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VERTICAL INTEGRATION AND SHARED FACILITIES IN UNREGULATED INDUSTRIES*

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In this paper we analyze the equilibrium market structure, following liberalization, of an industry involving an essential facility. Two alternative modes of market entry are considered, in conjunction with vertical integration, namely: (i) full entry, which means building a new and more efficient facility at a positive fixed cost; and (ii) partial entry, which means purchasing existing capacity from the incumbent, at a fixed price per unit that is freely negotiated between the incumbent and the entrant. We show that vertical integration is a dominant strategy for each firm under either entry mode, and that upstream firms choose to share the incumbent's facility when the entrant's fixed cost exceeds a positive threshold. In addition, welfare analysis shows that in many situations the market can efficiently solve the trade-off between fixed-cost savings and softened downstream competition, thus providing a rationale for the liberalization of such industries. Several competition policy implications are discussed. *Journal of Economic Literature* Classification Numbers: L4, O34.

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I INTRODUCTION

OVER THE LAST FEW DECADES there has been a global trend towards market liberalization and the introduction of competition among services that were previously provided by monopolists; examples include electricity, railways, telecommunications, water, oil and gas supply, airports and ports, all of which are now open to competition. Such markets are characterized by the presence of vertical integration and essential facilities (i.e. a facility that is essential for downstream production). Nowadays it is common to see port operators integrated with shipping companies, oil transportation and electricity companies integrated with distribution companies, and hospitals and/or health plans operating in partnership with medical clinics and physicians.[1]

In this paper we examine the problem of entering the essential-facility market, in conjunction with vertical integration and foreclosure. We consider a situation in which there are initially two independent downstream firms competing a-là-Cournot, and an nonintegrated monopoly upstream that owns an essential facility of unlimited capacity. The industry is then liberalized, so upstream Cournot competition and vertical integration are now allowed; and there is a potential upstream entrant facing two possible entry modes: (i) full entry, by building a new and more efficient facility at a positive fixed cost (hereinafter referred to as facility-based competition); or (ii) partial entry, by purchasing existing capacity from the incumbent at a fixed price per unit, freely negotiated between the incumbent and the entrant (hereinafter referred to as facility sharing).[2] This paper studies the equilibrium market structure and welfare implications, following liberalization, of two alternative modes of market entry in conjunction with vertical integration.

The post-liberalization equilibrium market structure is as follows. Facility-based competition and full vertical integration occurs when the fixed cost is below a positive

[1]In many countries vertical integration is increasingly the rule rather than the exception in service sectors such as electricity, telecommunications, natural gas, water and sewerage. See Newbery (1999) for a review of this topic.

[2]We do not allow for general nonlinear pricing. As suggested by one referee, a justification for linear pricing is the imposition of nondiscriminatory open access. In that sense, the model considered here can be thought of as one in which the only regulation is nondiscriminatory open access. Nonetheless, most results are robust to two-part tariffs.

threshold, whereas facility sharing and full vertical integration occurs otherwise. Vertical integration is a dominant strategy for each firm under either entry mode, since it avoids double marginalization and counters the integrated firm's incentive to foreclose by buying input from the nonintegrated upstream firm and hence easing downstream competition.[3] Facility-based competition occurs when the fixed cost is below a given threshold, since the incumbent's gain from preventing competition by stopping the entrant from building its own facility is outweighed by the loss from selling capacity at a price below marginal cost. If the fixed cost is large enough, however, the essential facility is shared since the per-unit price of capacity rises with the fixed cost, yet it never reaches the monopoly price. This result stems from the fact that the entrant trades off the benefit of building its own facility and having a low marginal cost, with that of sharing the incumbent's facility and facing a higher marginal cost given the per-unit capacity price, but avoiding the fixed cost of building the essential facility.

The welfare analysis gives rise to two key lessons. First, in many cases the market can efficiently solve the trade-off between avoiding fixed costs and softening downstream competition. For instance, facility-based competition is the socially preferred outcome whenever it is observed; and facility sharing is always observed and efficient when fixed costs are high. Second, the relationship between fixed costs and social welfare is non-monotonic, so competition authorities have to be careful when reviewing antitrust cases. From society's point of view, facility sharing avoids duplication of facilities and thus generates a cost saving equal to the fixed cost, but this results in higher variable production costs since the entrant's facility would have been more efficient. The intensity of competition is also diminished since the per unit capacity price is higher than the entrant's marginal cost when building its own facility.

These results have interesting policy implications.[4] First, facility-based competition and full vertical integration is pro-competitive and efficient. This means that when a market is liberalized, competition authorities should not oppose facility-based competition and

[3]This form of raising rival's costs strategy was first studied formally by Salop and Sheffman (1987).

[4]The policy prescriptions suggested here should be qualified by the fact that we only consider two different entry modes; no collusion is assumed; nonlinear pricing of capacity is not allowed; and the incumbent cannot degrade the quality of access to its facility.

vertical integration. Second, under facility sharing, vertical integration by the incumbent is not necessarily associated with restricted access to the incumbent's facility. Third, our results challenge the widely held belief that if the owner of an essential facility charges a downstream competitor a different price than it charges its vertically integrated affiliate, it acquires market power. Our results suggest that, depending on the relative efficiency of the entrant's facility, charging a price above the incumbent's upstream marginal cost of production may be socially efficient. Fourth, after liberalization the access charge is sometimes set below the incumbent's marginal cost, which is socially efficient.

As far as we are aware, this is the first time vertical integration has been analyzed in conjunction with facility sharing, although each of these phenomena has been extensively studied separately. From the standpoint of vertical integration, the paper's closest relation is the study by Gaudet and Van Long (1997). These authors show that vertical integration is a dominant strategy when the number of upstream and downstream firms is less than five, and that multiple equilibria exist for more than four firms, with full vertical integration being one of the possibilities. Gaudet and Van Long derive their results by assuming Cournot competition upstream and downstream, together with zero marginal costs and a linear demand function. As in this paper, they also do not impose market foreclosure. Other related papers on market foreclosure deal with vertical integration under different types of competition and various assumptions. These mainly deal with foreclosure under Bertrand competition with homogeneous goods upstream thereby eliminating the efficiency gains arising from avoiding double-marginalization. For instance, the well-know study by Ordover, Saloner and Salop (1990) assumes Bertrand competition and shows that vertical foreclosure is possible in equilibrium under certain trading restrictions, so vertical integration can have anticompetitive effects.[5] Other papers, such as Salinger (1988), do not assume away double marginalization but impose the coexistence of integrated and unintegrated firms. Apart from the different assumptions made by most papers in this literature, none of the foreclosure papers considers capacity sharing and an endogenous entry mode.

In a closely related paper, Chen and Ross (2000) show that facility sharing in a monopolistic market may deter more aggressive entry that would reduce the incumbent's

[5]This result is also derived by Chen (2001) under less restrictive trading restrictions.

market power; nonetheless, facility sharing always results in a larger output than that chosen by a monopoly, albeit less than under facility-based competition.[6] Gale (1994) shows that co-tenancy under a use-or-lose provision achieves a constrained Pareto optimum under open entry conditions, in which each party may utilize any unused portion of the other party's capacity by assuming the variable costs attributable to the additional production, but no portion of the fixed costs. In this setting, the equilibrium price converges to the long-run average cost. Neither of these two papers deals with vertical integration and foreclosure; and the latter does not allow for an endogenous entry mode either, i.e. it looks only at capacity choice by firms that form a co-tenancy prior to building capacity, without allowing a firm to enter the market by building capacity outside of co-tenancy.

Lastly, our paper is also related to the literature dealing with excess capacity as a deterrent instrument. Spence (1977) and Dixit (1980), for example, show how excess capacity is used as a deterrent instrument. In such papers, however, the incumbent chooses its capacity, whereas our paper starts from a situation in which the incumbent's capacity is already in place and there is excess capacity.

The rest of the paper is organized as follows. Section II presents the model; then in section III, we derive the equilibrium market structure under full and partial entry, together with the optimal entry mode and the welfare consequences of the equilibrium market structure. Two extensions are briefly discussed in section IV, namely: (i) the existence of capacity constraints under facility sharing; and (ii) a different bargaining assumption in determining the per-unit price of capacity. Concluding remarks are presented in the final section.

[6]The facility sharing contract used by Chen and Ross to derive this result is different from ours. Their contract allows the incumbent to set both the unit capacity price and the maximum quantity of capacity that the entrant can buy. They show that the optimal per-unit capacity price is a negative number large enough to induce the entrant to stay out of the market. In order to avoid this problem, they assume a non-negative per-unit capacity price based on the feasibility of implementing that type of contract in the real world.

