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A NOTE ON 'THIRD-DEGREE PRICE DISCRIMINATION WITH INTERDEPENDENT DEMANDS'*

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By incorporating symmetric interdependence into linear demands, this note shows that monopolistic third-degree price discrimination can improve social welfare even if total output remains the same, a result that counters to one of the main results presented in Layson [this *Journal*, 1998]. It is also shown that this positive change in social welfare benefits the monopoly side but that consumers' surplus decreases as a whole.

I. INTRODUCTION

THE PURPOSE OF THIS NOTE IS to examine the welfare effects of monopolistic third-degree price discrimination, assuming symmetric interdependent linear demands and constant marginal cost. In the literature, it is well known that when group demands are linear, and all the markets are served either under uniform pricing or under price discrimination, price discrimination does not change total output and decreases Marshallian social welfare.[1] The

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[1] Both Pigou [1920, Part 2, Ch.16, Sect.2 and Appendix D] and Robinson [1933, Book 3, Ch.13, Sect.5] noticed this relationship. Schmalensee [1981] confirmed this conjecture assuming that demand in one market is independent from prices in other markets and that marginal cost is constant. Successively, Varian [1985] and Schwartz [1990] adopted a representative consumer approach to take a further step. Schwartz's [1990] revealed-preference argument can be seen more general than Varian's [1985] algebraical analysis in that the former shows that the conjecture holds for any total cost function that depends only on total output, not on its distribution among markets.

usual explanation for this welfare deterioration proceeds as follows. First of all, in monopoly, (uniform) pricing itself is of course inefficient. To make matters worse, third-degree price discrimination means that consumers whose willingness-to-pay is the same pay different prices just because they belong to different groups. Nonetheless, price discrimination could still be welfare improving if it resulted in a large enough increase in total output. With linear demands, however, it leaves total output unchanged, suggesting that price discrimination should decrease welfare.

However, this explanation ignores the possibility that an increase in monopolistic profit by price discrimination may outweigh a loss in consumers' surplus. When group demand is such that willingness-to-pay is affected by a change in the ratio of supply to the two markets and, can thus be enhanced by price discrimination, the regime change from uniform pricing to price discrimination may benefit the monopolist sufficiently to improve social welfare even if total output remains the same. *Complementarity* between group demands seems to induce this effect. In a recent paper in this *Journal*, Layson [1998] deals with the case of interdependent demands. He states that "if the demands are linear functions of prices and there is cross-price symmetry, then price discrimination lowers welfare" (p.521). However, his argument solely relies on the traditional lesson that "a robust sufficient condition for price discrimination to lower aggregate welfare is that total output remains the same or falls under price discrimination" (p.521). In fact, the author just verifies that "[f]or the linear demand case with cross-price symmetry, the output effect of price discrimination is zero regardless of the slope of marginal cost" (p.520), and derives the above conclusion. He does not directly investigate the welfare change.

The point of this note is to show, with a simple specification of linear demands with symmetric interdependency and constant marginal cost, that this conclusion is not correct and that social welfare can improve even if total output is unaffected by price discrimination.[2]

II. THE MODEL

A monopolist produces a single final product and sells it directly to consumers. She is assumed to be faced with zero fixed cost and zero marginal cost.[3] The set of consumers is

[2] Adachi [2002] shows that social welfare can improve also in the case of linear demands with consumption externalities within groups.

[3] The zero marginal cost assumption can be seen just as assuming a constant marginal cost: in this case prices and consumers' willingness-to-pay are interpreted as net of the cost, which does not invalidate the main thrust of the results.

exogenously partitioned into two groups or sub-markets, 1 and 2, which are identifiably different. For price discrimination to be effective, resale between the two markets is assumed to be impossible.

Let the amount supplied in market i=1,2 be q_i. It is assumed that the monopolist is faced with the following linear inverse demand functions:

$$p_i = a_i - q_i + \eta \cdot q_j \text{ in market } i \neq j \ (i,j=1,2) \tag{1}$$

Here η is a constant that exhibits an effect of symmetric interdependency between markets. For each demand to be stable, it is assumed that $-1<\eta<1$, and without loss of generality $a_1>a_2>0$. When η is positive (negative), the two markets are complementary (substitutable). Since without any interdependency (i.e., if $\eta=0$) the highest willingness-to-pay in market 1 is higher than that in market 2 (that is, $a_1>a_2$), I call market 1 (2) the strong (weak) market as in Layson [1994].

By linearity, consumers' surplus in market i ($i \neq j$) under the regime $r=U,D$ (Uniform priding or price Discrimination) is $(q_i^r)^2/2$, where the amount supplied in market i under the regime r is q_i^r. The monopolist's profit, on the other hand, is the sum of the profit from each market i=1,2, $p_i^r q_i^r$, where the price in market i under the regime r is p_i^r. As usual, Marshallian social welfare is defined as the sum of the consumers' surplus and the monopolist's profit.

