j = 2, 3, ..., K \\
q_F^j &= \frac{1}{K+1}\frac{a-c}{2} \text{ for } j = 1, 2, ..., K \\
P^j &= a - (1 + \frac{j}{K+1})\frac{a-c}{2} \text{ for } j = 1, 2, ..., K
\end{aligned}$$

Proposition 1 shows that the leader allocates all its production to bin 1 and lets the follower do the discrimination. By doing this, the leader enjoys high profits from the high-end customers. The follower spreads its output equally among the bins. In order to explain the intuition for why the leader allocates all its production to bin 1, assume for now that $K = 2$. In addition to preempting the high-end consumer, the leader would preempt the low-end as well. But if the leader were to deviate from allocating all of its output to bin 1 and allocate some output to bin 2, then this would induce the follower to allocate more output to bin 1. When the follower decides how much to allocate to bin 1, it takes into account the negative impact of an expansion in bin 1 output on the profits of bin 2. If the leader expands output in bin 2, then bin 2 becomes less important to the follower which causes it to allocate more output to bin 1. The negative effect on the leader's bin 1 consumers would offset the gains to the leader from obtaining bin 2 consumers. This contrasts with the standard Stackelberg dynamics in which when the leader expands output, the follower reduces output. Here, the follower lowers output in bin 2; the problem for the leader is that the follower expands output in bin 1.

Other implications of Proposition 1 are as follows: First, as the number of the bins increases price converges to the marginal costs of the firms and the follower allocates essentially no output in bin 1. This is a vertical separation where the leader grabs the high-value consumers and leaves the lower ones to the follower. Second, one can show that if a monopolist can price discriminate, then it produces $q_M^1 = q_M^2 = \frac{a-c}{3}$. So, although in the standard Stackelberg model the leader produces the monopoly output (without price discrimination), if it can price discriminate, it produces less than the monopoly output (with price discrimination).[2] Finally, the price charged by the leader is higher than the Stackelberg competition price and the output-weighted price charged by the follower[3] is the same as the Stackelberg competition price.

Proposition 2*Suppose that price discrimination is permitted in a symmetric linear Stackelberg competition duopoly model with K bins. Profits of both the leader and the follower are higher compared to profits they would get in the standard Stackelberg competition. Although the consumer welfare is lower, the total welfare is higher. The profits, consumer welfare, and total welfare are:*

$$\begin{aligned}
\pi_L &= \frac{K}{K+1}\frac{(a-c)^2}{4} > \frac{1}{2}\frac{(a-c)^2}{4} = \pi_L^S \\
\pi_F &= \frac{K}{K+1}\frac{(a-c)^2}{8} > \frac{1}{2}\frac{(a-c)^2}{8} = \pi_F^S \\
CW &= \frac{K^2+5K+3}{(K+1)^2}\frac{(a-c)^2}{8} < \frac{9}{4}\frac{(a-c)^2}{8} = CW^S \\
TW &= \frac{4K^2+8K+3}{(K+1)^2}\frac{(a-c)^2}{8} > \frac{15}{4}\frac{(a-c)^2}{8} = TW^S
\end{aligned}$$

Proposition 2 shows the welfare effects of price discrimination. Because of the strategic interactions between the leader and the follower it might have been the case that the profits of both firms decrease as the degree of price discrimination increases. But it turns out that in our model the profits of both firms increase. Indeed, the profit of the leader increases even though the quantity that it produces remains the same. The reason for this is that the follower abandons the high-value consumers to focus on the low-value ones. Finally, note that total welfare monotonically converges to the socially optimal level of welfare, as K goes to infinity.

Appendix

The quantity of j^{th} fare provided by the leader and the follower are q_L^j and q_F^j, respectively. Let $Q_L^j \equiv \sum_{i=1}^j q_L^i$, $Q_F^j \equiv \sum_{i=1}^j q_F^i$, $Q^j \equiv Q_L^j + Q_F^j$, and $\alpha \equiv a - c$. We first consider the decision problem of the follower. The profit of the follower is given by:

[2] Note that in the case of price discrimination the leader produces the monopoly output without price discrimination.