II THE BASIC MODEL

The market structure considered initially has two independent downstream firms, D_1 and D_2 and one nonintegrated upstream firm U_1--referred to as the incumbent-- which owns an essential facility. There is also a firm $U2$ --referred to as the entrant-- which is deciding whether to enter the upstream market. This requires either buying access to the incumbent's facility,[7] or else building a new facility at a fixed cost K. Each upstream firm U_i, $i = 1, 2$, supplies a homogeneous input to downstream firms denoted by z_i, and each downstream firm produces a final good denoted by q_i. Upstream firms thus face a derived demand in the form of the input required by downstream firms. For simplicity, and in keeping with common practice, a constant-returns-to-scale fixed proportions technology downstream is assumed in which one unit of the final good requires one unit of the input z_i. Thus, $q_i = z_i$.

The only cost faced by downstream firms in producing a unit of the final good is the price paid for a unit of input; in other words, the cost of other inputs in the downstream market is normalized to zero, and all downstream firms are assumed to be symmetric. Thus, downstream firm D_i 's marginal cost is the price paid for each unit of input, hereinafter denoted by c_i.

Production of z_i units of input requires y_i units of capacity from the essential facility. We also assume a constant-returns-to-scale fixed proportions technology in input production, such that each unit of input requires one unit of capacity. Thus, $s_2^P(1) = -\frac{1(a + r - 2m_1)}{12b}$.

The facility, combined with other inputs, can produce units of capacity according to the cost function $m_i y_i$, where m_i is the marginal cost of each unit of capacity. As the transfer price for each unit of capacity is assumed to be set at the efficient level, the marginal cost of one unit of input faced by a firm owning an essential facility is m_i. In what follows we assume that $m_1 \geq m_2$; i.e. the entrant's facility is at least as efficient as the

[7] The incumbent's capacity is assumed to be unlimited. The section on robustness discusses the consequences of having limited capacity under facility sharing.

incumbent's facility.[8]

The timing of decisions is as follows. At stage 1, the market is liberalized and the entrant decides whether or not to enter. At stage 2, the incumbent makes a take-it-or-leave-it offer to supply as many units of capacity as the entrant desires at a specified per-unit capacity price or access charge r. The entrant either accepts the offer and does not build a new facility, or else rejects the offer and does builds one. At stage 3, upstream firms have an initial opportunity to acquire one of the downstream firms, D_1 and D_2 If there is a merger, this is assumed to be between firms U_i and D_i, with the merged firm denoted by F_i. At stage 4, upstream firms choose the amount of input to be produced; and at the final stage, downstream firms choose the amount of final good to be produced.[9]

III THE ANALYSIS

Preliminaries

The inverse demand function faced by downstream firms is assumed to be of the following form:[10]

$$P(Q) = a - bQ, a \geq 0 and b \geq 0,$$

where $Q = q_i + q_j$.

Since downstream firms compete à-la-Cournot, firm D_i chooses the final-good production level that maximizes its profit, which is given by: $\pi_{D_i}(n) = (a - bQ)q_i - c_i(n)q_i$, where $n \in \{0,1,2\}$ denotes the number of integrated firms. It is straightforward to verify that in equilibrium each downstream firm D_i produces $q_i(n) = \frac{1}{3b}(a - 2c_i(n) + c_j(n))$ units of the final good and makes a profit of:

[8]We envisage a situation in which new technologies have become available since the last time the incumbent firm upgraded its facility.

[9]The results are robust to Bertrand competition with differentiated goods downstream.

[10]Although most results do not depend on this assumption, the simplification allows us to obtain closed-form solutions that greatly facilitate the welfare analysis.

$$\pi_{D_i}(n) = \frac{(a - 2c_i(n) + c_j(n))^2}{9b},$$

where $c_i(n)$ is the marginal cost faced by firm i and $c_j(n)$ that form firm j .

To ensure positive quantities in equilibrium for all relevant non-negative marginal costs, we assume that $a \geq 2m_1$.

In what follows, let $Z = z_1 + z_2$ be total input production, and note that as downstream firms transform upstream input into final output on a one-to-one basis, in equilibrium $Z = Q$.

Lastly, under either entry mode there are four possible market structures to be considered: (i) no integration; (ii) full integration --i.e. integration by firms U_i and D_i for $i = 1,2$; (iii) integration by firms U_1 and D_1 only; and (iv) integration by firms U_2 and D_2 only.

Full Entry: Facility-Based Competition

In this section we assume that each upstream firm owns a facility, so upstream firm i 's marginal cost of production is m_i, for $i = 1,2$.

If no integration takes place, downstream firms buy the input in the market at the market clearing-price, so each downstream firm pays the same price for each unit of input. That is, $c_1(0) = c_2(0) = c(0)$, where 0 stands for zero vertically integrated firms. In this case the market clearing-price, which is the inverse of $q_1(0) + q_2(0)$, takes the form $c(0) = a - \frac{3}{2}bZ(0)$, where $Z(0)$ is the level of input production when no firm is vertically integrated. Thus, upstream firm U_i chooses $z_i(0)$ to maximize $\left(a - \frac{3}{2}bZ(0) - m_i\right)z_i(0)$.

Given quantity competition, it can readily be verified that, in equilibrium, upstream firm U_i produces $z_i^F(0) = \frac{2}{9b}(a - 2m_i + m_j)$, where F stands for full entry. Hence, upstream firm U_i 's profit is given by:

$$\pi_{U_i}^F(0) = \frac{2(a - 2m_i + m_j)^2}{27b} - K_i,$$

where $K_1 = 0$ and $K_2 = K$, and downstream firm D_i 's profit is given by:

$$\pi^F_{D_i}(0) = \frac{(2a - m_i - m_j)^2}{81b}.$$

Next consider a full integration situation. As there will be no demand for inputs from independent upstream firms, this market configuration corresponds to a standard Cournot duopoly in which the vertically integrated firm F_i 's marginal cost is m_i. In this case the equilibrium quantities are given by: $q_i^F(2) = z_i^F(2) = \frac{1}{3b}(a - 2m_i + m_j)$, and firm F_i 's profit is:

$$\pi^F_{F_i}(2) = \frac{(a - 2m_i + m_j)^2}{9b} - K_i$$

where 2 stands for two vertically integrated firms.

Lastly, we consider the case in which only firms U_i and D_i merge to form the vertically integrated firm F_i. In this situation, the integrated and the nonintegrated downstream firms simultaneously determine the quantities of the final output to be produced. This stage is preceded by the upstream production stage, during which upstream firms compete on the basis of quantities, taking account of the derived demand resulting from final-good production decisions at the next stage. The decision variable of the non-integrated upstream firm U_j is the quantity of the upstream good to produce, z_j. At this stage, the decision that matters for the integrated firm F_i is its net sales to the nonintegrated sector, hereinafter denoted by s_i. We allow the quantity of the input traded between the non-integrated firm and the integrated one to be determined endogenously with no a priori restrictions on the direction of this trade. In other words, the integrated firm may, if it so chooses, sell inputs to the nonintegrated downstream firm or buy inputs from the nonintegrated upstream firm, so s_i may either be positive or negative. The total profit of the integrated firm F_i is $(a - bQ - m_i)q_i + (c - m_i)s_i$, the profit of the nonintegrated upstream firm U_j is $(c - m_j)z_j - K_j$, and that of the nonintegrated downstream firm D_j is $(a - bQ - c)q_j$.

The following lemma is formally proven in the appendix, as are all subsequent lemmas and propositions.

***Lemma 1** When only firms U_i and D_i integrate, the input market clearing price is: $c_i^F(1)=\frac{1}{16}\left(5a+5m_i+6m_j\right)$; the output market clearing price is: $p_i^F(1)=\frac{1}{16}\left(7a+7m_i+2m_j\right)$; and the integrated firm's net sales are:* $s_i^F(1)=-\frac{1\left(a+m_i-2m_j\right)}{12b}$.

This lemma shows that the integrated upstream firm buys inputs from the unintegrated counterpart at the market-clearing price $c_i^F(1)$, which exceeds its own upstream marginal cost of production. This is done for a strategic reason, namely to raise the input price paid by the non-integrated downstream firm, which, in turn, reduces the intensity of competition in the downstream market. This is known as the *raising rivals' costs* strategy (Salop and Sheffman, 1987), and it has been studied for the case of symmetric Cournot oligopolies by Gaudet and Van Long (1997).

