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A NOTE ON THIRD-DEGREE PRICE DISCRIMINATION AND OUTPUT*

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It is known that monopolistic third-degree price discrimination decreases aggregate welfare if total output falls. In contrast to Adachi [this Journal, 2002], this note shows that welfare must decrease under price discrimination even if total output remains constant.

VARIAN [1985] AND SCHWARTZ [1990] PROVED THAT, very generally, monopolistic third-degree price discrimination decreases aggregate welfare if total output *falls* (a conjecture which dates back to the work of A. C. Pigou, [1920]). In particular, these authors adopted a representative consumer approach (by assuming quasi-linear preferences[1]) and used revealed-preference arguments. *En passant*, their proofs (Varian, [1985: p.875] and Schwartz, [1990: p.1261]) also show that welfare cannot improve if total output is unchanged. However, in a recent paper in this Journal, Adachi [2002], by using linear interdependent demands ('goods' are formally *q*-complements: see e.g. Deaton and Muellbauer, [1980: p.57]), appears to suggest that welfare may increase even if total output does not change.

In further contrast to the latter result,[2] this note aims to prove that welfare must *decrease* under price discrimination *if total output remains constant*. In fact, our argument is based on a simple and well-known result of consumer theory. To see it, let us refer to the model in Schwartz [1990]. Consumers have quasi-linear preferences, and thus we can treat aggregate consumer behavior as if it were due to a "representative consumer" with indirect utility function $v(\boldsymbol{p}) + y_0$, where $\boldsymbol{p}$ is the vector of prices that the monopolist requires for his product (which is homogeneous, so that total cost depends only on total output) in n (possibly different and interdependent) markets, and y_0 is the total endowment of the *numéraire* (the Marshallian composite commodity).[3] By quasi-linearity, $v(\cdot)$ is convex and aggregate (social) welfare can be written as $W(\boldsymbol{p}) = \pi(\boldsymbol{p}) + v(\boldsymbol{p}) + y_0$, where $\pi(\boldsymbol{p})$ is the monopolist's profit function. We wish to compare the case of a uniform price vector $\boldsymbol{p}^u$ ($\boldsymbol{p}^u = p^u \boldsymbol{\iota}$, where $\boldsymbol{\iota}$ is the relevant unit vector) with the case of a discriminatory price vector $\boldsymbol{p}^d$ ($p_i^d \neq p_j^d$ for at least some $i \neq j$, $i, j = 1, n$), under the assumption that total output does

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[1] A standard assumption in the literature, that also validates, as it is well known, the use of Marshallian surplus as a welfare measure.

[2] The problem with it is in the way the change in the Marshallian consumer surplus is computed (as it is well known, this change is measured by a line integral).

[3] Bold characters are used to indicate vectors; conventionally, $\boldsymbol{x}$ is a column vector and $\boldsymbol{x}'$ indicates its transpose (a row vector).

not change (i.e., $\iota'\boldsymbol{q}(\boldsymbol{p}^u) = \iota'\boldsymbol{q}(\boldsymbol{p}^d)$, where $\boldsymbol{q}(\boldsymbol{p})$ is the demand system faced by the monopolist).

Let us compute the change in profit while moving from uniform to discriminatory pricing. Since

$$\Delta\pi = \pi(\boldsymbol{p}^d) - \pi(\boldsymbol{p}^u), \tag{1}$$

it is easy to see that:

$$\Delta\pi = \boldsymbol{p}^{d}{}'\boldsymbol{q}^d - \boldsymbol{p}^{u}{}'\boldsymbol{q}^u = \boldsymbol{p}^{d}{}'\boldsymbol{q}^d - \boldsymbol{p}^{u}{}'\boldsymbol{q}^d = (\boldsymbol{p}^d - \boldsymbol{p}^u)'\boldsymbol{q}^d = \Delta p, \tag{2}$$

where Δp is the Paasche price variation relevant for the representative consumer. Now, consider the relevant Hicksian equivalent variation EV: by definition, $EV = v(\boldsymbol{p}^u) - v(\boldsymbol{p}^d)$. Since it is well known (see e.g. Deaton and Muellbauer, [1980], paragraph 7.1) that the Hicksian equivalent variation EV is larger than Δp (unless in the very special case of zero substitution effects, in which case EV and Δp are equal),[4] it follows that:

$$\Delta v = v(\boldsymbol{p}^d) - v(\boldsymbol{p}^u) = -EV < -\Delta p, \tag{3}$$

and thus:

$$\Delta W = \Delta\pi + \Delta v = \Delta p - EV < 0. \tag{4}$$

The intuition behind the previous results is actually straightforward: (3) shows that, if total output remains constant, by discriminating through linear prices the monopolist gains (loses) no more (no less) than the representative consumer loses (gains). Thus it must be the case that $\Delta W \leq 0$ (for our purpose it is unnecessary to assume that $\boldsymbol{p}^u$ and $\boldsymbol{p}^d$ maximize the monopolist's profit respectively under uniform and differentiated pricing). Moreover, if the representative consumer's preferences exhibit some scope for the substitution of the monopolist's commodity (the general case), then they are unable to just exchange the variation in the (gross) consumer surplus (part of it "gets lost" in the process, since the representative consumer trades off the monopolist's commodity with the *numéraire*). Thus $\Delta W < 0$ when moving from uniform to discriminatory pricing. Note that our argument applies to *any* price change (from uniform pricing), *provided* that total output remains constant (in fact, it applies to any price change such that $\Delta\pi \leq \Delta p$ holds).

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[4] This amounts to assume that $v(\boldsymbol{p}^u)$ is (locally) strictly convex, an assumption analogous to the one made in Schwartz [1990: p. 1260].