A Note on "Strategic Choice of Flexible Production Technologies and Welfare Implications"[1]

Sudheer Gupta

Faculty of Management, McGill University

1001 Sherbrooke St. W., Montreal

Canada H3A 1G5

e-mail: gupta@management.mcgill.ca

Abstract

This note corrects certain errors in welfare calculations and graphs of a paper by Roller and Tombak (JIE, 1990). The corrections necessitate a re-interpretation of some of their results. Contrary to authors' assertion (Proposition 1), consumer surplus is not always the highest in (FMS, FMS) state. Producing in both markets imposes a negative externality that grows as products become more substitutable, leading to a reduction in output levels. Secondly, corrected graphs reveal that the difference in producers' and total surplus between (DE, DE) and (FMS, FMS) states is not as large as depicted by the authors.

[1] I thank Professor Richard Loulou for his invaluable encouragement, guidance, and support. Any errors are solely mine.

I. Introduction

Roller and Tombak (1990) (hereafter referred to as R&T) provided an insightful and interesting analysis of a manufacturer's decision to invest in a flexible or a dedicated technology in a competitive setting. The model provided an important and useful departure from the traditional focus in the economics literature on examining the value of manufacturing flexibility in relation to changes in scale of production or in enhancing the manufacturer's ability to cope with demand (volume) uncertainties. As the burgeoning literature on flexibility has stressed, the ability to produce multiple, differentiated products is one of the central advantages of new flexible technologies, and economic analyses of this aspect of manufacturing flexibility have started to trickle in (eg, Roller and Tombak, 1993; Eaton and Schmitt, 1994). However, there are certain errors in the analysis presented by R&T which necessitate a re-examination and re-interpretation of some of their results. One of these errors, in the determination of equilibria of the technology choice game, was corrected by Kim, Roller and Tombak (1992). This note corrects the remaining errors in their welfare calculations, graphs, and propositions. For the benefit of the readers, we provide a brief summary of R&T's model along with the corrected analysis and results.

The model consists of a two-stage game between two manufacturers who make their decisions at each stage simultaneously and in full knowledge of the actions taken by both at the previous stage. The first stage decision involves a choice between a dedicated (DE) and a flexible technology (FMS); manufacturers compete on quantities in a Cournot fashion in the second stage. Choice of DE implies that a manufacturer can produce only one of two products, A or B, whereas FMS allows him to produce both. Following a 'non-address' approach for consumer preferences, following linear demand system is obtained:

$$P^j = \alpha - Q^j - \lambda Q^i, \; i \neq j, \; i,j = A \text{ or } B$$

The second stage profit function for manufacturer i in state j is:

$$\pi_{i,j} = P^A Q_{i,j}^A + P^B Q_{i,j}^B - F_k - C(Q_{i,j}^A + Q_{i,j}^B)$$

where state j depends on first-stage technology choice decisions; C is the marginal cost, assumed equal for both manufacturers; and, F_k is the fixed cost of technology chosen by the manufacturer. Set $F_{DE} = 1$, $F_{FMS} = 1 + s$, $s \geq 0$. Also, set $t = \alpha - C$, a measure of the market size. See R&T for details.

II. Characterization of Equilibria

R&T solve for subgame perfect equilibria of this two-stage game by backward induction. First, the second-stage Cournot quantity game is solved conditioning on the first-stage choice of technologies. For each of the three possible technology states (ie, (DE, DE), (FMS, FMS) or Mixed), the second-stage Cournot equilibrium output quantities and prices for each of the two products A and B, and profits for manufacturers 1 and 2, are given in Figure $A1$ (Q_i^j represents the quantity produced by manufacturer i, $i = 1, 2$, of product j, $j = A$ or B).