Firm U_j 's profit is given by:

$$\pi_{U_j}^F(1)=\frac{25(a-2m_j+m_i)^2}{384b}-K_j,$$

the independent downstream producer D_j obtains:

$$\pi_{D_j}^F(1)=\frac{(a-2m_j+m_i)^2}{64b},$$

and the integrated firm F_i obtains:

$$\pi_{F_i}^F(1)=\frac{\left(7a-9m_i+2m_j\right)^2}{256b}-\frac{\left(a+m_i-2m_j\right)\left(5a-11m_i+6m_j\right)}{192b}-K_i.$$

The integrated firm's profit has two terms: the first is the profit obtained from final good production, and the second is the cost of adopting the *raising rivals' cost* strategy. As expected, the more intense the downstream competition, i.e. the lower m_i and the larger b, the higher the cost of raising the downstream competitor's marginal cost. This is because the gain from reduced competition downstream caused by raising the rival's cost is small relative to the cost of buying inputs at a higher price.

Use of the *raising rivals' cost* strategy by the integrated firm is costly for the nonintegrated firm, which has to pay more for the input than when the integrated firm does not pursue that strategy. This increases the nonintegrated firm's incentive to vertically integrate since this would make the *raising rivals' cost* strategy unprofitable for the competitor. Thus, when the quantity of the input traded between the nonintegrated firm and the integrated firm is determined endogenously, there are two reasons for vertical integration: the standard one, which is to avoid double marginalization, and a new one, which is to avoid the consequences of competing with an integrated firm that takes steps to raise the downstream rival's marginal cost. Profit comparisons across different market configurations provide the following result.

Proposition 2 *Under facility-based competition, in equilibrium there is full vertical integration for all* $m_2 \in [0, m_1]$.

This proposition establishes that the only equilibrium market configuration under facility-based competition is for both upstream firms to vertically integrate. This occurs even though the firms are better-off if no one integrates; i.e. $\pi_{U_i}^F(0) + \pi_{D_i}^F(0) > \pi_{F_i}^F(2)$ for $i = 1, 2$. The firms thus face a prisoner's dilemma situation for while vertical integration is the unique equilibrium, each firm would be better-off if no one integrated. The reason is that by reducing the cost of the input into the downstream production process, vertical integration will increase competition in the downstream market, thus mitigating the gains from eliminating double marginalization.[11]

[11] Gaudet and Van Long (1997) show that when the marginal cost is zero for all upstream firms and there are more than four firms upstream and downstream, this result no longer holds. The reason is that when there are several firms in the downstream market the gain from reducing competition in that market cannot offset the increased marginal cost of production

Partial Entry: Facility Sharing

In this section we derive the equilibrium market structure when the entrant shares the incumbent's facility. In order to do so, the terms of the sharing contract have to be made explicit, mainly we assume that the incumbent sells as many units of capacity y as the entrant wants at a given per-unit price of capacity or access charge r. For the time being this price is treated as a parameter and in the next section the optimal r for the incumbent firm will be obtained. In order to guarantee non-negative quantities under each possible market configuration or a non-negative demand for capacity, we assume that r is never above the monopoly price $\frac{a+m_1}{2}$. In the next section we derive the optimal access charge r and show that in equilibrium this never exceeds the monopoly price.

Once the two firms have agreed to share the incumbent's facility under the terms of the contract, each upstream firm has to decide how many units of input to produce, and then each downstream firm chooses the level of final good production. The analysis is much the same as under facility-based competition, although now the entrant's marginal cost of production is r instead of m_2 --i.e. the price paid for each unit of capacity-- and the incumbent firm's profit function has a new term representing the profit it makes from selling units of capacity, namely $(r-m_1)y$. For the sake of brevity, we will focus only on the incumbent's production decision.

Consider first the situation when no firm is vertically integrated. The incumbent chooses z_1 to maximize $\left(a-\frac{3}{2}bZ-r\right)z_1+(r-m_1)y$. It is useful to keep in mind that $y=z_2$ since the production of one unit of input requires one unit of capacity.

It can easily be verified that the entrant's profit is $\pi_{U_2}^{P}(0)=\frac{2}{27b}(a-2r+m_1)^2$; the incumbent's profit is:

$$\pi_{U_1}^{P}(0)=\frac{2}{27b}\left[(a-2m_1+r)^2+3(r-m_1)(a-2r+m_1)\right];$$

and firm D_i 's profit for $i=1,2$ is given by:

$$\pi_{D_i}^{P}(0)=\frac{(2a-m_1-r)^2}{81b},$$

where P stands for partial entry or facility sharing.

An increase in the per-unit price of capacity on the one hand increases the mark-up per-unit of capacity sold $r-m_1$ and decreases upstream competition; and, on the other hand, decreases the total amount of capacity units sold. From the incumbent's standpoint the former dominates the latter for all $r \leq \frac{a+m_1}{2}$, which is the monopoly price.

Next we consider full vertical integration. The incumbent's problem is to choose z_1 to maximize $(a-bZ-m_1)z_1+(r-m_1)y$.

It is easy to see that firm F_2 's profit is:

$$\pi_{F_2}^{P}(2)=\frac{(a-2r+m_1)^2}{9b},$$

and firm F_1 's profit is given by:

$$\pi_{F_1}^{P}(2)=\frac{1}{9b}\left[(a-2m_1+r)^2+3(r-m_1)(a-2r+m_1)\right].$$

The incumbent faces the same trade-off as when no integration occurs, so profit rises with the access charge whenever the latter is below the monopoly price.

Now we consider the case in which the incumbent does not integrate but the entrant does. The incumbent's profit is equal to $\left(c_2^P(1)-m_1\right)z_1+\left(r-m_1\right)y$, where $c_2^P(1)$ is the input market-clearing price when only firms U_2 and D_2 integrate vertically.

As the first-order conditions are the same as under facility-based competition, the following can be shown to be true.

Lemma 3 *If only the entrant integrates, then: (i) the input market clearing price is:* $c_2^P(1)=\frac{1}{16}(5a+5r+6m_1)$, *and the output market clearing price is:* $p_2^P(1)=\frac{1}{16}(7a+7r+2m_1)$*; and (ii) net sales are:* $s_2^P(1)=-\frac{1(a+r-2m_1)}{12b}$ *and input production is given by:* $z_1^P(1)=\frac{5(a+r-2m_1)}{24b}$ *and* $z_2^P(1)=\frac{(17a+14m_1-31r)}{48b}$.

There are two points to notice here. First, the amount of input that the integrated firm F_2 buys from the nonintegrated upstream firm U_1 rises with the per-unit price of capacity r. This is because the cost to the entrant of using the *raising rivals' cost* strategy decreases as its marginal cost of production rises, because the opportunity cost of buying input in the market is higher. Thus, when negotiating the per-unit capacity price, the incumbent needs to bear in mind that a higher access charge makes the entrant behave more aggressively in terms of buying inputs from the incumbent to mitigate downstream competition. Secondly, the amount of input produced by the integrated firm is larger, and that produced by the nonintegrated firm is smaller, relative to output levels when both firms integrate. The reason is that the unintegrated firm does not avoid double marginalization and does not counter the cost of the raising rivals' cost strategy used by the integrated firm. Thus, for a given capacity price, the incumbent sells more units when only the entrant integrates vertically.

It readily follows from lemma 3 that the incumbent's profit is given by:

$$\pi_{U_1}^P(1)=\frac{25(a+r-2m_1)^2}{384b}+(r-m_1)\frac{(17a+14m_1-31r)}{48b},$$

the independent downstream firm D_1 obtains:

$$\pi_{D_1}^P(1)=\frac{(a+r-2m_1)^2}{64b},$$

and the integrated firm F_2 earns:

$$\pi_{F_2}^P(1)=\frac{(7a-9r+2m_1)^2}{256b}-\frac{(5a-11r+6m_1)(a+r-2m_1)}{192b}.$$

The incumbent's total profit rises with r for all $r \le \frac{93a+130m_1}{223}$, which is lower than the monopoly price. The reason is that a higher price per-unit of capacity induces the entrant to buy fewer units of capacity and more units of input, but the combined purchases decrease.

Lastly we consider the case in which the entrant remains as an independent firm while the incumbent integrates vertically. In this case the incumbent chooses z_1 to maximize $(a-bQ-m_1)q_1+(c_1^P(1)-r)s_1+(r-m_1)y$, where s_1 is the integrated firm's net sales to the nonintegrated sector and $c_1^P(1)$ is the input market-clearing price when only firms U_1 and D_1 integrate vertically. The following can be readily shown.

Lemma 4 *If only the incumbent integrates, then: (i) the input market clearing price is:* $c_1^P(1) = \frac{1}{16}(5a + 5m_1 + 6r) > m_1$ *and the output market clearing price is:* $p_1^P(1) = \frac{1}{16}(7a + 7m_1 + 2r)$ *; and (ii) net sales are:* $s_1^P(1) = -\frac{1(a+m_1-2r)}{12b}$ *and input production is given by:* $z_2^P(1) = \frac{5(a+m_1-2r)}{24b}$ *and* $z_1^P(1) = \frac{(17a+14r-31m_1)}{48b}$.

