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BUNDLING AND MENUS OF TWO-PART TARIFFS: COMMENT

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This comment, following Kolay and Shaffer [2003] (hereafter K&S), examines the problem of a monopolist that is unable to distinguish between high and low-demand consumers and considers two pricing strategies: two-part tariffs and bundling. Assuming linear demand curves, the profit-maximizing menu for each pricing strategy is solved and for each strategy necessary and sufficient conditions for the monopolist to serve only the high-demand consumers are characterized. By not considering such corner solutions, K&S understate the welfare benefits of bundling.

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I. INTRODUCTION

IN A RECENT ISSUE OF THIS JOURNAL, Kolay and Shaffer [2003] (hereafter K&S) considered the case of a monopolist selling to two types of consumers with different valuations of its product, where the monopolist cannot distinguish between consumer types. They compared two pricing mechanisms: a menu of bundles consisting of packages of fixed quantities and a menu of two-part tariffs consisting of fixed fees and per-unit prices that allows each consumer to choose the quantity s/he purchases. Assuming that it is always optimal for the monopolist to serve both consumer types, they showed that, absent cost considerations, the monopolist earns strictly higher profit under the bundling strategy, with ambiguous implications for social welfare. They also showed that if the monopolist could distinguish between consumer types (or if it only serves the high-demand consumers) then the two pricing mechanisms yield the same levels of profit and have the same implications for social welfare.

K&S did not, however, consider the important case in which it is optimal for the monopolist to serve both consumer types only if it can offer price-quantity packages.[1] By not considering this case, K&S understate the welfare benefits of bundling, and consequently, overstate the welfare benefits of two-part tariff pricing.[2]

In this comment, assuming linear demand curves, it is shown that while in the case of a profit-maximizing bundling mechanism the correct solution always emerges from the analysis of K&S, this is not necessarily the case under two-part tariffs. By deriving the necessary and sufficient conditions for corner solutions to arise (i.e., the monopolist serves only the high demand consumers) an error in one of K&S's numerical examples is pointed out. Although, as K&S report, bundling and menus of two-part tariffs yield the same level of social welfare for the case of parallel linear demand functions when the monopolist wants to serve both types of consumers, the parameters they chose to illustrate this fall within the identified region in which it is optimal for the monopolist to serve both types of consumers only if it sells packages. Hence, in their example for this case, social welfare is actually higher under bundling.

II. THE MONOPOLIST'S PROBLEM

Let us consider a profit-maximizing monopoly that produces a single product at constant marginal cost c and faces a linear demand composed of two types of consumers: consumers of type 1 with taste parameters (α_1, θ_1) that occur in proportion λ, $\lambda \in [0,1]$, and consumers of type 2 with taste parameters (α_2, θ_2) that occur in proportion $1-\lambda$. A consumer of type $i, i = 1,2$ has the following linear demand curve: $q_i(p) = D_i(p_i) = \dfrac{\theta_i - p_i}{\alpha_i}$. We assume that $c < \theta_1 \leq \theta_2$ and $\dfrac{\theta_2}{\alpha_2} \geq \dfrac{\theta_1}{\alpha_1}$, which implies that consumers of type 1 are the low-demand consumers.[3]

I(i). *Menus of Two-Part Tariffs*

Under two-part tariff pricing, K&S solved the following problem:

(1) $$\max_{p_1, p_2, F_1, F_2} \lambda(p_1 - c)q_1(p_1) + \lambda F_1 + (1-\lambda)(p_2 - c)q_2(p_2) + (1-\lambda)F_2,$$

subject to two constraints: (i) the low-demand consumer buys a positive quantity, and (ii) the high-demand consumer chooses to purchase under (p_2, F_2) rather than (p_1, F_1), where p_i denotes the per-unit price and F_i is the fixed fee, $i = 1,2$.

They obtained the following well-known general results: (i) there is no pricing distortion at the top, (ii) the low-demand consumer derives no net surplus, and (iii) the high-demand consumer is indifferent between (p_2, F_2) and (p_1, F_1).

Under the class of linear demand curves, their results imply that:

(2) $p_2^* = c$, $F_1 = \frac{(\theta_1 - p_1)^2}{2\alpha_1}$, and $F_2 = \frac{(\theta_2 - c)^2}{2\alpha_2} - \left(\frac{(\theta_2 - p_1)^2}{2\alpha_2} - F_1\right)$.[4]

Thus, if the monopolist serves both types its profit as a function of p_1 is:

(3) $\pi^{2PT}(p_1) = \lambda\pi_1^{2PT}(p_1) + (1-\lambda)\pi_2^{2PT}(p_1)$, where

$$\pi_1^{2PT}(p_1) = \left(\frac{(p_1 - c)(\theta_1 - p_1)}{\alpha_1} + \frac{(\theta_1 - p_1)^2}{2\alpha_1}\right)$$

and

$$\pi_2^{2PT}(p_1) = \left(\frac{(\theta_2 - c)^2}{2\alpha_2} + \frac{(\theta_1 - p_1)^2}{2\alpha_1} - \frac{(\theta_2 - p_1)^2}{2\alpha_2}\right).$$