	(DE, DE)	(FMS, FMS)	(FMS, DE)	(DE, FMS)
Q_1^{A*}, Q_2^{A*}	$\frac{t}{2+\lambda}, 0$	$\frac{t}{3(1+\lambda)}, \frac{t}{3(1+\lambda)}$	$\frac{t}{2(1+\lambda)}, 0$	$\frac{t}{3}, \frac{t(2-\lambda)}{6(1+\lambda)}$
$Q_{1,}^{B*} Q_2^{B*}$	$0, \frac{t}{2+\lambda}$	$\frac{t}{3(1+\lambda)}, \frac{t}{3(1+\lambda)}$	$\frac{t(2-\lambda)}{6(1+\lambda)}, \frac{t}{3}$	$0, \frac{t}{2(1+\lambda)}$
p^{A*}	$C + \frac{t}{2+\lambda}$	$C + \frac{t}{3}$	$C + \frac{t(3-\lambda)}{6}$	$C + \frac{t}{3}$
p^{B*}	$C + \frac{t}{2+\lambda}$	$C + \frac{t}{3}$	$C + \frac{t}{3}$	$C + \frac{t(3-\lambda)}{6}$
π_1^*	$[\frac{t}{2+\lambda}]^2 - 1$	$\frac{2t^2}{9(1+\lambda)} - (1+s)$	$\frac{t^2(13-5\lambda)}{36(1+\lambda)} - (1+s)$	$\frac{t^2}{9} - 1$
π_2^*	$[\frac{t}{2+\lambda}]^2 - 1$	$\frac{2t^2}{9(1+\lambda)} - (1+s)$	$\frac{t^2}{9} - 1$	$\frac{t^2(13-5\lambda)}{36(1+\lambda)} - (1+s)$

Figure $A1$: Equilibrium Expressions for Various Technology States

See (1)–(6) in R&T for the equilibrium conditions. It is easy to see that mixed equilibria cannot exist in the technology choice game (in pure strategies), as pointed out by Kim, Roller, and Tombak (1992), correcting an error in R&T. A corrected plot of these conditions defining the equilibrium regions is reproduced in Figure $2'$.

Thus, high t and low λ encourage FMS adoption; there are no parameter values for which mixed equilibria could exist; and, there is a narrow range over which both (FMS, FMS) and (DE, DE) are equilibria. We also plot manufacturers' equilibrium profits (at $t = 10, s = 0.5$) in Figure $A2$.

The figure clearly shows an individual manufacturer's incentive to invest in FMS, assuming a status quo of dedicated technologies for both manufacturers. The ability to produce multiple products leads to a significant increase in the profits of the manufacturer adopting FMS, at the expense of the other manufacturer. As both manufacturers adopt FMS, the result is an increase in competition in both markets and a reduction in manufacturer profits. The welfare effects are summarized in the next section.