The two remarks made for lemma 3 also apply here.

It follows from lemma 4 that the entrant's profit is given by:

$$\pi_{U_2}^P(1) = \frac{25(a - 2r + m_1)^2}{384b},$$

the independent downstream firm D_2 earns:

$$\pi_{D_2}^P(1) = \frac{(a - 2r + m_1)^2}{64b},$$

and the integrated firm F_1 's profit is:

$$\pi_{F_1}^P(1) = \frac{(7a-9m_1+2r)^2}{256b} - \frac{(5a-11m_1+6r)(a+m_1-2r)}{192b} + (r - m_1)\frac{5(a-2r+m_1)}{24b}$$

An increase in the price of capacity increases the mark-up per unit of capacity sold $r - m_1$, and decreases upstream competition and the net sales to the unintegrated sector. It also decreases the total amount of units of capacity sold. From the incumbent's perspective, the former effect dominates the latter for all $r \leq \frac{a+m_1}{2}$ since there is an extra benefit from raising the access charge, namely a decrease in net sales to the unintegrated sector.

As under facility-based competition, vertical integration avoids double marginalization and counters the incentives of the integrated firm to soften downstream competition by purchasing inputs from the unintegrated sector. However, under facility sharing, on the one hand the entrant is more aggressive in terms of buying inputs from the incumbent when this is not integrated and hence the incumbent has more incentive to counter this effect by integrating vertically. On the other hand, by integrating vertically, the incumbent sells fewer units of capacity to the entrant than when the latter is integrated and the incumbent is not. Thus, the incumbent firm has a countervailing incentive to integrate vertically that arises from selling fewer units of excess capacity to the entrant. The next proposition shows that this countervailing incentive is not enough to overcome the benefits of avoiding double marginalization, and the more intensive use of the raising rivals' cost strategy by the entrant.

Proposition 5 *If facility sharing occurs in equilibrium there is full vertical integration for all* $r \in \left[0, \frac{a+m_1}{2}\right]$.

Surprisingly, in equilibrium, the entry mode does not alter the equilibrium market structure; vertical integration remains the unique market equilibrium structure when facility-based competition occurs and the access charge is set below the monopoly price. Thus, facility sharing does not modify the prisoner's dilemma-type problem faced by firms when deciding whether or not to integrate.[12]

The Optimal Entry Mode

In this section we derive the optimal entry mode. By sharing its facility with the entrant, the incumbent becomes the sole provider of capacity and attempts to exploit its monopoly power. This is limited, however, by the fact that the entrant can build its own facility if the access charge is set too high. How much the incumbent charges for access depends mainly on the following: the fixed cost of building a new facility and the efficiency thereof, the type of access contracts that are allowed, and the incumbent's bargaining power.

For simplicity, as mentioned in the previous section, we consider a facility-sharing contract in which the incumbent sells however many units of capacity the entrant desires at a fixed per-unit capacity price, r. Following Chen and Ross (2001), we assume that the per-unit capacity price is determined by bargaining between the entrant and the incumbent, where the latter makes a take-it-or-leave-it offer to the entrant.[13] Furthermore, following normal practice in the vertical integration literature, we assume that upstream firms make

[12]This result is partly driven by the assumption that there is no capacity constraint under facility sharing. This will be discussed in more detail in Section.

[13]The consequences of this assumption are discussed in Section.

take-it-or-leave-it offers to downstream firms. This implies that the upstream firm obtains the total profit from integration minus the downstream profit when no integration occurs, both conditional on the rival's strategy.

Under these assumptions, the entrant prefers to share the incumbent's facility than build a new one, if and only if the rent from vertical integration and facility sharing is at least as large as the rent from vertical integration and facility-based competition. In other words, for a given r:

$$\pi_{F_2}^{P}(2)-\pi_{D_2}^{P}(1)\geq \pi_{F_2}^{F}(2)-\pi_{D_2}^{F}(1).$$

This constraint is referred to as the entrant's facility-sharing constraint.

Note that for any given r the entrant's willingness to pay for each unit of capacity rises with the fixed cost, since by purchasing access to the incumbent's facility the entrant's cost savings increase. When the fixed cost is zero, the most that the entrant is willing to pay for each unit of capacity is the upstream marginal cost of production of a new facility, m_2. Moreover, the entrant's rent from integrating vertically when the incumbent is integrated decreases with r since this implies a larger marginal cost of production for the entrant.

The incumbent chooses a capacity price that solves the following problem:

$$\begin{aligned}&\max_{r\in\left[0,\frac{a+m_1}{2}\right]} \pi_{F_1}^{P}(2)-\pi_{D_1}^{P}(1)\\ &\text{subject to } \pi_{F_2}^{P}(2)-\pi_{D_2}^{P}(1)\geq \pi_{F_2}^{F}(2)-\pi_{D_2}^{F}(1),\end{aligned} \tag{1}$$

where the objective function is the incumbent's rent under full vertical integration and facility sharing --i.e. the combined profits minus the profit that the downstream firm D_1 makes when it stays unintegrated and the entrant is vertically integrated.

It is straightforward to show that the incumbent's rent rises with r for all $r\leq\frac{151a+178m_1}{329}$ and decreases otherwise. Hence, if the entrant's facility sharing constraint is nonbinding at $r=\frac{151a+178m_1}{329}$, the access charge that maximizes the incumbent's rent is $\frac{151a+178m_1}{329}$. If the constraint is binding at $r=\frac{151a+178m_1}{329}$, the access charge that maximizes the incumbent's rent is the highest charge that exactly satisfies the entrant's constraint, which is given by:

$$\frac{a+m_1}{2}-12\sqrt{\frac{b\left(\overline{K}-K\right)}{55}},$$

where $\overline{K}$ is the largest fixed cost at which the entrant makes non-negative profits under facility-based competition and full integration.[14] Thus, the optimal per-unit capacity price, denoted by $r(K)$, is given by:

$$r(K)=\begin{cases}\frac{a+m_1}{2}-12\sqrt{\frac{b\left(\overline{K}-K\right)}{55}} & \text{if } K\leq\hat{K},\\ \frac{151a+178m_1}{329} & \text{if } K>\hat{K},\end{cases}$$

where $\hat{K}$ is the lowest fixed cost under which the entrant's facility sharing constraint is nonbinding when the incumbent charges $\frac{151a+178m_1}{329}$ per-unit of capacity.[15]

There are several points worth highlighting with regard to the optimal access charge. First, when the fixed cost is zero, $r(K)=m_2$ since any capacity price greater than the new facility's marginal cost yields lower profits than building a new facility. Secondly, the entrant is willing to pay up to the monopoly price $\frac{a+m_1}{2}$ when the fixed cost is such that the entrant earns zero rents from building a new facility $\overline{K}$. Thirdly, the per-unit capacity price rises with the fixed cost K since the entrant's cost saving from sharing the incumbent's facility increases. Fourthly, the optimal per-unit capacity price is smaller than m_1 when the fixed cost is sufficiently low, which in turn implies that the incumbent sells capacity units at a price below its marginal cost of production when facility sharing occurs. And fifthly, the access charge is always set at a level below the monopoly price, because the profit made by the downstream firm D_1 when it stays unintegrated, and the entrant is vertically integrated, increases with the access charge r. This results from the fact that the higher the entrant's marginal cost, the weaker is downstream competition for the downstream firm D_1.

Given the optimal per-unit capacity price $r(K)$, the incumbent is better-off sharing its facility with the entrant when $\pi^P_{F_1}(2)-\pi^P_{D_1}(1)$ evaluated at $r(K)$ is greater than the rent from

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[14]Formally, $\overline{K}$ satisfies the following $\pi^F_{F_2}(2)-\pi^F_{D_2}(1)=0$ and is given by $\overline{K}\equiv\frac{55}{576b}\left(a-2m_2+m_1\right)^2$.

[15]Note that $\hat{K}<\overline{K}$.

facility-based competition, $\pi_{F_1}^{F}(2)-\pi_{D_1}^{F}(1)$. The next proposition provides the conditions for this to hold.

