THE JOURNAL OF INDUSTRIAL ECONOMICS WEBNOTE
Volume LIV September 2006 No.3

PATENT LENGTH AND THE TIMING OF INNOVATIVE ACTIVITY

JOSHUA S. GANS AND STEPHEN P. KING*

The standard result in patent policy, as demonstrated by Gilbert and Shapiro (1990), is that infinitely lived but very narrow patents are optimal as deadweight losses are minimised and spread through time but inventors can still recover their R&D expenditures. By extending their innovative environment to include timing as an important choice, we demonstrate that a finitely lived, but broader, patent can be socially desirable. This is because a patent breadth is a better instrument than length to encourage socially optimal timing. Thus, patents need not be infinitely long in order to encourage a greater number of inventions. *Journal of Economic Literature* Classification Numbers: L4, O34.

Keywords. innovation, patent length, patent breath, timing, R&D expenditures.

---

* Melbourne Business School and IPRIA, University of Melbourne (Gans) and Australian Competition and Consumer Commission (King). We are grateful to the co-editor and two anonymous referees for help comments and to IPRIA and the Australian Research Council for financial assistance. Of course, all views expressed are those of the authors and do not necessarily reflect any of the above organisations. All correspondence to: Joshua Gans (J.Gans@unimelb.edu.au). The latest version of this paper is available at: www.mbs.edu/jgans.

THE JOURNAL OF INDUSTRIAL ECONOMICS WEBNOTE
Volume LIV September 2006 No.3

I INTRODUCTION

TO DATE, THE LITERATURE on patent design has focused on how this impacts on whether innovations are pursued or not but not on their timing.[1] The goal of this paper is to consider innovation timing explicitly and consider what this implies for patent design: namely, the length and breadth of patents.

With regard to encouraging innovative activity per se, Nordhaus (1969) argued that patent length should be set so that, on average, the marginal social losses from innovations made unprofitable equal the marginal social gain from a shorter period of monopoly per innovation. Gilbert and Shapiro (1990) – hereafter GS – argued that patent breadth was another instrument that policy-makers had to limit deadweight losses. A narrower patent would allow entry of closer substitute products that constrain a patent-holder's prices and profits.

GS explored what mix of length and breadth may be optimal. They noted that both instruments could be used to achieve the goal of encouraging more innovations – and so were substitutes – but that each had a different temporal implication for deadweight losses. Adopting a longer patent with a narrow scope would spread those losses over time whereas a short patent with broad scope would concentrate them earlier. With concave social welfare and positive discounting, long, narrow patents would be optimal.

Rather than moving beyond GS by considering specific product and technologically competitive environments as others have done,[2] we consider a natural

[1] In contrast, timing is a principal focus of the literature on technology racing (Gilbert and Newbery, 1981; Reinganum, 1983, 1989; Katz and Shapiro, 1987).

[2] Most notably Klemperer (1990) who found that broad patents discouraged socially sub-optimal product substitution by consumers. Related contributions include Gallini (1992) who sees broad patents as a way of discouraging costly imitation or Denicolo (1996) who explores how length and breadth impact on the nature of competition in patent races. Matutes, Regibeau and Rockett (1996) consider the impact of design on the diffusion of innovations. Finally, there is a host of contributions that consider the sequential innovation environment. For instance, O'Donoghue, Scotchmer and Thisse (1998) consider how patent length and breadth can impact upon the rate of innovative activity. In their case, firm's decisions to innovate do not involve a timing dimension but competition can impact on the rate of effective technological change. See Lampe and Niblett (2003) for a recent survey.

change to the process of innovation; something that would apply across a wide class of innovations. For GS, innovation is a simple payment in terms of once off R&D costs that produces an innovation (and patent) instantaneously. We suppose instead that the inventor chooses the timing of innovative activity and faces additional costs (in terms of different profitability) if an innovation occurs sooner rather than later. This type of invention process is commonly used in models of technological competition (e.g., Katz and Shapiro, 1987).