[3] i.e., $P_F = \sum_{i=1}^K \frac{P^i q_F^i}{\sum_{i=1}^K q_F^i}$

$$\begin{aligned} \pi_F(q_L) &= q_F^1(P^1-c) + q_F^2(P^2-c) + \ldots + q_F^K(P^K-c) \\ &= q_F^1(\alpha - Q^1) + q_F^2(\alpha - Q^2) + \ldots + q_F^K(\alpha - Q^K) \end{aligned} \tag{2}$$

Differentiating with respect to the decision variables q_F^j for $j = 1, 2..., K$ gives the first order conditions for profit maximization:

$$\frac{\partial \pi_F}{\partial q_F^j} = \alpha - (Q_L^j + Q_F^K + q_F^j) = 0 \text{ for } j = 1, 2, .., K \tag{3}$$

Then,

$$0 = \frac{\partial \pi_F}{\partial q_F^j} - \frac{\partial \pi_F}{\partial q_F^{j-1}} = q_F^{j-1} - q_L^j - q_F^j \tag{4}$$

This implies that for all $j = 2, 3, ..., K$

$$q_L^j = q_F^{j-1} - q_F^j \tag{5}$$

Moreover, by equation (3) we have:

$$q_L^1 = \alpha - q_F^1 - Q_F^K \tag{6}$$

Now, we solve the maximization problem of the leader. The profit of the leader is given by:

$$\begin{aligned} \pi_L &= q_L^1(P^1-c) + q_L^2(P^2-c) + \ldots + q_L^K(P^K-c) \\ &= q_L^1(\alpha - Q^1) + q_L^2(\alpha - Q^2) + \ldots + q_L^K(\alpha - Q^K) \end{aligned} \tag{7}$$

Substituting q_L^j and q_L^1 from equations (5) and (6) respectively gives:

$$
\begin{aligned}
\pi_L &= (\alpha - q_F^1 - Q_F^K)(\alpha - Q^1) + (q_F^1 - q_F^2)(\alpha - Q^2) + ... + (q_F^{K-1} - q_F^K)(\alpha - Q^K) \quad (8)\\
&= (\alpha - q_F^1 - Q_F^K)(\alpha - ((\alpha - q_F^1 - Q_F^K) + q_F^1)) + \\
&\quad (q_F^1 - q_F^2)(\alpha - ((\alpha - q_F^1 - Q_F^K) + q_F^1 + Q_F^1)) + \\
&\quad \vdots \\
&\quad (q_F^{K-1} - q_F^K)(\alpha - ((\alpha - q_F^1 - Q_F^K) + q_F^1 + Q_F^{K-1})) \\
&= (\alpha - q_F^1 - Q_F^K) Q_F^K + \\
&\quad (q_F^1 - q_F^2)(Q_F^K - Q_F^1) + \\
&\quad \vdots \\
&\quad (q_F^{K-1} - q_F^K)(Q_F^K - Q_F^{K-1}) \\
&= ((\alpha - q_F^1 - Q_F^K) + (q_F^1 - q_F^2) + (q_F^2 - q_F^3) + ... + (q_F^{K-1} - q_F^K)) Q_F^K - \\
&\quad ((q_F^1 - q_F^2) Q_F^1 + (q_F^2 - q_F^3) Q_F^2 + ... + (q_F^{K-1} - q_F^K) Q_F^{K-1}) \\
&= (\alpha - Q_F^K - q_F^K) Q_F^K - ((q_F^1 - q_F^2) Q_F^1 + (q_F^2 - q_F^3) Q_F^2 + ... + (q_F^{K-1} - q_F^K) Q_F^{K-1}) \\
&= (\alpha - Q_F^K) Q_F^K - \sum_{k=1}^{K} (q_F^k)^2
\end{aligned}
$$

By (8) we derive the first order conditions for the leader as:

$$
\frac{\partial \pi_L}{\partial q_F^j} = \alpha - 2(Q_F^K + q_F^j) = 0 \text{ for } j = 1, 2, ..., K \tag{9}
$$

Hence,

$$
Q_F^K + q_F^j = \frac{\alpha}{2} \text{ for } j = 1, 2, ..., K \tag{10}
$$

After summing (10) over j solving for Q_F^K gives:

$$
Q_F^K = \frac{K}{K+1} \frac{\alpha}{2} \tag{11}
$$

By (10) and (11) the follower's optimal quantity for the fare class $j = 1, 2, ..., K$ is given by:

$$
q_F^j = \frac{1}{K+1} \frac{a-c}{2} \tag{12}
$$

By (5), (6), (11), and (12) the leader's optimal quantities for the fare class 1 is given by:

$$q_L^1 = \frac{a-c}{2} \quad (13)$$

$$q_L^j = 0 \text{ for } j = 2, 3, ..., K \quad (14)$$

Given the quantities, both the calculation of prices and the proof of the Proposition 2 are trivial; we leave them to the reader.

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