Proposition 6 *There exists $K^* \in [0,\hat{K})$, with $K^*=0$ when $m_2=m_1$, such that in equilibrium: (i) facility-based competition and full vertical integration occurs when $K \le K^*$; (ii) facility sharing and full vertical integration occurs when $K^* < K \le \overline{K}$; and (iii) no entry takes place when $K > \overline{K}$.*

The reason for this is as follows: when $K > \overline{K}$, entry is a noncredible threat since the entrant's profit from facility-based competition is negative. Thus, the incumbent ignores that threat for example by offering to share its facility at a capacity price that is higher than the monopoly price. When $K^* < K \le \overline{K}$, facility sharing takes place. By sharing the facility with the entrant, the incumbent curbs potential competition by the entrant by preventing the entry of a more efficient upstream firm since $m_2 \le m_1$, and charging an access price above the entrant's upstream marginal cost --i.e. $r(K) > m_2$. These two effects cause a reduction in total output, i.e. $Q^F(2) > Q^P(2)$.[16] Furthermore, because the optimal per-unit capacity price or access charge rises with the fixed cost, when the latter is sufficiently large the incumbent clearly makes money by selling capacity. Nonetheless, one can show that for values of K at the lower end of range $(K^*, \overline{K}]$, the optimal per-unit price is lower than m_1. This means the incumbent is willing to sell capacity at a unit price below marginal cost to deter the entrant from building its own facility, and thereby obtains the benefits of curbing competition. However, when the fixed cost is sufficiently small, $K \le K^*$, deterring the entrant from building its own facility by charging for access below marginal cost is too costly, since the looses arising from selling capacity at less than marginal cost cannot be

[16]This is similar to what Chen and Ross (2000) call the collusion effect. There are two main differences between our paper and theirs. Firstly, Chen and Ross set the price per-unit of capacity to zero, so collusion is achieved by restricting the amount of capacity sold to a level below what the entrant would buy if it were to build a facility. Secondly, they assume that the incumbent and the entrant have the same upstream marginal cost of production.

compensated by the gains from curbing competition. Facility-based competition thus occurs.

In short, the incumbent faces a basic trade-off between curbing competition by dissuading the entrant from building its own facility, versus selling units of capacity at a per-unit price that is below the marginal cost. The basic trade-off faced by the entrant, in contrast, is between building its own facility and having a low marginal cost, versus sharing the incumbent's facility and having a higher marginal cost $r(K)$, but a fixed cost saving equal to K.

Welfare Analysis

In this section we analyze the efficiency of facility-based competition compared to facility sharing. First, notice that facility sharing is always more efficient socially than no entry because the former expands output beyond the pre-liberalization level without incurring any new fixed cost. So, a priori and compared to no entry, the competition authority might be expected to support agreements to share the incumbent's facility, since these would benefit consumers, the entrant and society at large. This logic is flawed, however because facility sharing may take the place of more aggressive entry that would involve building a new and more efficient facility.

Facility sharing in comparison to facility-based competition affects total welfare through two channels. Firstly, it avoids duplication of facilities, and hence generates a cost saving of K; but this results in a higher variable cost of production since $m_1 \geq m_2$. Secondly, it lowers the intensity of competition since the unit price of capacity is higher than the entrant's marginal cost when it builds its own facility. The first effect can either increase or decrease welfare while the second decreases it. To determine the net effect, we define

$$\Delta W(K) \equiv W^F(2) - W^P(2),$$

where $W^F(2) \equiv \pi_{F_1}^F(2) + \pi_{F_2}^F(2) + \frac{b\left[Q^F(2)\right]^2}{2} + K$ is total welfare when the entrant builds its own facility before considering the fixed costs directly, and $W^P(2) \equiv \pi_{F_2}^P(2) + \pi_{F_2}^P(2) + \frac{b\left[Q^P(2)\right]^2}{2}$ is total welfare when the entrant builds a new facility.

It is simple to verify that $\Delta W(K)$ increases with K for all $K \leq \hat{K}$ and is constant with K for all $K > \hat{K}$. At higher levels of fixed cost, the entrant would be willing to pay more for access, making a facility sharing agreement less socially efficient; before considering the fixed cost. Nonetheless, this ends when the fixed cost is high enough to make the entrant's facility-sharing constraint non-binding, in which case the price of capacity is set to the level that maximizes the incumbent's rent. Then, for any given value of K, facility sharing improves welfare if and only if $K > \Delta W(K)$.

Before finding the condition under which $K > \Delta W(K)$ holds, at least three comments that are worth making. First, the total output produced under facility-based competition is larger than that under facility sharing, because for all $K > 0$ the per-unit price of capacity $r(K)$ is set to a level higher than the entrant's facility marginal cost m_2. This means that consumers' welfare is lower under facility sharing. Second, for $K > \hat{K}$, $\Delta W(K) - K$ decreases with the fixed cost since the capacity price is independent of it. Thus, there is a fixed cost cutoff at which facility sharing starts to yields a higher welfare. In what follows, this cutoff is denoted by K^{**} and is formally defined as the smallest fixed cost such that $\Delta W(K) - K \geq 0$ when the optimal per-unit capacity price is set at $\frac{151a+178m_1}{329}$.[17] Third, a key determinant of $\Delta W(K)$ is the difference between the efficiency of the incumbent's and entrant's facilities, which is given by $\Delta m \equiv m_1 - m_2$. The more efficient the entrant's facility relative to the existing one, the more likely it is that facility-based competition will generate greater total welfare than facility sharing. The reason is that the social cost of curbing competition through facility sharing is higher the more efficient is the entrant's facility. Thus, the following proposition holds, as shown in the appendix.

[17] The existence of K^{**} is guaranteed by the fact that the optimal access charge is below the monopoly price. This implies that the fixed cost saving obtained by sharing the incumbent's facility more than compensates for the social cost of softening competition.

Proposition 7 *(i) If* $\Delta m > 0.313(a - m_1)$ *, there exists* $K^{**} \in (\hat{K}, \overline{K})$ *such that facility-based competition is more efficient than facility sharing for all* $K \leq K^{**}$*; (ii) if* $0.313(a - m_1) \geq \Delta m > 0.262(a - m_1)$*, there exist* $K_L \in (0, K^*)$ *and* $K_H \in (0, \hat{K})$ *with* $K_H > K_L$ *such that facility-based competition is more efficient than facility sharing for all* $K \in [0, K_L] \cup [K_H, K^{**}]$*; (iii) if* $\Delta m \leq 0.262(a - m_1)$*, there exists* $K_L \in (0, K^*)$ *such that facility-based competition is more efficient than facility sharing for all* $K \in [0, K_L]$.

This proposition leads to several interesting results. When the relative efficiency of the entrant's facility is sufficiently large, i.e. $\Delta m > 0.313(a - m_1)$, facility-based competition is efficient for all $K \leq K^{**}$. The reason is that the social cost of curbing competition is high, so unless the fixed cost saving from sharing the incumbent's facility is also large, facility-based competition is socially efficient. This result, coupled with the fact that facility-based competition is observed only when $K \leq K^*$, implies that it is always efficient when it occurs. Moreover, because $K^{**} > K^*$, there is a range for the fixed cost given by $(K^*, K^{**}]$ such that facility-based competition is efficient but not observed. Hence, the market can efficiently solve the trade-off between saving on fixed costs and curbing competition when the fixed costs are either relatively small $K \leq K^*$ or relatively large $K > K^{**}$.

When the relative efficiency of the entrant's facility is small enough, i.e. $\Delta m \leq 0.262(a - m_1)$, facility-based competition is efficient for all $K < K_L$, whereas facility sharing is efficient otherwise. Because the social cost of curbing competition is now small, the fixed cost saving obtained by sharing the incumbent's facility does not need to be large to make facility sharing socially efficient. Once again, when observed, facility-based competition is efficient.

Finally, when the relative efficiency of the entrant's facility is neither sufficiently small nor sufficiently large, i.e. $0.313(a - m_1) \geq \Delta m > 0.262(a - m_1)$, the relationship between total welfare and fixed costs is non-monotonic. To gain a better understanding of this, it is useful to consider why there is a K_L and K_H, both lower than $\hat{K}$, such that total welfare is the same under either entry mode at these cutoffs. When the cost of building a new facility is zero, the optimal price per-unit of capacity $r(K)$ is m_2 and hence total welfare should be the same under either entry mode. Yet, because the incumbent's facility is less efficient than the entrant's, sharing the incumbent's facility causes total industry output to be produced less efficiently, and thus with a higher variable cost of production.[18] It follows from this that when the fixed cost is small, facility-based competition is the socially preferred outcome. When the cost of building a new facility is $\hat{K}$, then $r(\hat{K}) >> m_1$. Thus, the social cost of curbing competition exceeds the saving on fixed costs. These two results jointly explain why facility sharing is welfare-enhancing for $K \in (K_L, K_H]$ while facility-based competition is efficient for $K \in (K_H, K^{**}] \cup [0, K_L]$. Lastly, we note that for $K > \hat{K}$, $r(K)$ is independent of K so the social cost of curbing competition remains the same while the fixed cost saving keeps increasing with K. Thus, for $K > K^{**}$, facility sharing is the socially preferred outcome. In this case, it is also true that facility-based competition, when observed, is the socially preferred outcome.