The price p_1^* that fulfills the first-order condition for maximization of π^{2PT} is:

(4) $$p_1^* = \frac{(1-\lambda)(\theta_2\alpha_1 - \theta_1\alpha_2) + \lambda\alpha_2 c}{(1-\lambda)(\alpha_1 - \alpha_2) + \lambda\alpha_2},$$

which implies that $q_1^*(p_1^*) = \frac{\lambda\alpha_2\alpha_1^{-1}(\theta_1 - c) - (1-\lambda)(\theta_2 - \theta_1)}{(1-\lambda)(\alpha_1 - \alpha_2) + \lambda\alpha_2}$. Since the second-order condition implies that the denominator of q_1^* is positive, a necessary and sufficient condition for $p_1^* < \theta_1$ or for $q_1^*(p_1^*) > 0$ is $\frac{\theta_2 - \theta_1}{\theta_1 - c} < \frac{\lambda\alpha_2}{(1-\lambda)\alpha_1}$.

Notice, however, that the solution to the monopolist's problem in (1) subject to constraints (i) and (ii) is necessary but not sufficient to establish whether the monopolist would want to serve both consumer types or only the high-demand consumers. Although choosing p_1 such that type 1 consumers choose $q_1 = 0$ is feasible in this program, the ensuing monopoly profits are not the same as they would be if the monopolist simply refused to sell to these consumers. The reason is that at the price $p_1 = \theta_1$, one gets that $F_2 = \frac{(\theta_2 - c)^2}{2\alpha_2} - \frac{(\theta_2 - \theta_1)^2}{2\alpha_2}$, which is lower than

$\frac{(\theta_2 - c)^2}{2\alpha_2}$. This implies that F_2 does not fully extract the high-demand consumers' rent.

By substituting (4) into (3) and rearranging terms, one obtains that the monopolist's profit, if it serves both consumer types, is:

(5) $$\pi^{2PT}(p_1^*) = \pi^* = (1-\lambda)\frac{(\theta_2 - c)^2}{2\alpha_2} + A \text{ , where}$$

$$A = \frac{(1-\lambda)(1-2\lambda)(\theta_2-\theta_1)^2 - 2\lambda(1-\lambda)(\theta_2-\theta_1)(\theta_1-c) + \lambda^2\alpha_1^{-1}\alpha_2(\theta_1-c)^2}{2\left((1-\lambda)(\alpha_1-\alpha_2)+\lambda\alpha_2\right)}.$$

However, if the monopolist sells only to the high-demand consumers its profit will be $(1-\lambda)\frac{(\theta_2 - c)^2}{2\alpha_2}$, which is equal to the first term of equation (5). Thus, it is optimal for the monopolist to serve both consumer types if and only if $p_1^* < \theta_1$ and $A > 0$. Letting $y = \frac{\theta_2 - \theta_1}{\theta_1 - c}$, the conditions for this to occur can be written as:

(6) $$y < \frac{\lambda\alpha_2}{(1-\lambda)\alpha_1} \text{ and } (1-\lambda)(1-2\lambda)y^2 - 2\lambda(1-\lambda)y + \frac{\lambda^2\alpha_2}{\alpha_1} > 0.$$

This completes the analysis of the monopolist's problem under two-part tariff pricing.

I(ii). *Bundling*

Under bundling, K&S solved the following problem:

(7) $$\max_{q_1, q_2, T_1, T_2} \lambda(T_1 - cq_1) + (1-\lambda)(T_2 - cq_2),$$

subject to: (i) the low-demand consumer buys a positive quantity, and (ii) the high-demand consumer prefers the bundle q_2 at price T_2 rather than q_1 at price T_1.

They obtained the following well-known general results: (i) there is no quantity distortion at the top, (ii) the low-demand consumer derives no net surplus ,and (iii) the high-demand consumer is indifferent between (q_1, T_1) and (q_2, T_2).