First, let us check for what values of parameters the tow markets are open under both uniform pricing and price discrimination. The option of only opening the weak market is never chosen since the profit from opening only the strong market, $a_1^2/4$ (the associated output and price are $a_1/2$ and $a_1/2$, respectively), is always larger than the profit from that option, $a_2^2/4$. It is assumed that the monopolist does not open the weak market unless by doing so she is strictly better off that by keeping it closed. This condition is satisfied if η is large enough.

Lemma 1. Both markets are open under either regime if and only if $\eta(\alpha)<\eta<1$, *where* $\alpha \equiv a_1/a_2$ *and* $\eta(\alpha)=-\alpha+(1-\alpha^2)/2$.

Proof. see appendix

Note that since η(α) is upper-bounded by one half, the weak market is open under *either regime whenever 1/2<η<1.*

III. THE RESULTS

This section examines whether the change in social welfare can be positive when both markets are open under either regime, and if so, when. First, the following proposition is obtained.

Proposition 1. Suppose that the condition η(α)<η<1 is satisfied. Then, (i) the regime change from uniform pricing to price discrimination leads to a reduction in equilibrium output in the strong market and an increase in equilibrium output in the weak market ($q_1^U > q_1^D$ and $q_2^D > q_2^U$). (ii) Total output is the same through the change in the regime. (iii) Discriminatory prices do not depend on the parameter of interdependency, η.

Proof. see appendix

It can be verified that symmetry of interdependency is crucial for parts (ii) and (iii) to hold, but not for part (i).

The change in Marshallian social welfare is $\Delta W = \sum_{i=1,2}(\Delta CS_i + \Delta \pi_i)$, where, for *i*=1,2, $\Delta CS_i \equiv [(q_i^D)^2 - (q_i^U)^2]/2$ (a change in consumers' surplus in market *i*), and $\Delta \pi_i \equiv p_i^D q_i^D - p_i^U q_i^U$ (a change in the monopolist's profit in market *i*). First, the following lemma that comes solely from the fact that is larger than is presented.

Lemma 2. For any η ∈(η(α),1), a regime change from uniform pricing to price discrimination decreases (increases) consumers' surplus in the strong (weak) market.

Proof. Since $\Delta CS_i \equiv [(q_i^D)^2 - (q_i^U)^2]/2$ *for i=1,2,* $q_1^U > q_1^D$, *and* $q_2^D > q_2^U$, *we have* $\Delta CS_1 < 0$ *and* $\Delta CS_1 > 0$. *QED*

So, when price discrimination neither increases nor decreases total output, it necessarily decreases (increases) consumers' surplus in the strong (weak) market. Needless to say, as long as total output remains the same, Pareto improvement is never expected since after price discrimination, some customers in the strong market necessarily give up their consumption ($q_1^D < q_1^U$).[4] Next, the following proposition is obtained.

Proposition 2. Price discrimination decreases the sum of the consumers' surpluses ($\Delta CS_1 + \Delta CS_2 < 0$) irrespective of the value of interdependency, $\eta \in (\eta(\alpha), 1)$.

Proof. Since $\Delta CS_i \equiv [(q_i^D)^2 - (q_i^U)^2]/2$ *for i=1,2 , it is that*

$$\begin{aligned} \Delta CS_1 + \Delta CS_2 &= [(q_1^D + q_2^D)^2 - 2q_1^D q_2^D - (q_1^U + q_2^U)^2 + 2q_1^U q_2^U]/2 \\ &= -q_1^D q_2^D + q_1^U q_2^U \quad (\because \text{Part (i) of Proposition 2}) \\ &< 0 \quad (\because \text{Parts (i) and (ii) of Proposition 2}). \end{aligned}$$

Thus, we are done. QED

However, the following proposition shows that the change in welfare can be positive because an increase in the monopolist's profit can be enough to cover the loss in consumers' surplus.

Proposition 3. Price discrimination improves social welfare ($\Delta W > 0$) if and only if the value of the interdependency exceeds one half ($1/2 < \eta$ (<1)).

Proof. Suppose that $1/2 < \eta$ (<1). Note that this is sufficient for the weak market to be open under uniform pricing as has already been explained. The changes in profit and in consumers' surplus are

$$\Delta\pi_1 + \Delta\pi_2 = (a_1 - a_2)^2 / 8(1+\eta) \qquad (2)$$

and

$$\Delta CS_1 + \Delta CS_2 = -3(a_1 - a_2)^2 / 16(1+\eta)^2 \qquad (3)$$

respectively. Therefore, $\Delta W = (2\eta - 1)(a_1 - a_2)^2/16(1+\eta)^2 > 0$. The converse is clearly true as well.

See the denominators of (2) and (3). When η is positive, or 1+η is larger than one, a change in η has the first power effect for the change in the monopolist's profit ($\Delta\pi_1 + \Delta\pi_2$), while it has a smaller (the second power) effect for the change in the sum of consumers'

[4] For Pareto improvable price discrimination, see Nahata, Ostaszewski and Sahoo [1990].

surpluses ($\Delta CS_1+\Delta CS_2$). This difference comes from the fact that while the monopolist's discriminatory prices are not affected by the change in η (part (iii) of Proposition 2) due to symmetric interdependency, the change affects consumers' surpluses through a quadratic term, $(q_i^r)^2$. At the same time, note that the monopolist's profit and consumers' surplus in each market got to infinity as η approaches to one. This is because the effect of interdependency is linearly incorporated into market demand. So, when is near unity, this result should not be overemphasized. The contrast between Propositions 4 and 5 is also important: with interdependency, the positive change in social welfare only benefits the monopoly side.