There are two key lessons to be derived from the welfare analysis. Firstly, in many situations the market can efficiently solve the trade-off between fixed cost savings and softening downstream competition. In fact, whenever facility-based competition is observed it is socially efficient. Secondly, as the relationship between fixed costs and social welfare is non-monotonic, the competition authorities need to be careful when analyzing antitrust cases.

[18] $K_L = 0$ when $m_1 = m_2$.

IV ROBUSTNESS

In order to reduce the complexity of the issue studied here, we have made several simplifying assumptions, and a thorough analysis of robustness would require a paper in its own right. We will therefore focus on the following two issues: (i) a capacity limit is imposed on the incumbent's facility; and (ii) the entrant is assumed to have all the bargaining power.[19]

Capacity Constraints

The results derived in propositions and were obtained under the assumption that each facility has unlimited capacity, which in some circumstances is unrealistic. In this section we explore how the results change when the incumbent cannot satisfy the entrant's demand for capacity under each possible market structure, but when each upstream firm has its own facility there are no capacity constraints.

Under facility-based competition, the unique market equilibrium is still full vertical integration since there are no capacity restrictions; but under facility sharing the capacity constraint does prevents the incumbent from reaping the benefits of vertical integration since that means expanding output beyond available capacity. Furthermore, the opportunity cost to the incumbent of using a unit of capacity at a of cost m_1 is now the price that the integrated sector is willing to pay for that unit, which may be greater than m_1. In fact, it can be shown that, vertical integration by the entrant and nonvertical integration by the incumbent is the *unique* equilibrium market structure under facility sharing.

As was the case under unlimited capacity, the per-unit capacity price depends on the level of fixed cost, so, when the latter is sufficiently large, sharing the incumbent's facility is chosen rather than facility-based competition. Thus the main differences with the no-capacity-constraint case are that the incumbent will never choose to integrate with a downstream firm upon entry and that the fixed-cost threshold for facility-based competition is positive irrespective of the value of m_2.

[19]The formal details of the analysis in this section are available on request.

Bargaining Power

It is fairly obvious that our results partly depend on the assumption that the incumbent has all the bargaining power. We now analyze the opposite case in which the entrant has all the bargaining power and makes a take-it-or-leave-it offer to the incumbent to buy as many units of capacity as it wants at per-unit price of capacity r.

Because the entrant has all the bargaining power, on entry it will make an offer to the incumbent to buy as many units of capacity as it wants at a price that leaves the incumbent indifferent between accepting and rejecting the proposal to share its facility with the entrant. In other words, the entrant will offer the lowest possible access charge such that:

$$\pi_{F_1}^{F}(2)-\pi_{D_1}^{F}(1)=\pi_{F_1}^{P}(2)-\pi_{D_1}^{P}(1).$$

It is easy to verify that this price, denoted by r^*, is given by:

$$\frac{151a+178m_1}{329}-\frac{\sqrt{4706(a-m_1)^2+18095\left[(a-m_2)^2-2(m_1-m_2)^2\right]}}{329}.$$

The first point to notice here is that the optimal per-unit capacity price is independent of the fixed cost, since the incumbent has no fixed cost. The second point is that the optimal access charge decreases with m_2 so $r^* \leq m_1$ for all $m_2 \leq m_1$, since $r^* = m_1$ when $m_2 = m_1$. Thus, when $m_2 = m_1$, the entrant can save the fixed cost by sharing the incumbent's facility, and obtain the input for the same price as when it builds its own facility. Thus, facility sharing is the unique equilibrium and is efficient. When $m_2 < m_1$, the incumbent is willing to share its facility at a per-unit capacity price below marginal cost because it can thus curb competition in a way that outweighs the loss sustained by selling units of capacity for less than marginal cost m_1. This means that there is a level of fixed cost above which facility sharing is the only equilibrium, while facility-based competition is the unique equilibrium otherwise.[20]

Because the price per-unit of capacity is independent of the fixed cost, facility-based

[20]It can easily be shown that the fixed cost cutoff is given by $\frac{55(a+m_1-m_2-r^*)(r^*-m_2)}{144b}$.

competition is socially efficient when the fixed cost is below a certain threshold, while facility sharing is the socially preferred entry mode otherwise. In this case it is also true that facility-based competition, when observed, is socially efficient.

This suggests that when neither the entrant nor the incumbent has all the bargaining power the result would be qualitatively the same as when the incumbent has all the bargaining power. Nonetheless, the optimal per-unit price of capacity would be smaller, which in turn implies that facility sharing plus full integration would occur more often and social welfare would be larger.

V CONCLUSIONS

This paper has studied the market equilibrium and the welfare implications of liberalization in an industry initially characterized by an upstream monopoly owning an essential facility. We have shown that facility-based competition and full vertical integration occurs when the fixed cost is below a positive fixed-cost threshold, with facility sharing and full vertical integration occurring otherwise. Two key lessons were obtained in terms of the welfare implications of liberalization,. First, in many cases, the market can efficiently solve the trade-off between saving fixed costs and softening downstream competition. For instance, whenever facility-based competition is observed it is the socially preferred outcome; and when fixed costs are high, facility sharing is always observed and efficient. Second, the relationship between fixed costs and social welfare is non-monotonic, so the competition authorities must be careful when analyzing antitrust cases.

These results provide a rationale for the liberalization of industries with natural monopoly characteristics as a complementary measure, if not as an alternative to direct regulation. In fact, unregulated privatization is a relevant policy design under facility-based competition. In practice, however, identifying the industries to which this can be applied requires a case by case analysis that goes beyond the scope of this paper. Liberalization may also be useful as an antitrust remedy, especially when fixed costs are low and the relative efficiency of a new facility is large. Some caveats are in order, however. First, we have assumed that the incumbent cannot deter entry, which is not necessarily true particularly in network industries. Second, we did not consider the issue of degrading the quality of access to the incumbent's facility, a strategy that may undermine the benefits of liberalization and usually results in some type of regulation in the industry.[21] And, third, we have ignored other anticompetitive practices that could be facilitated by vertical integration, such as collusion.

[21]See, Beard *et al* (2001) on this topic.

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APPENDIX

Proof of Lemma 1.

From the equilibrium conditions for the downstream market it follows that the optimal quantities are given by $q_j^F(1) = \frac{1}{3b}(a - 2c + m_i)$ and $q_i^F(1) = \frac{1}{3b}(a - 2m_i + c)$, where 1 indicates that there is just one integrated upstream firm. The market demand for the upstream input comes from the nonintegrated downstream firm D_j. This firm will be supplied by the nonintegrated upstream firm U_j that produces z_j and potentially by the integrated upstream firm U_i, which have net sales of s_i. The competition at the upstream stage is therefore subject to the derived inverse demand

$$c_j(1) = \frac{a + m_i - 3b(s_i + z_j)}{2}. \qquad (2)$$

Using the envelope theorem, the equilibrium conditions are:

$$a + m_i - 2m_j - 3bs_i - 6bz_j \leq 0, \qquad (3)$$

$$\frac{2}{3}c(1) - \frac{1}{3}a - \frac{1}{3}m_i + s_i\left(-\frac{3}{2}b\right) \leq 0. \qquad (4)$$

It readily follows from these equilibrium conditions that the optimal quantities are:

$$z_j^F(1) = \frac{5(a - 2m_j + m_i)}{24b},$$
$$s_i^F(1) = -\frac{1(a + m_i - 2m_j)}{12b},$$
$$z_i^F(1) = \frac{17a - 31m_i + 14m_j}{48b}.$$

Thus, the input price is $c_i^F(1) = \frac{1}{16}(5a + 5m_i + 6m_j)$ and the final good price is $p_i^F(1) = \frac{1}{16}(7a + 7m_i + 2m_j)$.

Proof of Proposition 2.

Firstly, consider the best response of firms U_i and D_i to nonintegration by firms U_j and D_j. Note that

$$\pi^F_{F_i}(1)-\left[\pi^F_{U_i}(0)+\pi^F_{D_i}(0)\right]= \frac{869a^2-2390am_i+652am_j+1349m_i^2-308m_im_j-172m_j^2}{20736b}.$$

Also note that if $m_i = m_j = m$, then $\pi^F_{F_i}(1)-\left[\pi^F_{U_i}(0)+\pi^F_{D_i}(0)\right]=\frac{869}{20736}\frac{a^2-2am+m^2}{b}>0$. Suppose $i=1$ and $j=2$. Then, $\pi^F_{F_1}(1)-\left[\pi^F_{U_1}(0)+\pi^F_{D_1}(0)\right]$ is strictly increasing in m_2 and at $m_2=0$ is equal to $\frac{869a^2-2390am_1+1349m_1^2}{20736b}$. Notice that this is decreasing in m_1 and that at $m_1=\frac{a}{2}$ is equal to $\frac{5}{9216}\frac{a^2}{b}>0$. Thus, vertical integration is the best response to nonintegration by firms U_2 and D_2 for all m_1 and m_2.