Under the class of linear demand curves, these results imply that:

(8) $$q_2^* = \frac{\theta_2 - c}{\alpha_2},\ T_1 = \frac{(2\theta_1 - \alpha_1 q_1)q_1}{2}, \text{ and } T_2 = \frac{\theta_2^2 - c^2}{2\alpha_2} - \left(\frac{(2\theta_2 - \alpha_2 q_1)q_1}{2} - T_1\right).$$

Thus, if the monopolist serves both types its profit as a function of q_1, is: [5]

(9) $$\pi^B(q_1) = \lambda\pi_1^B(q_1) + (1-\lambda)\pi_2^B(q_1), \text{ where}$$

$$\pi_1^B(q_1) = \left(\frac{(2\theta_1 - \alpha_1 q_1)q_1}{2} - cq_1\right)$$

and

$$\pi_2^B(q_1) = \left(\frac{\theta_2^2 - c^2}{2\alpha_2} + \frac{(2\theta_1 - \alpha_1 q_1)q_1}{2} - \frac{(2\theta_2 - \alpha_2 q_1)q_1}{2} - \frac{c(\theta_2 - c)}{\alpha_2}\right).$$

The quantity q_1^* that fulfills the first-order condition for maximization of π^B is:

(10) $$q_1^* = \frac{\theta_1 - (1-\lambda)\theta_2 - \lambda c}{\alpha_1 - (1-\lambda)\alpha_2} = \frac{\lambda(\theta_1 - c) - (1-\lambda)(\theta_2 - \theta_1)}{\alpha_1 - (1-\lambda)\alpha_2}.$$

Since the second-order condition implies that the denominator of q_1^* is positive, $q_1^* > 0$ if and only if $y < \frac{\lambda}{(1-\lambda)}$, where $y = \frac{\theta_2 - \theta_1}{\theta_1 - c}$ was previously defined.

It is interesting that the solution in this case, unlike that found above for the case of two-part tariff pricing, is in fact both necessary and sufficient to establish whether the monopolist would want to serve both types of consumers or only the high-demand consumers. It is easy to see, for example, that not serving the type 1 consumers by choosing $q_1 = 0$ is not only feasible in this program but also yields the same monopoly profits as by simply refusing to make an offer to these consumers.

Thus, it is optimal for the monopolist to serve both consumer types under bundling if and only if $q_1^* > 0$, or in other words, if and only if $y < \frac{\lambda}{(1-\lambda)}$.

Using this latter condition and the conditions in (6), allows us to characterize necessary and sufficient conditions for corner solutions to arise:

Proposition 1. Necessary and sufficient conditions for corner solutions to arise are:

(i) If $\lambda \neq 0.5$ and all second-order conditions for maximization hold, then there exists a value y_1, $y_1 = \frac{\lambda(1-\lambda) - \lambda\sqrt{(1-\lambda)^2 - (1-\lambda)(1-2\lambda)\alpha_1^{-1}\alpha_2}}{(1-\lambda)(1-2\lambda)}$, such that if $y < y_1$, then it is optimal for the monopolist to serve both consumer types. If $y_1 < y < \frac{\lambda}{(1-\lambda)}$, then it is optimal for the monopolist to serve both consumer types only in the case of bundling.[6] If $y > \frac{\lambda}{(1-\lambda)}$, then it is never optimal for the monopolist to serve both consumer types.

(ii) If $\lambda = 0.5$ and all second-order conditions for maximization hold, then it is optimal for the monopolist to serve both consumer types if $y < \frac{\alpha_2}{2\alpha_1}$. If $\frac{\alpha_2}{2\alpha_1} < y < 1$, then it is optimal for the monopolist to serve both consumer types only in the case of bundling.[7] If $y > 1$, then it is never optimal for the monopolist to serve both consumer types.

(iii) It is never optimal for the monopolist to serve both types of consumers under two-part tariff pricing but not under bundling.

Proof. The proof of (iii), which follows from showing that $y_1 < \frac{\lambda}{1-\lambda}$ when $\lambda \neq 0.5$ and $\frac{\alpha_2}{2\alpha_1} < 1$ when $\lambda = 0.5$, is available on request. A sketch of the rest of the proof is provided in the following. In order to characterize the range of parameters that satisfy the second inequality in (6), we characterize the solutions of the following equation:

$$(11) \qquad (1-\lambda)(1-2\lambda)y^2 - 2\lambda(1-\lambda)y + \frac{\lambda^2\alpha_2}{\alpha_1} = 0\,.$$

If $\lambda > 0.5$, then equation (11) has two solutions but only one of them, denoted by y_1, is positive, where $y_1 = \frac{\lambda(1-\lambda) - \lambda\sqrt{(1-\lambda)^2 - (1-\lambda)(1-2\lambda)\alpha_1^{-1}\alpha_2}}{(1-\lambda)(1-2\lambda)}$. Since $0 < y_1 < \frac{\lambda\alpha_2}{(1-\lambda)\alpha_1}$, and $1-2\lambda < 0$ the inequalities in (6) are fulfilled (i.e. it is optimal for the monopolist to serve both consumer types) if and only if $y = \frac{\theta_2 - \theta_1}{\theta_1 - c} < y_1$.