This change means that a policy-maker needs to consider both the profitability of innovating (as in GS) and when an innovation actually occurs. We demonstrate that patent length and breadth have different impacts, as instruments, on the timing of innovative activity. Indeed, a broader patent can provide a means of accelerating innovation timing while at the same time compensating inventors for their activity and hence, reducing the need for a longer patent.

Section 2 presents a simple model with these features while Section 3 demonstrates our main result that finite length patents can be socially desirable. We show there the conditions under which the GS result fails to hold when timing is endogenous. A final section concludes.

II MODEL OF INNOVATION TIMING

We begin by presenting a model of innovation timing. An inventor can produce an innovation at any time, $t \geq 0$. Following Katz and Shapiro (1987), we assume that time periods have length Δ and explore continuous time solutions as Δ approaches 0. In each time period, the inventor decides whether to innovate or wait. Once an innovation is generated, it can earn economic value immediately. We assume that if the inventor innovates at time *T*, then the current costs of inventing are $K(T)e^{rT}$. The discount factor is $\delta = e^{-r\Delta}$. Thus, invention costs have a present value as viewed from time t = 0, of

$K(T)$. Like Katz and Shapiro we assume that investment costs decline with time ($d(K(T)e^{rT})/dT < 0$ and $d^2(K(T)e^{rT})/dT^2 > 0$) and that $\lim_{T\to\infty} K(T) = K \geq 0$.

We assume that if an innovation is not patented, the inventor earns no rents.[3] Once an innovation has been generated, while it is patented it can earn the inventor, $\pi(T)$, per period. The important feature here, that is a departure from the literature on patent design, is that inventor rents depend upon the time an innovation is generated. The basic idea is that no innovation has value in isolation. In particular, new ideas are coming into the economy that may complement or enhance the inventor's idea. Specifically, the inventor may be able to adjust the quality and economic value of the innovation prior to submitting a patent application that takes such changes into account. In this sense, the actual time a patent is filed may affect both inventor profits and, as we assume below, social welfare. In any case, the inventor does not have control over the rate at which these alternative ideas are become available but can forecast this to some degree. We make no assumption regarding the relationship of inventor rents to *T* except that π is quasi-concave.[4]

Social welfare from an innovation is simply $W(\pi(T),T)$ and is assumed, as in GS, to be non-increasing and concave in inventor profits and quasi-concave in *T*. Finally, we assume that $\lim_{T\to\infty} e^{-rT}\int_0^\infty W(\pi(T),T)e^{-rx}dx > K$ so that it is socially optimal to develop the innovation at some finite date.

First Best Timing

In this situation, what is the first best innovation timing? It is clear that this would involve an unpatented innovation, the timing of which is determined by:

[3] GS assume that a non-patented innovation can earn a positive rent to the innovator. Including this adds some additional notation but would not change any qualitative result below.

[4] To be sure, an assumption that innovation value depends upon the time of innovation stands in contrast to an assumption where the value of the innovation may depend upon the actual time, *t*, or, more generally, the time since innovation, *t* – *T*. In Gans and King (2004), we demonstrated that we can generalise our simple specification somewhat and preserve our main results.

$$\max_T e^{-rT}\int_0^\infty W(0,T)e^{-rx}dx - K(T) \quad (1)$$

The first best timing, T^*, will satisfy:

$$\int_0^\infty \left(\tfrac{\partial W(0,T^*)}{\partial T} - rW(0,T^*)\right)e^{-rx}dx = \tfrac{1}{r}\tfrac{\partial W(0,T^*)}{\partial T} - W(0,T^*) = K'(T^*) \quad (2)$$

Inventor Timing

Suppose, instead, that the inventor had an infinitely lived patent over the innovation. In this case, it would solve:

$$\max_T e^{-rT}\int_0^\infty \pi(T)e^{-rx}dx - K(T) \quad (3)$$

The privately optimal timing, $\hat{T}$, satisfies:

$$\int_0^\infty \left(\tfrac{\partial \pi(\hat{T})}{\partial T} - r\pi(\hat{T})\right)e^{-rx}dx = \tfrac{1}{r}\tfrac{\partial \pi(\hat{T})}{\partial T} - \pi(\hat{T}) = K'(\hat{T}) \quad (4)$$