Next suppose that $i=2$ and $j=1$. Then, $\pi^F_{F_2}(1)-\left[\pi^F_{U_2}(0)+\pi^F_{D_2}(0)\right]$ is strictly decreasing in m_2 and at $m_2=m_1$, is equal to $\frac{869}{20736}\frac{a^2-2am_1+m_1^2}{b}>0$. Thus, vertical integration is the best response to nonintegration by firms U_1 and D_1 for all m_1 and m_2.

Now consider the best response of firms U_i and D_i to integration by firms U_j and D_j. Notice that

$$\pi^F_{F_i}(2)-\left[\pi^F_{U_i}(1)+\pi^F_{D_i}(1)\right]=\frac{35}{1152}\frac{(a-2m_i+m_j)^2}{b}>0.$$

Given that the difference in joint profits is positive, vertical integration is the best response to integration by firms U_j and D_j for all m_i and m_j.

These two jointly imply that vertical integration is a dominant strategy.

Proof of Lemma 3.

Applying the envelope theorem, the first-order conditions are given by:

$$z_1 \;:\; a+r-2m_1-3bs_2-6bz_1=0$$
$$s_2 \;:\; \tfrac{1}{3}\left(a+r-3b(s_2+z_1)\right)-\tfrac{1}{3}a-\tfrac{1}{3}r-\tfrac{3}{2}bs_2=0.$$

Solving these gives:

$$z_1^P(1) = \frac{5(a+r-2m_1)}{24b},$$
$$s_2^P(1) = -\frac{1(a+r-2m_1)}{12b},$$
$$z_2^P(1) = \frac{(17a+14m_1-31r)}{48b}.$$

Substituting these values into the derived demand gives $c_2(1)$ and $p_2^P(1)$ as claimed.

Proof of Lemma 4.

Applying the envelope theorem, the first-order conditions are given by:

$$z_2 \;:\; a + m_1 - 2r - 3bs_1 - 6bz_2 = 0$$
$$s_1 \;:\; \tfrac{1}{3}(a + m_1 - 3b(s_1 + z_2)) - \tfrac{1}{3}a - \tfrac{1}{3}m - \tfrac{3}{2}bs_1 = 0$$

Solving these gives:

$$z_2^P(1) = \frac{5(a+m_1-2r)}{24b},$$
$$s_1^P(1) = -\frac{1(a+m_1-2r)}{12b},$$
$$z_1^P(1) = \frac{(17a+14r-31m_1)}{48b}.$$

Substituting these values into the derived demand gives $c_1(1)$ and $p_1^P(1)$ as claimed.

Proof of Proposition 5.

Firstly consider the best response of firms U_2 and D_2 to nonintegration by firms U_1 and D_1. Notice that

$$\pi_{F_2}^P(1) - \left[\pi_{U_2}^P(0) + \pi_{D_2}^P(0)\right] =$$
$$\frac{1}{20736b}\left(869a^2 - 2390ar + 652am_1 + 1349r^2 - 308rm_1 - 172m_1^2\right)$$

It can easily be verified that the difference in joint profits is continuous, strictly convex in r, strictly decreasing in r for all $r \leq \frac{2390a+308m_1}{2698}$, and equal to $\frac{1}{20736}\frac{869a^2+652am_1-172m_1^2}{b} > 0$ at $r = 0$. Note, moreover, that this expression has two real roots given by $.489m_1 + .511a$ and $-.261m_1 + 1.261a$. These two roots are positive and larger than $\frac{a+m_1}{2}$, which is the maximum value allowed for r. Thus, vertical integration is the best response to nonintegration by firms U_1 and D_1 for all $r \leq \frac{a+m_1}{2}$.

Next consider the best response of firms U_2 and D_2 to integration by firms U_1 and

D_1. Notice that

$$\pi^P_{F_2}(2) - \left[\pi^P_{U_2}(1) + \pi^P_{D_2}(1)\right] = \frac{35}{1152b}\left(a + m_1 - 2r\right)^2.$$

Given that the difference in joint profits is positive, vertical integration is the best response to integration by firms U_1 and D_1 for all $r \le m_1$.

Therefore, vertical integration is a dominant strategy for firms U_2 and D_2 all $r \le \frac{a+m_1}{2}$.

Now consider the best response of firms U_1 and D_1 to nonintegration by firms U_2 and D_2. Notice that

$$\pi^P_{F_1}(1) - \left[\pi^P_{U_1}(0) + \pi^P_{D_1}(0)\right] = \frac{1}{20736b}\left(869a^2 + 364ar - 2102am_1 + 404r^2 - 1172rm_1 + 1637m_1^2\right).$$

It can easily be verified that the difference in joint profits is continuous, strictly convex in r and equal to $\frac{869a^2 - 2390am_1 + 1349m_1^2}{20736b}$ when $r = 0$, which is always positive since $\frac{a}{2} \ge m_1$. Furthermore, this expression has no real roots for r. Thus, vertical integration is the best response to nonintegration by firms U_1 and D_1 for all $r \ge 0$.

Finally, consider the best response of firms U_1 and D_1 to integration by firms U_2 and D_2. Notice that

$$\pi^P_{F_1}(2) - \left[\pi^P_{U_1}(1) + \pi^P_{D_1}(1)\right] = \frac{35a^2 + 46ar - 116am_1 + 11r^2 - 68rm_1 + 92m_1^2}{1152b}.$$

This difference is a strictly convex function of r and has two real roots given by

$$\left\{r = -a + 2m_1, r = -\frac{35}{11}a + \frac{46}{11}m_1\right\}.$$

These roots are both negative since $m_1 \le \frac{a}{2}$. Thus, given the strict convexity of the profit function, vertical integration is the best response to integration by firms U_2 and D_2 for all $r > 0$.

Proof of Proposition 6:

The incumbent faces the following optimization problem:

$$\max_{r\in\left[0,\frac{a+m_1}{2}\right]} \pi^P_{F_1}(2)-\pi^P_{D_1}(1)$$
$$\text{subject to } \pi^P_{F_2}(2)-\pi^P_{D_2}(1)\geq \pi^F_{F_2}(2)-\pi^F_{D_2}(1),$$

where

$$\pi^P_{F_1}(2)-\pi^P_{D_1}(1)=\frac{55a^2-412am_1+302ar+28m_1^2+356rm_1-329r^2}{576b}$$

and that

$$\pi^P_{F_2}(2)-\pi^P_{D_2}(1)=\frac{55}{576}\frac{(a-2r+m_1)^2}{b} \text{ and } \pi^F_{F_2}(2)-\pi^F_{D_2}(1)=\frac{55}{576}\frac{(a-2m_2+m_1)^2}{b}-K$$

It is straightforward to show that, when the constraint is ignored, the solution to this problem is $r=\frac{151a+178m_1}{329}<\frac{a+m_1}{2}$. Thus, if the entrant's constraint is satisfied at $r=\frac{151a+178m_1}{329}$, the optimal solution for problem (1), denoted by $r(K)$, is given by $\frac{151a+178m_1}{329}$. Because the objective function is strictly concave and increases with r for all $r\leq\frac{151a+178m_1}{329}$ and the restriction is strictly convex and decreases with r for all $r\leq\frac{a+m_1}{2}$, the optimal $r(K)$ is set to the minimum of

$$\min\left\{\frac{a+m_1}{2}-\frac{1}{2}\left((a-2m_2+m_1)^2-\frac{576}{55}bK\right)^{\frac{1}{2}},\frac{151a+178m_1}{329}\right\},$$

where the first entry is the per-unit price of capacity that exactly satisfies the entrant's facility sharing constraint.