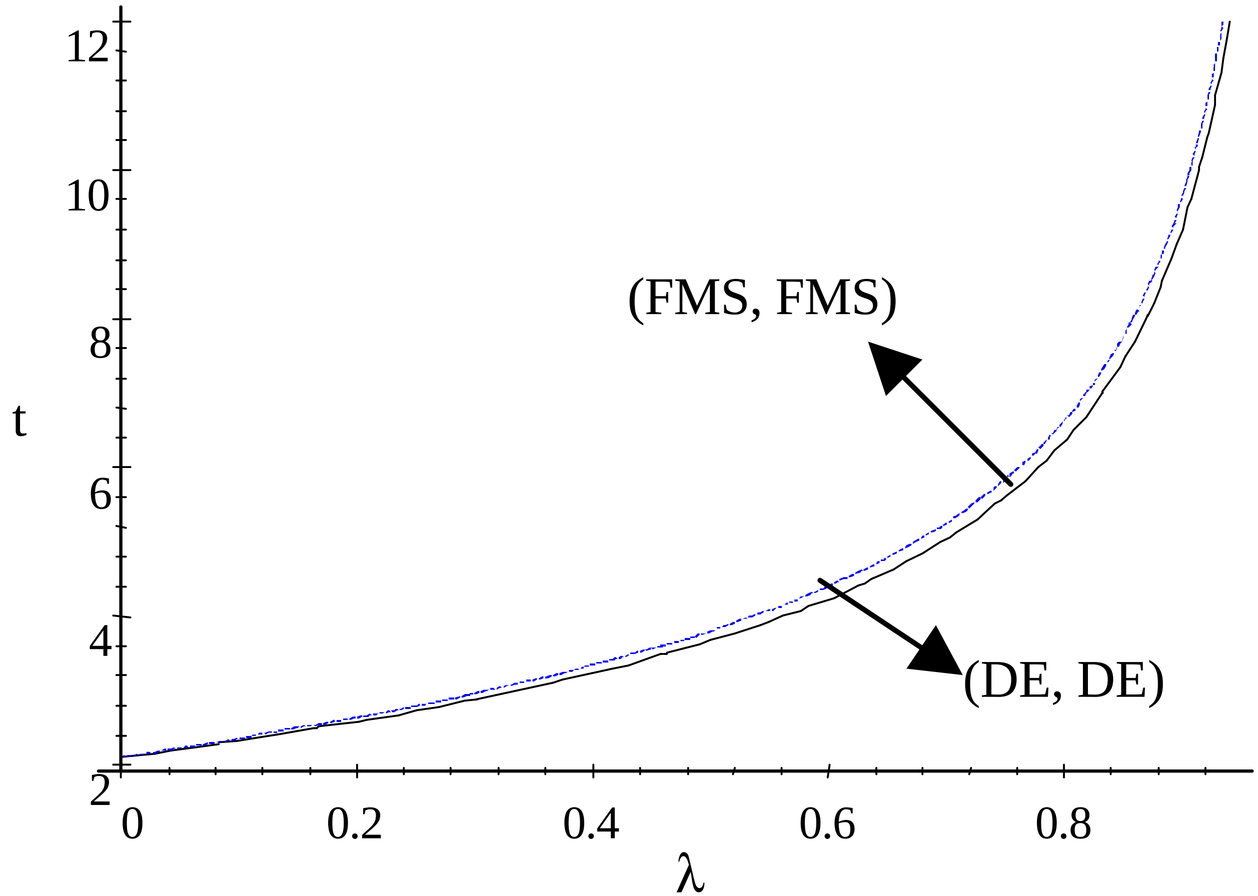

Figure 2' Equilibrium Regions (*s=0.5*)