It is interesting to note that the privately optimal timing may be delayed or accelerated relative to the socially optimal timing (see Katz and Shapiro, 1987). Specifically, $\hat{T} \geq T^*$ if $\tfrac{\partial \pi(T^*)}{\partial T} \geq r(\pi(T^*) + K'(T^*))$. This, in turn, implies that:

$$r\left(W(0,T^*) - \pi(T^*)\right) \geq \tfrac{\partial W(0,T^*)}{\partial T} - \tfrac{\partial \pi(T^*)}{\partial T} \text{ or } r\left(W(0,\hat{T}) - \pi(\hat{T})\right) \leq \tfrac{\partial W(0,\hat{T})}{\partial T} - \tfrac{\partial \pi(\hat{T})}{\partial T} \quad (5)$$

giving a necessary condition of $\tfrac{\partial W(0,\hat{T})}{\partial T} > \tfrac{\partial \pi(\hat{T})}{\partial T}$. This type of condition may naturally arise in a model where broader welfare depends upon other ideas at the time of innovation but those ideas do not fundamentally impact on the profits of the inventor. Notice that if $\tfrac{\partial W(0,\hat{T})}{\partial T} < \tfrac{\partial \pi(\hat{T})}{\partial T}$, then it is certainly the case that $\hat{T} < T^*$. This could arise in a situation where the inventor might gain an important first mover advantage from earlier innovation that otherwise does not have much impact on social welfare.

III OPTIMAL PATENT LENGTH AND BREADTH

We are now in a position to revisit the analysis of GS regarding the optimal length and breadth of patents. In their analysis, the optimal patent policy was the solution to the following social planner's problem:

$$\max_{\alpha,\tau} \Omega(\hat{T},\alpha) \equiv e^{-r\hat{T}} \left(\int_0^\tau W(\pi(\hat{T},\alpha),\hat{T})e^{-rt}dt + \int_\tau^\infty W(0,\hat{T})e^{-rt}dt \right) - K(\hat{T})$$

subject to:

$$e^{-r\hat{T}} \int_0^\tau \pi(\hat{T},\alpha)e^{-rt}dt \geq K(\hat{T}) \text{ (PC)}$$

$$\hat{T} \in \arg\max_T e^{-rT} \int_0^\tau \pi(T,\alpha)e^{-rt}dt - K(T) \text{ (IC)}$$

where here we add the (IC) constraint as timing as well as participation is chosen by the inventor. α is a measure of the breadth of patent protection and it is assumed that $\pi(.)$ is increasing in α and were $\alpha = 0$ is a normalisation describing the minimum feasible patent breadth. That is, $\alpha = 0$ implies that with an infinite patent length the inventor breaks even: i.e., $\pi(\hat{T},0) = rK(\hat{T})$ where $\hat{T}$ is chosen to maximise inventor profits given the infinite patent length and $\alpha = 0$.

Exogenous Timing

Let $\phi(T,\tau)$ be the total profit flow required so that an innovation generated at time T satisfies (PC).Then:

$$(1-e^{-r\tau})\phi(T,\tau) = re^{rT}K(\hat{T}) \tag{6}$$

Taking the derivative of (6) with respect to τ gives: $e^{-r\tau}r\phi(T,\tau) + \frac{\partial\phi}{\partial\tau}(1-e^{-r\tau}) = 0$. If patent length is set at τ, and α such that $\pi(T,\alpha) = \phi(T,\tau)$, total welfare becomes a function, $\Omega(T,\phi(T,\tau))$. Now consider how social welfare changes with patent length, τ:

$$\frac{d\Omega}{d\tau}=\frac{\partial\Omega}{\partial\tau}+\frac{\partial\Omega}{\partial\phi}\left(\frac{\partial\phi}{\partial\tau}+\frac{\partial\phi}{\partial T}\frac{\partial T}{\partial\tau}\right)+\frac{\partial\Omega}{\partial T}\left(\frac{\partial T}{\partial\tau}+\frac{\partial T}{\partial\phi}\frac{\partial\phi}{\partial\tau}\right) \tag{7}$$