If $\lambda < 0.5$, and the second-order conditions hold, equation (11) has two positive solutions:

$$(12) \qquad y_{1,2} = \frac{\lambda(1-\lambda) \mp \lambda\sqrt{(1-\lambda)^2 - (1-\lambda)(1-2\lambda)\frac{\alpha_2}{\alpha_1}}}{(1-\lambda)(1-2\lambda)}\,.$$

However, since $(1-2\lambda) > 0$ and $y_1 < \frac{\lambda\alpha_2}{(1-\lambda)\alpha_1} < y_2$, it is optimal for the monopolist, in this case as well, to serve both consumer types if and only if $y < y_1$.

If $\lambda = 0.5$, then the second inequality of (6) becomes $\frac{0.25\alpha_2}{\alpha_1} - 0.5y > 0$. Thus, it is optimal for the monopolist to serve both types if and only if $y < \frac{\alpha_2}{2\alpha_1}$ Q.E.D.

Proposition 1 characterizes necessary and sufficient conditions for all possible outcomes under monopoly pricing with bundling and two-part tariffs. This is important because it highlights the possibility that the monopolist may, under some conditions, want to serve both consumer types under bundling but not under two-part pricing, and that it is never optimal for the firm to serve both types only under two-part pricing. Thus, the true welfare benefits of bundling are relatively understated when attention is focused, as in K&S, only on the case in which both types are served.

To illustrate, the numerical example presented on pages 397 and 402 of K&S uses the following values: $\theta_2 = 40, \theta_1 = 20, \alpha_2 = \alpha_1 = 1, \lambda = \frac{2}{3}$, and $c = 6$, which imply the following inequalities: $\frac{\lambda\alpha_2}{(1-\lambda)\alpha_1} = 2 > \frac{\theta_2 - \theta_1}{\theta_1 - c} = y = 1.428 > y_1 = 0.828$. Therefore, as one can see from Proposition 1, K&S have erred in characterizing this case. In fact, since $y_1 < y < \frac{\lambda}{(1-\lambda)} = 2$ in their example, it is optimal for the monopolist to serve both consumer types only in the case of bundling. As a result, the monopolist's profit is 192.66, which is higher than K&S' reported value of 131.33.

III. CONCLUSION

In this comment, assuming linear demand curves, we completely characterize the optimum solution (corner or interior) for the monopolist's profit maximization problem under both a menu of two-part tariffs and price-quantity packages. We show that there exists a scenario wherein the monopolist finds it optimal to serve both types of consumers only when it offers price-quantity packages. By ignoring this case, K&S understate the welfare benefits of bundling. This can be seen in their example for the case of parallel linear demands. In this example, K&S erroneously conclude that social welfare is independent of the pricing strategy used. However, as it has been shown above, the monopolist would not want to serve both consumer types in their example under a two-part tariff strategy, whereas it would want to serve both consumer types under bundling. Therefore, in their example, output and hence welfare will be higher under bundling.[8]

REFERENCES

Kolay, S. and Shaffer, G., 2003, 'Bundling and Menus of Two-Part Tariffs,' *The Journal of Industrial Economics*, 51, pp. 383-403.

END NOTES

[1]It is never optimal to serve both types with two-part pricing but only one type with bundling.

[2]K&S's conclusion that bundling is more profitable than two-part tariffs still holds in this case.

[3]Due to the heterogeneity of the consumers, at least one of the inequalities is strict.

[4]Notice that the second term in F_2 is the type 2 consumer's information rent.

[5]Notice that the second term in T_2 is the type 2 consumer's information rent.

[6]Notice, however, there is a range of parameters such that $y_1 < y < \frac{\lambda\alpha_2}{(1-\lambda)\alpha_1}$, in which it is optimal for the monopolist to serve only type 2 consumers under two-part tariff pricing even though $p_1^* < \theta_1$.

[7]Notice, however, there is a range of parameters such that $\frac{\alpha_2}{2\alpha_1} < y < \frac{\alpha_2}{\alpha_1}$, in which it is optimal for the monopolist to sell only to type 2 consumers under two-part tariff pricing even though $p_1^* < \theta_1$.

[8]In the case of parallel demand curves, for parameters such that the monopolist would always want to serve both consumer types, the two pricing mechanisms will lead to the same level of social welfare, as noted by K &S in Proposition 2 of their paper. To illustrate this, suppose that θ_1 = 30, θ_2 = 50, λ = 3/4, and c = 6. Then the quantities consumed by each type are indeed identical across the two strategies: $q_1^*(p_1^*)$ = $\hat{q}_1$ = 17.33 and $q_2^*(p_2^*)$ = $\hat{q}_2$ = 44. The per-unit prices and payments under the optimal menu of two-part tariffs are p_1^* = 12.67, p_2^* = 6, T_1^* = 369.78 and T_2^* = 685.33, and the monopolist's maximized profit is π^{2PT} = 304.67. The prices under the optimal menu of price-quantity packages are $\hat{T}_1$ = 369.78 and $\hat{T}_2$ = 885.33, and the monopolist's maximized profit is π^B = 354.67.