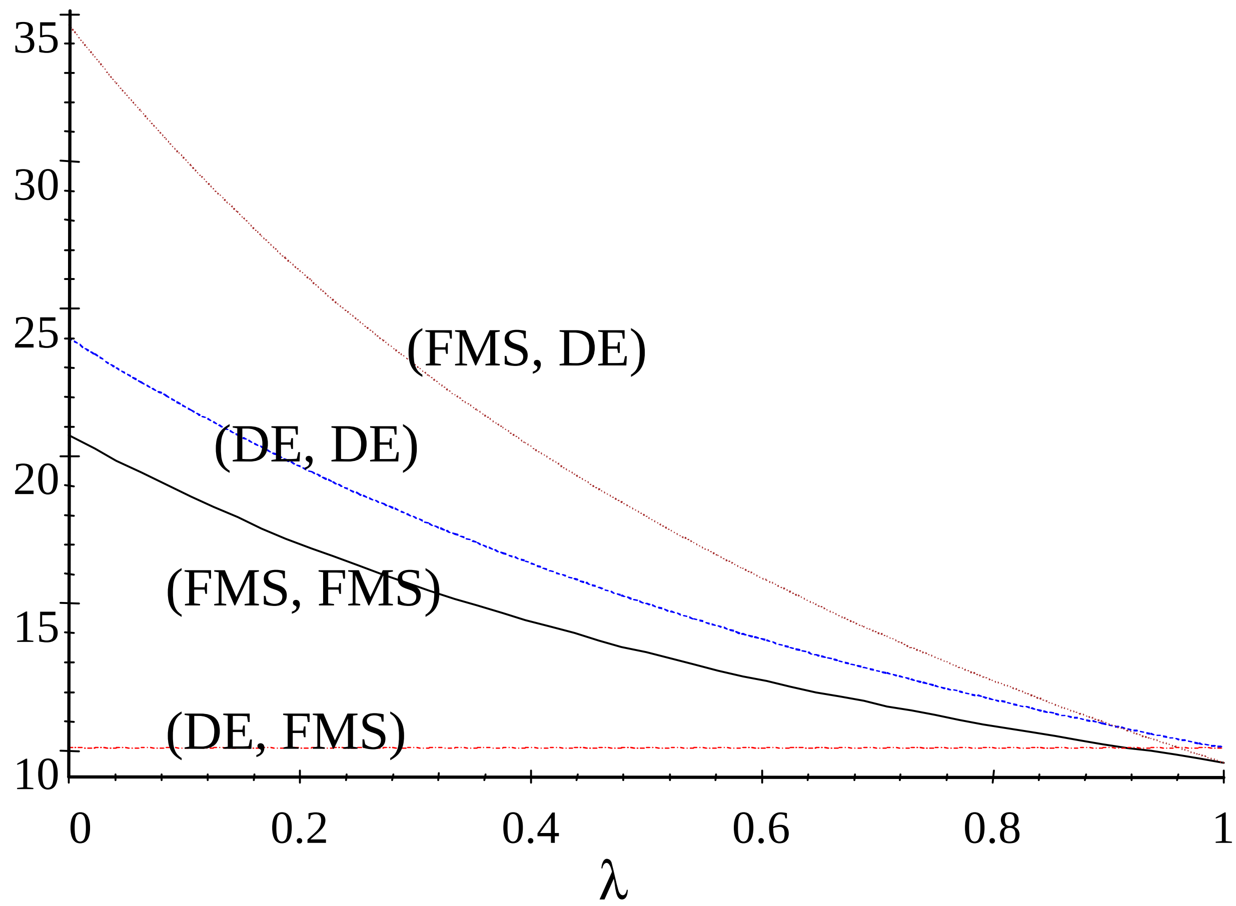

Figure A2 Equilibrium Manufacturer Profits (*s=0.5, t=10*)

III. Welfare Implications

1. Consumer Surplus: As mentioned by R&T, the consumer surplus in both markets for a given technology state j is given by [2]:

$$CS_j = \frac{1}{2}((Q_j^{A*})^2 + (Q_j^{B*})^2)$$

where Q_j^{A*}, Q_j^{B*} are equilibrium quantities produced of products A and B respectively in state j. However, there is an error in R&T's calculation of surpluses in the mixed case. Using Figure $A1$, the corrected expressions for consumer surplus are given in Figure $3'$, and a corrected plot (for $s = 0.5, t = 10$) is reproduced in Figure $4'$:

state	CS_j
(FMS, FMS)	$\frac{4t^2}{9(1+\lambda)^2}$
Mixed	$\frac{t^2(25+8\lambda+\lambda^2)}{72(1+\lambda)^2}$
(DE, DE)	$\frac{t^2}{(2+\lambda)^2}$

Figure $3'$: Consumer Surplus

[2] The following expression applies as long as the cross-partial derivatives of the demand functions are equal, as is the case here (I thank an anonymous referee for this observation).