When T is fixed, GS demonstrated that (7) was everywhere positive. That is,

$$\begin{aligned}\frac{d\Omega}{d\tau}&=\frac{\partial\Omega}{\partial\tau}+\frac{\partial\Omega}{\partial\phi}\frac{\partial\phi}{\partial\tau}\\&=e^{-rT}\left(\left(W(\phi(\tau),T)-W(0,T)\right)e^{-r\tau}+\tfrac{\partial W(\phi(\tau),T)}{\partial\alpha}\phi'(\tau)(1-e^{-r\tau})/r\right)\\&=e^{-r(T+\tau)}\left(W(\phi(\tau),T)-W(0,T)+\tfrac{\partial W(\phi(\tau),T)}{\partial\alpha}\phi(\tau)\right)>0\end{aligned} \tag{8}$$

where the third step comes from the definition of ϕ and the final sign comes from the earlier assumption that W is concave in π. Thus, GS demonstrate that the social optimal patent policy involves setting α so that $\pi = 0$ and τ infinitely large to satisfy (PC).

Before turning to consider the impact of endogenous timing, it is useful to reflect on the intuition for this result. Basically, innovations are desirable and greater patent length and breadth can ensure that they are privately profitable. If the marginal social loss associated with greater monopoly rents becomes higher the greater those rents are (that is W is concave in π), it is particular costly to incur those social losses sooner rather than later. For this reason, a longer patent life can spread those losses over time but also redistribute them forward in a socially desirable manner. Hence, patent length is a less costly instrument to use to encourage innovation than patent breadth.

Endogenous Timing

When timing is endogenous, the two instruments of patent policy cannot be traded-off in the same manner as above as the planner will be concerned both with the level of innovative activity as well as its timing. To this end, there is one key issue: how effective are patent length and breadth as instruments to influence innovation timing? Looking at the (IC) constraint, notice that the first order condition is:

$$\int_0^{\tau}\left(\tfrac{\partial\pi(\hat{T},\alpha)}{\partial T}-r\pi(\hat{T},\alpha)\right)e^{-rx}dx=K'(\hat{T}) \tag{9}$$

Notice that changing breadth (α) will cause earlier innovation if:

$$\frac{\partial^2 \pi(\hat{T},\alpha)}{\partial T \partial \alpha} < r \frac{\partial \pi(\hat{T},\alpha)}{\partial \alpha} \tag{10}$$

The right hand side of (10) is positive by definition. In addition, it is reasonable to assume that the marginal benefit from greater patent breadth becomes smaller as the innovation is delayed (perhaps because more substitute products become available over time). For this reason, patent breadth will be an effective instrument for influencing innovation timing.

In relation to considering patent length (τ), an increase in τ will cause earlier innovation if:

$$\frac{\partial \pi(\hat{T},\alpha)}{\partial T} \geq r\pi(\hat{T},\alpha) \tag{11}$$

Notice that if $K(T)$ were increasing in T, then (11) would always hold strictly. If $K(T)$ were non-increasing in T it would never hold. In this case, increasing patent length would either *not* be effective at altering innovation timing or would result in delayed innovation.

To take this further, consider an extreme case where (PC) never binds – i.e., $K = 0$. In this case, as patent length does not impact on timing and participation is not an issue it can optimally be set arbitrarily close to zero. Instead, positive patent breadth (α) can be used to accelerate timing (if that is desired) trading off the social value of earlier innovation with the small short-term cost of higher monopoly power. However, as τ becomes close to 0, this latter cost becomes negligible and so α will be set so that $\hat{T} = T^*$.

This provides a strong indication that, when timing is endogenous, the GS result regarding the desirability of an infinite patent length and very narrow scope may not hold in general. Indeed, the following proposition characterises when a finite patent length is optimal.

Proposition. *Let* $\hat{T}(\infty,0)$ *be the privately chosen innovation timing when* $(\tau,\alpha) = (\infty,0)$. *Then a finite patent life is socially optimal if* $\hat{T}(\infty,0) \geq T^*$ *and* $K'(\hat{T}(\infty,0)) \leq 0$.