APPENDIX

Proof of Lemma 1

First, suppose the monopolist (now, not necessarily optimally) decides to open both markets under uniform pricing. The monopolist's outputs under uniform pricing are determined as follows:

$$q_1^U = K(\eta)a_1 + L(\eta)a_2 \tag{A1}$$

and

$$q_2^U = L(\eta)a_1 + K(\eta)a_2, \tag{A2}$$

where $K(\eta)\equiv(3-\eta)/4\Delta(\eta)$, $L(\eta)\equiv(3\eta-1)/4\Delta(\eta)$ and $\Delta(\eta)=1-\eta^2(>0)$.

Of course, the above outputs should be non-negative. Since $(3-\eta)a_1+(3\eta-1)a_2>0$ when $-1<\eta<1$, q_1^U is always positive. However, should be zero if

$$-1<\eta\leq(a_1-3a_2)/(3a_1-a_2)$$

holds. This upper bound is either positive or negative corresponding to whether (a_1-3a_2) is positive or negative. In this region, the monopolist is better off by opening the strong market only.

Since the price p^U is $(a_1+a_2)/4$ (incorporate (A1) or (A2) into (1)), the associated profit when both markets are open is

$$\frac{a_1 + a_2}{4} \cdot \frac{1+\eta}{2\Delta(\eta)}(a_1 + a_2)$$

(Note that although the homotype market formulation makes the above uniform price not depend on η, generally uniform price depends on η.)

So the monopolist is better off shutting the weak market if and only if

$$\frac{a_1^2}{4} \geq \frac{(1+\eta)(a_1 + a_2)^2}{8\Delta(\eta)}$$
$$\Leftrightarrow \quad -1 < \eta \leq -\frac{a_2}{a_1} + \frac{1}{2}[1-(\frac{a_2}{a_1})^2].$$

Let us define α as a_1/a_2 and η(α) as $-\alpha+(1-\alpha^2)/2$. Since it is verified that

$$\frac{a_1 - 3a_2}{3a_1 - a_2} < -\frac{a_2}{a_1} + \frac{1}{2}[1-(\frac{a_2}{a_1})^2],$$

the monopolist opens both markets if and only if η(α)<η<1.

Next, let us consider the regime of price discrimination. The outputs under price discrimination when the monopolist (not necessarily optimally) decides to open both markets are determined as follows.

$$q_1^D = \frac{1}{2\Delta(\eta)}a_1 + \frac{\eta}{2\Delta(\eta)}a_2 \tag{A3}$$

and

$$q_2^D = \frac{\eta}{2\Delta(\eta)}a_1 + \frac{1}{2\Delta(\eta)}a_2. \tag{A4}$$

Note that the outputs should be non-negative, since $q_i^D \geq 0$ (i=1,2) should hold. So, if $-1<\eta\leq -a_2/a_1$ (=-α), then $q_2^D=0$; i.e., by making the output in the weak market zero, the monopolist can eliminate the negative effect on the strong market, so she becomes better off by opening the strong market only. When she open both markets, the associated discriminatory prices are (i, j=1,2, $i \neq j$)

$$p_i^D = a_i - q_i^D + \eta \cdot q_j^D \tag{A5}$$
$$= a_i / 2.$$

(Interestingly, p_i^D is equal to the price in market 1 when market 2 is not open. So, under price discrimination with linear demands which have symmetric interdependency, whether the weak market is open or not, the price in the strong markets remains the same. The monopolist is affected only through changes in outputs.)

The monopolist's profit when both markets are open under price discrimination is

$$\frac{a_1^2 + 2\eta a_1 a_2 + a_2^2}{4\Delta(\eta)}.$$

Since we have

$$\frac{a_1^2 + 2\eta a_1 a_2 + a_2^2}{4\Delta(\eta)} \geq \frac{a_1^2}{4}$$
$$\Leftrightarrow$$
$$(a_1\eta + a_2)^2 \geq 0,$$

we know that both markets open in the region -α<η<1. In sum, if and only if -1<η≤-α, only the strong market opens, and if and only if -α<η<1, both markets open.

Now since -α<η(α), it is immediate to see that Lemma 1 holds. QED

(Note that as the value of interdependency approaches to its least upper bound, the monopolist's profit under either regime goes to infinity since outputs in both markets got to infinity.)

Proof of Proposition 2

Parts (i) and (ii) are derived by direct calculation using (A1)-(A4):

$$q_1^U - q_1^D = ((a_1 - a_2)/(4(1+\eta)))$$
$$= q_2^D - q_2^U > 0.$$

As for part (iii), see (A5). QED

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