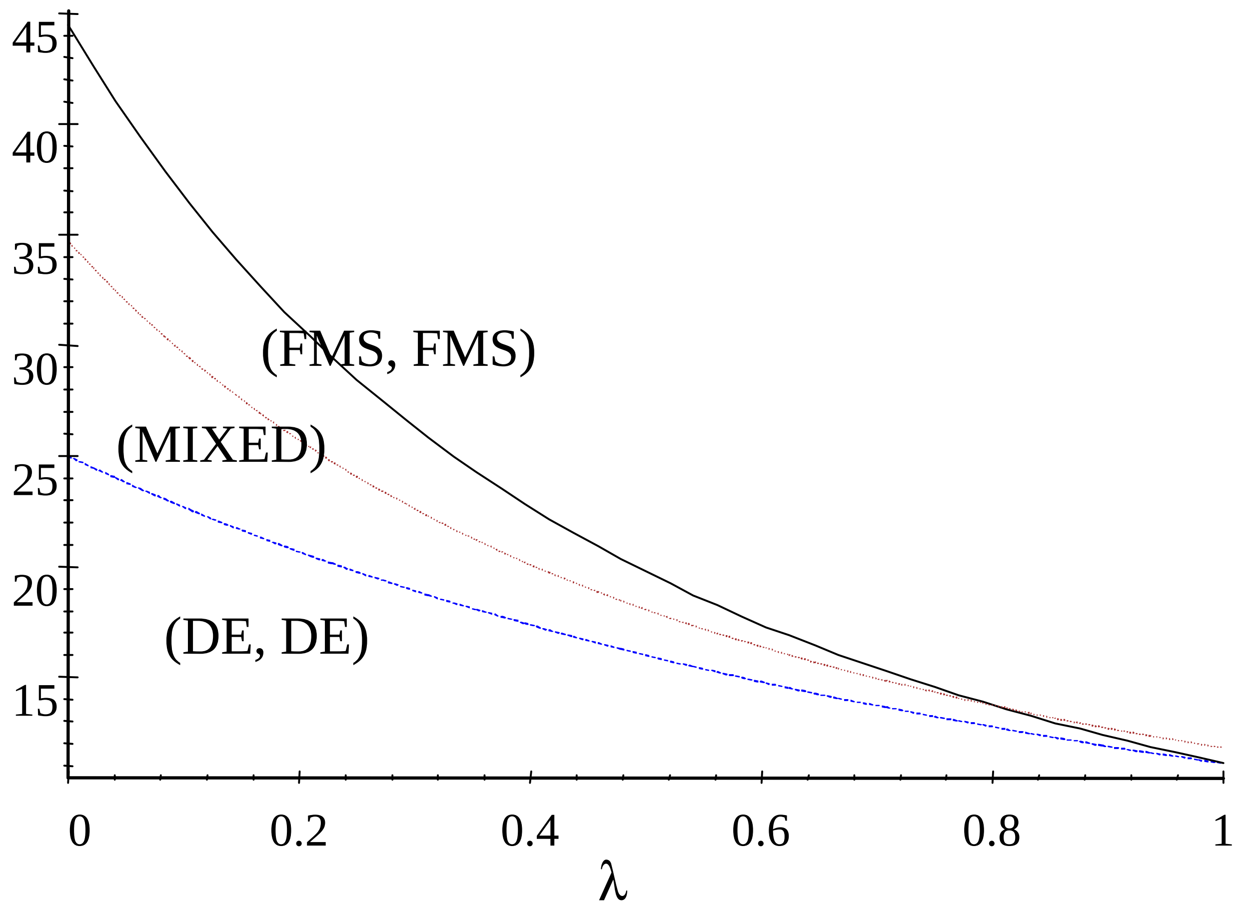

Figure 4' Consumer Surplus (*s=0.5*, *t=10*)

In this example, the consumer surplus is obviously not the highest in (FMS, FMS) over the whole range, which means R&T's Proposition 1 needs to be modified.

Proposition 1 (Roller and Tombak, 1990). *(a) Consumer surplus is always the lowest in a (DE, DE) equilibrium, and (b) Consumer surplus is the highest in (FMS, FMS) equilibrium only for* $\lambda \leq 0.796$.

Proof: (a) $CS(\text{mixed}) - CS(\text{DE, DE}) = \frac{t^2(25+8\lambda+\lambda^2)}{72(1+\lambda)^2} - \frac{t^2}{(2+\lambda)^2} = \frac{t^2(28-12\lambda-11\lambda^2+12\lambda^3+\lambda^4)}{72(1+\lambda)^2(2+\lambda)^2}$ which is ≥ 0 for all λ, t, s. And,

$CS(\text{FMS, FMS}) - CS(\text{DE, DE}) = \frac{4t^2}{9(1+\lambda)^2} - \frac{t^2}{(2+\lambda)^2} = \frac{t^2(7+5\lambda)(1-\lambda)}{9(1+\lambda)^2(2+\lambda)^2}$ which is ≥ 0 for all λ, t, s (equal at $\lambda = 1$).

(b) $CS(\text{FMS, FMS}) - CS(\text{mixed}) = \frac{4t^2}{9(1+\lambda)^2} - \frac{t^2(25+8\lambda+\lambda^2)}{72(1+\lambda)^2} = \frac{t^2(7-8\lambda-\lambda^2)}{72(1+\lambda)^2}$ which is ≥ 0 as long as $7 - 8\lambda - \lambda^2 \geq 0$, or $\lambda \leq 0.796$. Q.E.D.