The proof is in an appendix. Intuitively, raising α above 0 can only accelerate innovation timing. When this is socially desirable at an infinite patent length, then the proposition demonstrates that it is socially desirable it raise α. This, in turn, relaxes (PC) and makes it optimal to have a finite patent length which it self contributes to the accelerated innovation timing (when $K'(\hat{T}(\infty,0)) \leq 0$).

This can be seen even more clearly with the following special case. Katz and Shapiro (1987) assume that innovator profits (and social welfare) do not depend upon time. In this situation, invention timing is the solution to: $\max_T \frac{1}{r} e^{-rT} \pi (1-e^{-r\tau}) - K(T)$ with first order condition: $-e^{-r\hat{T}}(1-e^{-r\tau})\pi = K'(\hat{T})$. Notice that, in this case, an increase in τ leads to a decrease in $\hat{T}$.

With this specification (and π and W independent of time) it is always the case that $\hat{T} > T^*$ even for an infinitely lived patent. Therefore, even when setting this type of patent length, it may be desirable for the social planner to set patent breadth (α) above a minimum of $\underline{\alpha}$ where $\underline{\alpha}$ is such that $\pi(\underline{\alpha}) = re^{r\hat{T}} K(\hat{T})$. This would allow socially desirable acceleration of patent timing at the cost of greater deadweight losses; a choice that could optimal in some environments.

In contrast, when $K'(\hat{T}(\infty,0)) > 0$, it is possible that a reduction in patent length could lead to a decrease in $\hat{T}$. Thus, a finite patent length (even when combined with greater patent breadth) may not lead to accelerated innovation timing in a way that increases social welfare. Hence, we cannot rule out an infinite patent length as socially optimal.

When $\hat{T}(\infty,0) < T^*$ and $K'(\hat{T}(\infty,0)) \leq 0$, an infinite patent length and narrow patent breadth does become socially optimal. This is because increasing α will lead to accelerated innovation timing that, in this case, reduces social welfare. Of course, if $K'(\hat{T}(\infty,0)) > 0$, then patent length could become an instrument for improving (i.e., delaying) innovation timing and so a finite patent length could be socially desirable.

IV CONCLUSION

This paper has made a seemingly minor, but realistic, adjustment to the environment considered by GS. We have demonstrated that their result that socially optimal patents should be long lived but narrow may not hold. Specifically, patent length is an imperfect instrument for guiding optimal patent timing and hence, there is a need to rely upon breadth as an alternative. The reason for this is that in GS's environment only the number of innovations was at issue and patent length and breadth were two substitute instruments to encourage this. Here there are two dimensions to social welfare – number and timing of innovations – and hence, both patent breadth and length can be used to improve welfare. Our results suggest that patent policies with patent scope beyond its narrowest conceivable application are likely to be socially desirable.

APPENDIX: PROOF OF PROPOSITION

Taken together, (9) and (10) imply that (7) can be rewritten as:

$$\frac{d\Omega}{d\tau} = \underbrace{\frac{\partial\Omega}{\partial\tau} + \frac{\partial\Omega}{\partial\phi}\frac{\partial\phi}{\partial\tau}}_{>0} + \frac{\partial\Omega}{\partial T}\underbrace{\frac{\partial T}{\partial\phi}\frac{\partial\phi}{\partial\tau}}_{>0}$$

$$= e^{-r(T+\tau)}\left(W(\phi(\tau,T),T) - W(0,T) - \tfrac{\partial W(\phi(\tau,T),T)}{\partial\alpha}\phi(\tau,T)\right) \qquad (12)$$

$$+\frac{\partial\Omega}{\partial T}\frac{\frac{\partial^2\pi(\hat{T},\alpha)}{\partial T\partial\alpha} - r\frac{\partial\pi(\hat{T},\alpha)}{\partial\alpha}}{\frac{\partial^2\pi(\hat{T},\alpha)}{\partial T^2} - r\frac{\partial\pi(\hat{T},\alpha)}{\partial T}}\frac{r\phi(T,\tau)e^{-r(T+\tau)}}{(1-e^{-r(T+\tau)})}$$

A sufficient condition to preserve the Gilbert-Shapiro result would be for $\partial\Omega/\partial T \geq 0$ at $\alpha = 0$. This requires that at $\hat{T}$ such that $\frac{\partial\pi(\hat{T})}{\partial T} = r(\pi(\hat{T}) + K'(\hat{T}))$, $r\left(W(0,\hat{T}) - \pi(\hat{T})\right) \geq \frac{\partial W(0,\hat{T})}{\partial T} - \frac{\partial\pi(\hat{T})}{\partial T}$. As noted earlier, this condition implies that $\hat{T}(\infty,0) < T^*$ which we rule out.