Define $\hat{K}$ as the lowest capital level that solves the following

$$\frac{a+m_1}{2}-\frac{1}{2}\left((a-2m_2+m_1)^2-\frac{576}{55}bK\right)^{\frac{1}{2}}=\frac{151a+178m_1}{329}.$$

Thus,

$$\hat{K}=\frac{55\left(26878a^2+54485am_1+26878m_1^2-108241am_2+108241m_2^2-108241m_2m_1\right)}{15586704b}.$$

This means that $r(K)$ is equal to $\frac{a+m_1}{2}-\frac{1}{2}\left((a-2m_2+m_1)^2-\frac{576}{55}bK\right)^{\frac{1}{2}}$ for $K\leq\hat{K}$, and equal to $\frac{151a+178m_1}{329}$ for $K > \hat{K}$. $\hat{K}$ is lower than $\overline{K}$ since $\frac{a+m_1}{2} - \frac{1}{2}\left(\left(a-2m_2+m_1\right)^2 - \frac{576}{55}bK\right)^{\frac{1}{2}}$ at $K = \overline{K}$ is equal to $\frac{a+m_1}{2}$, which is higher than $\frac{151a+178m_1}{329}$.

For full integration and facility sharing to be an overall equilibrium, the incumbent must be better-off sharing its facility with the entrant than if the entrant firm U_2 builds its own facility. Formally, this implies that $\pi^P_{F_1}(2) - \pi^P_{D_1}(1) \geq \pi^F_{F_1}(2) - \pi^F_{D_1}(1)$.

Notice that $\left[\pi^P_{F_1}(2) - \pi^P_{D_1}(1)\right] - \left[\pi^F_{F_1}(2) - \pi^F_{D_1}(1)\right]$ is equal to

$$\frac{-192am_1 + 302ar - 192m_1^2 + 356rm_1 - 329r^2 - 110am_2 + 220m_1m_2 - 55m_2^2}{576b}, \qquad (5)$$

which is strictly concave and increases with r for all $r \leq \frac{151a+178m_1}{329}$.

Evaluating this at the optimal per-unit capacity price $r(K)$ for $K > \hat{K}$ one gets the following:

$$\frac{-9412am_1 \quad 22801a^2 - 31484m_1^2 - 36190am_2 \quad 72380m_1m_2 - 18095m_2^2}{189504b}.$$

Notice firstly that equation (5) is positive for all $K \geq \hat{K}$ since it is a strictly concave function of m_2 and has two real roots given by: $-2.50a + 3.50m_1$ and $.503a \quad .497m_1$. The former is negative while the latter is positive and larger than m_1. This ensures that equation is positive for all $m_2 \in [0, m_1]$. Notice also that equation (5) when evaluated at $r = m_2$, which is the optimal per-unit price of capacity when $K = 0$, gives

$$-\frac{\Delta m}{3}\frac{a + m_1 - 2m_2}{b}, \qquad (6)$$

which is negative for all $\Delta m > 0$, with $\Delta m \equiv m_1 - m_2$. Thus, since $r(K)$ increases with K for all $K \leq \hat{K}$ and $\left[\pi^P_{F_1}(2) - \pi^P_{D_1}(1)\right] - \left[\pi^F_{F_1}(2) - \pi^F_{D_1}(1)\right]$ increases with r for all $r \leq \frac{151a+178m_1}{329}$, there is a fixed cost level, denoted by K^*, with $K^* < \hat{K}$, such that the incumbent's rent is larger under a shared-facility agreement than under facility-based competition for all $K > K^*$. It also follows from this that $K^* > 0$ for all $\Delta m > 0$ and that $r(K) < m_1$ for all $K < K^*$ since equation (6) when evaluated at $r = m_1$ is positive for all $m_1 > m_2$.

Proof of Proposition 7.

Total welfare under facility-based competition and full integration, minus total welfare under facility sharing and full integration before subtracting the fixed cost, $\Delta W(K)$, is given by:

$$\frac{6m_1^2 - 14m_2m_1 + 6am_1 + 11m_2^2 - 8am_2 + 2ar + r^2 - 4rm_1}{18b}. \tag{7}$$

Note that this increases with r, and since $r(K)$ also rises with K for all $K \leq \hat{K}$, $\Delta W(K)$ increases with K. Furthermore, for $K \leq \hat{K}$, $\Delta W(K) - K$ is strictly convex in K, equal to $\frac{1}{3b}(a - 2m_2 + m_1)\Delta m \geq 0$ at $K = 0$ and to

$$\frac{99846m_1^2 - 293797m_1m_2 + 94105am_1 + 170093m_2^2 - 46389am_2 - 23858a^2}{742224b} \tag{8}$$

at $K = \hat{K}$.

Notice that equation (8) is strictly convex in m_2, strictly decreasing in m_2 for all $m_2 \leq m_1$ and equal to $-\frac{11929}{371112}\frac{(a-m_1)^2}{b} < 0$ at $m_2 = m_1$ while it is positive at $m_2 = 0$. Thus, it is easy to verify that $\Delta W(\hat{K}) - \hat{K} > 0$ for all $\Delta m > 0.262(a - m_1)$.

Furthermore, given the strict convexity of $\Delta W(K) - K$ for all $K \leq \hat{K}$ with respect to K, $\Delta W(K) - K$ has two roots, denoted by K_L and K_H respectively, given by:

$$\frac{55\left[\begin{array}{c}(100a - 155m_2 + 55m_1)\Delta m + 13(a - m_2)^2 \pm \\ (a - m_1)\left(169(a - m_1)^2 - (252a - 924m_2 + 672m_1)\Delta m\right)^{\frac{1}{2}}\end{array}\right]}{15876b}$$

Because the term under the square root is strictly concave in m_2, these roots are real for all $\Delta m \leq 0.313(a - m_1)$. This implies that welfare under full entry is larger than welfare under partial entry for all $K \leq K_L$ and $K > \min\{K_H, \hat{K}\}$ when Δm is smaller than $0.313(a - m_1)$, otherwise welfare is always larger under full entry for all $K \leq \hat{K}$. Notice also that K_L , when real, is positive since it reaches its minimum at $m_2 = m_1$, namely 0 .

Because $\Delta W(\hat{K}) - \hat{K} > 0$ for all $\Delta m > 0.262(a - m_1)$ and K_H is real for all $\Delta m < 0.313(a - m_1)$. It readily follows that $K_H \leq \hat{K}$ for all $\Delta m \geq 0.262(a - m_1)$.

Finally notice that $\Delta W(K)-K$ is strictly decreasing in K for all $K>\hat{K}$ since $r(K)$ is independent of K. Thus, when $r(K)=\frac{302a+356m_1}{658}$, $\Delta W(K)-K<0$ for all $K>K^{**}$, where $K^{**}=\hat{K}$ when $\Delta m \leq 0.262(a-m_1)$ and $K^{**}=\frac{99846m_1^2-293797m_1m_2+94105am_1+170093m_2^2-46389am_2-23858a^2}{742224b}$ when $\Delta m > 0.262(a-m_1)$. Before concluding the proof we need to find conditions, if any, under which $K^{**}-\overline{K}\leq 0$. A few steps of simple algebra show that $K^{**}-\overline{K}$ is given by:

$$\frac{347687m_1^2-123704m_2m_1-571670am_1-1360744m_2^2+2845192am_2-1136761a^2}{8906688b}.$$

Notice that $K^{**}-\overline{K}$ increases with m_2 for all $m_2\leq m_1$ and when evaluated at $m_2\leq m_1$ gives $-\frac{1136761}{8906688}\frac{(a-m_1)^2}{b}$, which is negative. Therefore, $K^{**}-\overline{K}<0$ for all $m_2\leq m_1$. Finally, it can be shown that $K_L\geq K^*$. Note firstly that for all $K\leq K^*$, the following holds at $r(K)$: $\pi_{F_2}^P(2)-\pi_{D_2}^P(1)=\pi_{F_2}^F(2)-\pi_{D_2}^F(1)$ and $\pi_{F_1}^F(2)-\pi_{D_1}^F(1)\geq\pi_{F_1}^P(2)-\pi_{D_1}^P(1)$. Hence,

$$\Delta W(K)-K\geq\frac{b\left[Q^F(2)\right]^2}{2}+\pi_{D_1}^F(1)+\pi_{D_2}^F(1)-\frac{b\left[Q^P(2)\right]^2}{2}-\pi_{D_1}^P(1)-\pi_{D_2}^P(1)$$

for all $K\leq K^*$.

By substituting the corresponding profit levels and quantities, it can be verified that

$$\Delta W(K)-K\geq\frac{-146am_2-8m_1m_2+77m_2^2+146ar(K)-77r(K)^2+8r(K)m_1}{576b}.$$

The right-hand side of this equation is strictly increasing with r for all $r\leq\frac{a+m_1}{2}$ and since the lowest value that the optimal per-unit price of capacity $r(K)$ can take is m_2, if the right-hand side is positive at $r(K)=m_2$, then it is positive at any $r(K)$. It can also be verified that the right-hand side is equal to 0 at $r(K)=m_2$ so $\Delta W(K)-K\geq 0$ for all $K\leq K^*$. This implies that $K_L\geq K^*$.

Combining all these results proves the proposition.