These modifications do not affect R&T's discussion of the desirability of FMS adoption from the consumer welfare standpoint, particularly in highly differentiated markets (in footnote 4, the partial derivative should now read: $\frac{\partial CS_{\text{mixed}}}{\partial \lambda} = -\frac{t^2(7+\lambda)}{12(1+\lambda)^3}$, which is negative). However, at high product substitutability, FMS adoption by both producers leads to a reduction in consumer welfare (as compared to the mixed case). This is somewhat counter-intuitive because production for both markets by the manufacturers would seem to benefit the consumers due to increased competition. When products are highly substitutable, producing in one market makes the other market more competitive. When a firm adopts FMS and starts producing for both markets, it takes into consideration the negative externality it is imposing on the other market while producing in one market. This externality grows as products become more subtitutable (i.e., as λ approaches 1). Thus, in the mixed case, the firm with dedicated technology produces a higher quantity as it does not face the negative externality on the other market.[3] Thus, there can be too much investment in FMS from a consumer surplus viewpoint.

[3] I thank an anonymous referee for suggesting this explanation.

2. Producer Surplus: R&T's results on producer surplus are unaffected, and proposition 2 holds. However, Figure 6 is not plotted correctly, and the discussion following it is somewhat erroneous. A corrected version is given below.

Note that Figure $6'$ plots the total producers' surplus; for individual firm profits, see Figure $A2$.

As determined from conditions (2) and (4) in R&T using $t = 10, s = 0.5$, (FMS, FMS) is an equilibrium for $\lambda \leq \lambda^* = 0.9139$, and (DE, DE) is an equilibrium for $\lambda \geq \lambda^{**} = 0.9065$. There are two equilibria in the small region $0.9065 \leq \lambda \leq 0.9139$, and a Prisoner's Dilemma situation obtains for $\lambda < \lambda^{**}$.

3. Total Surplus: The correct expression for total surplus in both markets for a given technology state j is given below:

$$TS_j = t(Q_j^{A*} + Q_j^{B*}) - \frac{1}{2}((Q_j^{A*})^2 + (Q_j^{B*})^2) - 2\lambda Q_j^{A*} Q_j^{B*} - F_j^T$$

where F_j^T is the total fixed cost of technology adoption by both manufacturers. Expressions for total surplus in each of the three states are given in Figure $7'$. See Figure $8'$ for a corrected plot ($s = 0.5, t = 10$).

state	TS_j
(FMS, FMS)	$\frac{4t^2(2+\lambda)}{9(1+\lambda)^2} - 2(1+s)$
Mixed	$\frac{t^2(59+40\lambda-\lambda^2)}{72(1+\lambda)^2} - (2+s)$
(DE, DE)	$\frac{3t^2}{(2+\lambda)^2} - 2$

