

REPLY TO PAOLO BERTOLETTI, ‘A NOTE ON THIRD-DEGREE PRICE DISCRIMINATION AND OUTPUT’*

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The aim of this reply to Bertolotti [this *Journal*, 2004] is to give a ‘rational expectations’ rationale for the demand system and the definition of consumer surplus in Adachi [this *Journal*, 2002].

I. INTRODUCTION

IN THE LITERATURE OF monopolistic third-degree price discrimination, it has been widely held that a change in aggregate Marshallian social welfare by price discrimination is negative if total output decreases.¹ Among others, Schwartz [1990] verifies this conjecture for any total cost function that depends only on total output, not on its distribution among markets. Letting ΔX and ΔW denote a change in aggregate output and in social welfare, respectively, we can express his argument as follows:

$$\Delta X < 0 \Rightarrow \Delta W < 0.$$

Bertolotti [2004] extends Schwartz's [1990] result; he shows that social welfare must

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¹ See e.g. Stole [2003, Subsection 3.1].

decrease by price discrimination even if total output remains constant. Again, using the same notation, we can describe his claim by:

$$\Delta X \leq 0 \Rightarrow \Delta W < 0.$$

On the contrary, in Adachi [2002] I point out that when symmetric consumption externality between groups is considered, monopolistic third-degree price discrimination can improve social welfare even if total output remains the same, which seems contradictory to Bertolotti's [2004] result. One might be tempted to say Bertolotti's [2004] model is not general enough, or Adachi's [2002] argument has some logical flaws. Neither is correct. To make clear this point, the present reply gives a "rational expectations" rationale for the demand system and the definition of consumer surplus used in Adachi [2002].

Originally, I wrote Adachi [2002] as a note on Layson [1998] with intent to correct one of his results; that monopolistic third-degree price discrimination never improves social welfare if total output remains the same. At that time I interpreted consumption externality between groups as interdependency between group demands. I now realize that this interpretation is a misleading one, and I can no longer say that Adachi [2002] corrected one of the main results of Layson [1998] because his (and Bertolotti's [2004]) micro foundation and mine are different. In particular, the statements below in Introduction of Adachi [2002] are no longer justified:

‘... In a recent paper in this *Journal*, Layson [1998] deals with the case of interdependent demands. He states that "if the demands are linear functions of prices and there is cross-price symmetry, then price discrimination lowers welfare" (p.521). However, his argument solely relies on the traditional lesson that "a robust sufficient condition for price discrimination to lower aggregate welfare is that total output remains the same or falls under price discrimination" (p.521). In fact, the author just verifies that "[f]or the linear demand case with cross-price symmetry, the output effect of price discrimination is zero regardless of the slope of marginal cost" (p.520), and derives the above conclusion. He does not directly investigate the welfare change.’

At the same time, however, the results in Adachi [2002] also make sense if what I explain in the following sections is understood. The bottom line is that in Adachi [2002] I wrote the demand system with the same notation as Layson's [1998], though Adachi's [2002] demand system is a conceptually different object from Layson's [1998] and

Bertoletti's [2004].

II. EACH CONSUMER HAS A UNIT DEMAND (CONSUMER HETEROGENEITY)

In the model of Adachi [2002], there is a continuum of consumers, and each consumer in market $i=1,2$ is indexed by $x_i \in [0, M]$ (M is so large a positive real number for a boundary solution to be excluded), and the expected number of customers who consume in market $j=1,2$ is denoted by $n_j^e \in [0, M]$. We assume that each individual can consume at most one unit of the good: if a consumer x_i in market i purchases the good, he gains gross utility $(a_i - x_i + \eta n_j^e)$, where $j \neq i$, by paying the price $p_i \geq 0$, and if he does not purchase, he gains nothing. This is where consumer heterogeneity arises, and η is a constant which exhibits the effect of a consumption externality between two markets.² For the law of demand to hold, we need to assume that $0 < \eta < 1$ (meaning that the focus is not on negative externality but on positive externality). Lemma 1 in Adachi [2002] shows the existence of the lower bound $\underline{\eta}(\alpha)$, where $\alpha \equiv a_2 / a_1$, for η to make the monopolist optimally supply a positive amount of the good to both markets under either the regime of uniform pricing or the regime of price discrimination.