Therefore, here $\partial\Omega/\partial T < 0$ at $(\tau,\alpha) = (\infty,0)$. Moreover, given the concavity of *W*, the first terms of (12) are arbitrarily small at this patent policy. Note also that at this policy:

$$\frac{\partial\Omega}{\partial T} = \frac{e^{-r(T+\tau)}}{r}\left(\frac{\partial W(0,T)}{\partial T} - rW(0,T) + (1-e^{r\tau})\left(\begin{array}{l} rW(rK(T)/(1-e^{-r(T+\tau)}),T) - \frac{\partial W(rK(T)/(1-e^{-r(T+\tau)}),T)}{\partial T} \\ -\frac{e^{r(T+\tau)}r(rK(T)-K'(T))}{(1-e^{r(T+\tau)})^2}\frac{\partial W(rK/(1-e^{-r(T+\tau)}),T)}{\partial\phi}\end{array}\right)\right)$$

(13)

Using L'Hopital's rule, we have:

$$\lim_{\tau\to\infty}\frac{\partial\Omega}{\partial T} = -e^{-rT}\left(rW(rK(T)/(1-e^{-rT}),T) - \tfrac{\partial W(rK(T)/(1-e^{-rT}),T)}{\partial T}\right) < 0 \qquad (14)$$

as $\hat{T} > T^*$. Thus, when timing is endogenous the socially optimal patent policy involves a finite patent life.

REFERENCES

Denicolo, V. (1996), "Patent Races and Optimal Patent Breadth and Length," *Journal of Industrial Economics*, 44, pp.249-265.

Gallini, N. (1992), 'Patent Policy and Costly Imitation', *Rand Journal of Economics*, 23, pp.52-63.

Gans, J.S. and S.P. King (2004), "Patent Length and the Timing of Innovative Activity," *Working Paper*, No.07/04, IPRIA (www.ipria.org).

Gilbert, R. and D. Newbery (1982), "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review*, 72 (3), pp.514–526.

Gilbert, R. and C. Shapiro (1990), "Optimal Patent Length and Breadth," *Rand Journal of Economics*, 21, pp.106-112.

Katz, M.L. and C. Shapiro (1987), "R&D Rivalry with Licensing or Imitation," *American Economic Review*, 77 (3), pp.402-420.

Klemperer, P. (1990), "How Broad Should the Scope of Patent Protection Be?" *Rand Journal of Economics*, 21, pp.113-130.

Lampe, R. and A. Niblett (2003), "The Economics of Patent Design: A Selective Survey," *Working Paper*, No.06/03, Intellectual Property Research Institute of Australia (www.ipria.org).

Matutes, C., P. Regibeau and K. Rockett (1996), "Optimal Patent Design and the Diffusion of Innovations," *RAND Journal of Economics*, 27 (1), pp.183-196.

Nordhaus, W. (1969), *Invention, Growth and Welfare: A Theoretical Treatment of Technological Change*, Cambridge Mass, MIT Press.

O'Donoghue, T., S. Scotchmer, J-F. Thisse (1998), "Patent Breadth, Patent Life, and the Pace of Technological Progress," *Journal of Economics and Management Strategy*, 7 (1), pp.1-32.

Reinganum, J.F. (1983), "Uncertain Innovation and the Persistence of Monopoly," *American Economic Review*, 73 (4), pp.741–748.

Reinganum, J.F. (1989), "On the Timing of Innovation," in R. Schmalansee and R.D. Willig, eds., *Handbook of Industrial Organization*, Vol. 1, Amsterdam: North Holland, pp.849–908.