Figure $7'$: Total Surplus

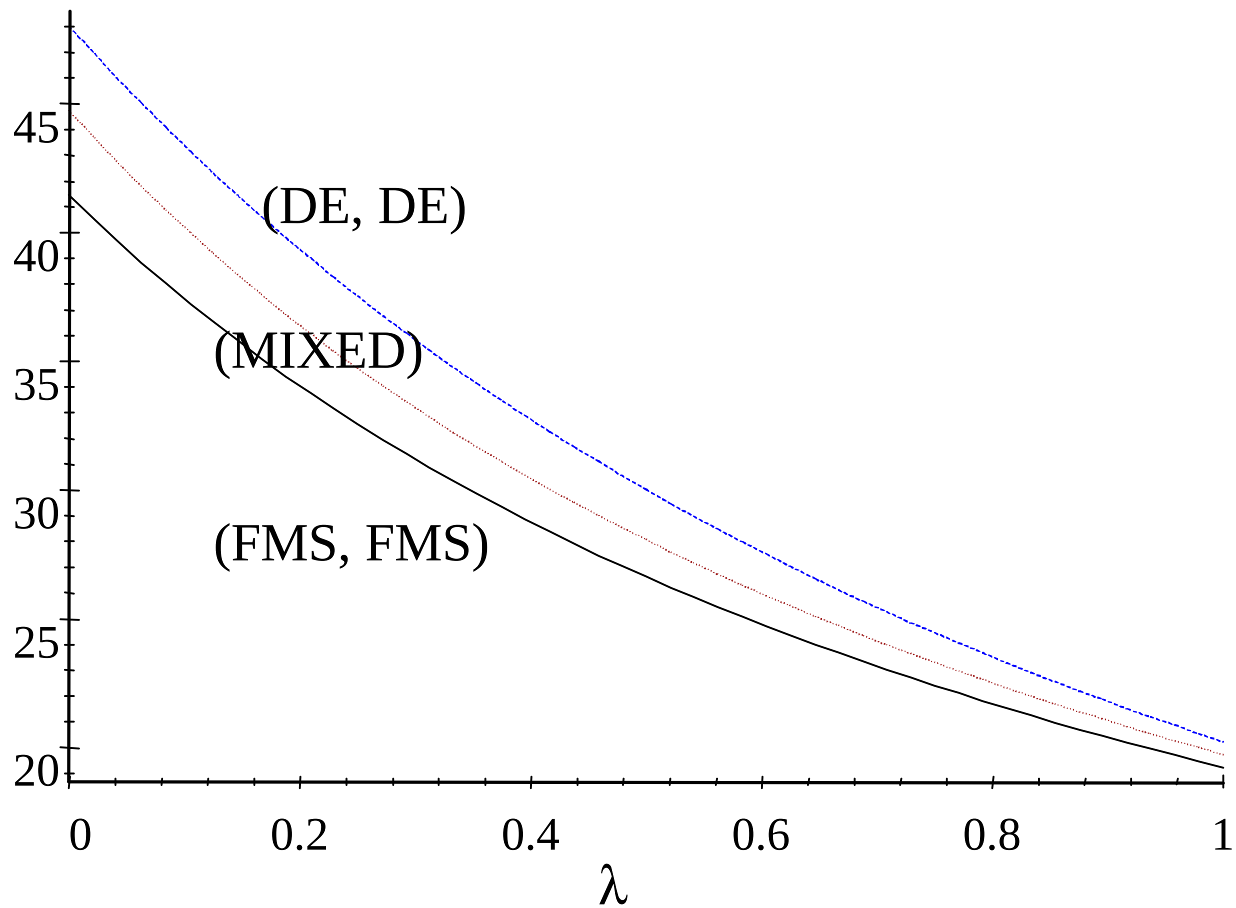

Figure 6' Producers' Surplus (*s=0.5*, *t=10*)