III. EQUILIBRIUM DEMAND SYSTEM

As is the case in the literature of network externality such as Katz and Shapiro [1985], we confine our attention to the Rational Expectations Equilibrium (hereafter, REE) market demands (that is, $n_i^e = q_i$ for $i=1,2$). The terms q_i and q_j which appears in the equation (1) in Adachi [2002],

$$p_i = a_i - q_i + \eta \cdot q_j \quad \text{in market } i \neq j \ (i, j=1,2), \quad (1)$$

² See Adachi [2004] for an analysis of the situation where there are consumption externalities within markets.

are such REE market demands.

Considering REE market demands does not mean that we treat an arbitrary pair satisfying

$$p_i^r = a_i - q_i^r + \eta \cdot q_j^r \quad \text{in market } i \neq j \text{ under regime } r=U, D, \quad (2)$$

where U means Uniform Pricing and D means Discriminatory Pricing. It rather means that we take the realized unique pair by the monopolist's profit maximization (the existence can be checked), as the pair of aggregate demands. Essentially, it is possible to think that the monopolist directly decides a supply schedule (q_1^r, q_2^r) , under equality constraint (2), to maximize her profit. By the uniqueness, the solution of the monopolist's optimization (profit maximization) without the constraint $(q_1^r, q_2^r) \in [0, M]^2$ is a unique alternative of the optimal supply schedule. Since in the following such q_i^r is verified to be non-negative (note that M can be arbitrary large) under the present model, it is assured that the pair indeed exists as optimal quantities for the monopolist.

IV. DEFINITION OF CONSUMER SURPLUS UNDER REE

In REE, the net utility of consumer x_i is expressed by $(a_i - q_i + \eta q_j - p_i^r)$, and Adachi [2002] defines the consumer surplus of each market i under the regime r by

$$CS_i^r \equiv \int_0^{q_i^r} [a_i - x_i - \eta q_j^r - p_i^r] dx_i = \frac{(q_i^r)^2}{2}, \quad (3)$$

which matches the definition of CS in Katz and Shapiro [1985, p.428, Equation (9)].

Notice here that we are integrating the net utility with respect to x_i . We do not use the demand system (1) when we calculate consumer surplus: our CS_i^r is not derived from the demand system. This is because the demand system (1) just determines the equilibrium amount of demand. Hence, it is not the basis of welfare evaluation. This is exactly what makes a crucial difference between the model with consumption externality such as Adachi [2002] and the ones without such as Bertolotti [2004] and Garela [2002]. We see this difference in the next section.

V. DEFINITION OF CONSUMER SURPLUS UNDER THE REPRESENTATIVE CONSUMER ASSUMPTION

To derive the market demands (1) and consumer surplus (3), we have supposed that each individual has a unit demand as explained in Section 2. Yet, there is another way to derive the market demands (1), which is based on the Representative Consumer Assumption (hereafter, RCA). The following accounts are based on Galera [2002].

This approach starts with assuming that the demand system is determined as a solution of a representative consumer's maximizing his utility, $U(q)-p \cdot q$. In RCA, this $U(q)-p \cdot q$ itself is defined as consumer surplus. If we assume the following functional form:

$$U(q_1, q_2) = a_1 q_1 + a_2 q_2 - (1/2)(q_1^2 + q_2^2 - 2\eta q_1 q_2),$$

then we can verify that maximizing utility with respect to q_1, q_2 yields the same demand system as in (1).

Now, let \overline{CS}_i^r be the consumer surplus of each market i under regime r with the above specification of the representative consumer's utility. Galera [2002] shows that for each r , the following relationship holds (assuming the monopolist supplies a positive amount of the good to both markets under either regime):

$$\overline{CS}_1^r + \overline{CS}_2^r = CS_1^r + CS_2^r - \eta q_1 q_2.$$

Let ΔCS be the change in the sum of consumer surplus due to the regime change from uniform pricing to price discrimination. Then, Galera [2002] shows that

$$\Delta CS = -(3/16)(a_2 - a_1)^2 / (1 + \eta) \tag{4}$$

whereas the corresponding object in Adachi [2002] is as follows:

$$\Delta CS = -(3/16)(a_2 - a_1)^2 / (1 + \eta)^2. \tag{5}$$

This appears in the proof of Proposition 3 in Adachi [2002], which states that 'price discrimination improves social welfare if and only if the value of the interdependency exceeds one half ($1/2 < \eta < 1$).' If we replace (5) by (4), then Proposition 3 does not hold any more, that is, for any $\alpha \in (0, 1)$, price discrimination cannot improve social welfare for

any $\eta \in (\eta(\alpha), 1)$. Moreover, Bertolotti [2004] generally shows that under RCA social welfare must decrease by price discrimination even if total output remains the same.

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