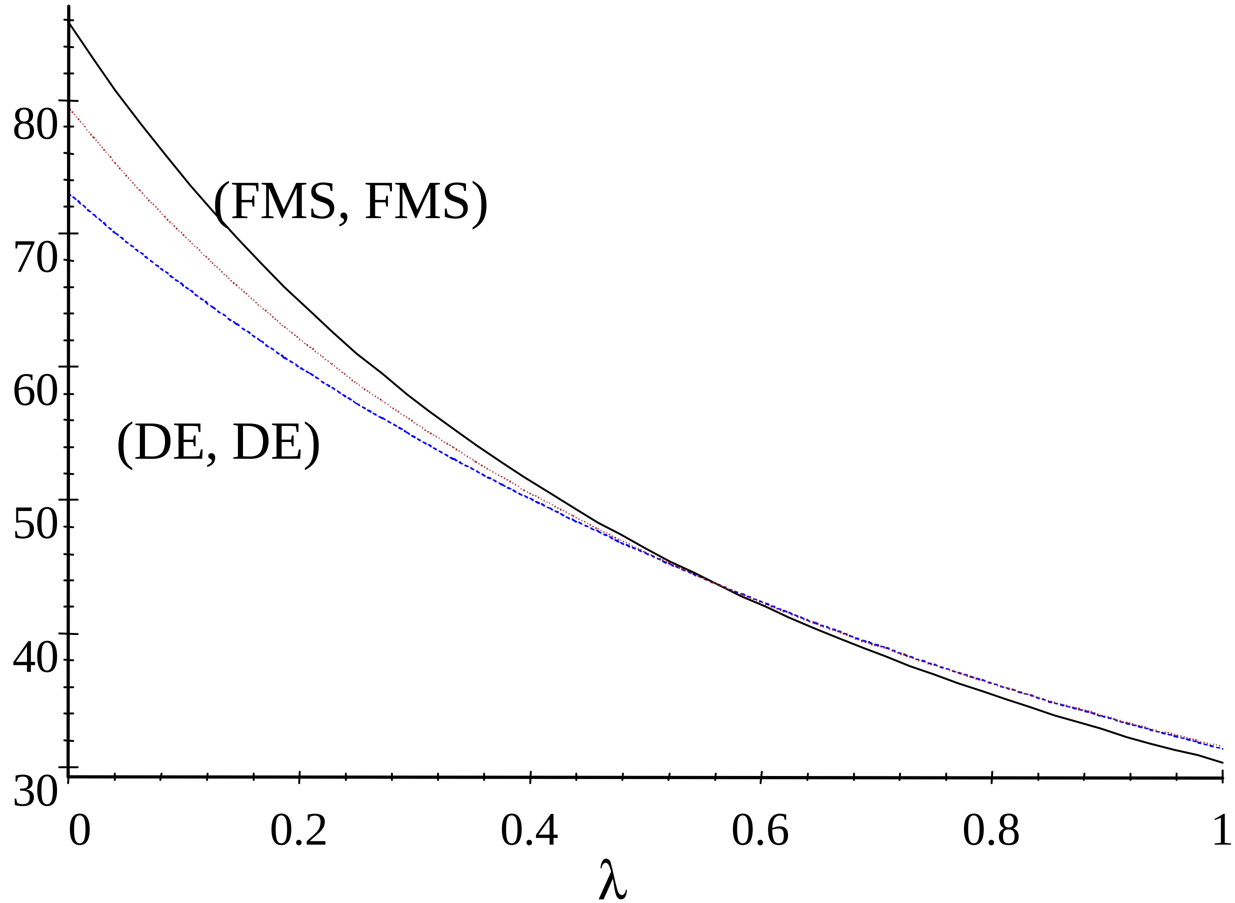

Figure 8' Total Surplus (*s=0.5*, *t=10*)

Therefore, while R&T's Proposition 3 holds qualitatively (i.e., (FMS, FMS) is the efficient market outcome when λ and s are low and t is high), the proof needs to be modified a little taking the correct expressions from Figure 7′: equation (7) should now read $TS_1 - TS_{\text{mixed}} = \frac{t^2(5-8\lambda+\lambda^2)}{72(1+\lambda)^2} - s$. Moreover, we see that the difference in total surplus between (FMS, FMS) and (DE, DE) states is not as large as depicted by R&T. For the example in figure 8′, (FMS, FMS) is efficient for $\lambda \leq \widehat{\lambda} = 0.555$, and (FMS, FMS) is an equilibrium of the technology choice game for $\lambda \leq \lambda^* = 0.9139$. However, for $\lambda > \widehat{\lambda}$, (DE, DE) is efficient; it is an equilibrium for $\lambda \geq \lambda^{**} = 0.9065$. This would imply that, from a total surplus standpoint, government subsidies to encourage investments in FMS are not necessary.

REFERENCES

EATON, B. C. and SCHMITT, N., 1994, 'Flexible Manufacturing and Market Structure', *American Economic Review*, 84, No. 4, pp. 875–888.

KIM, T., RÖLLER, L.-H. and TOMBAK, M. M., 1992, 'Strategic Choice of Flexible Production Technologies and Welfare Implications: Addendum et Corrigendum', *Journal of Industrial Economics*, XL, No. 2, pp. 233–235.

RÖLLER, L.-H. and TOMBAK, M. M., 1993, 'Competition and Investment in Flexible Technologies', *Management Science*, 39, No. 1, pp. 107–114.

RÖLLER, L.-H. and TOMBAK, M. M., 1990, 'Strategic Choice of Flexible Production Technologies and Welfare Implications', *Journal of Industrial Economics*, XXXVIII, No. 4, pp. 417–431.