Endogenous market price index, trade frictions and elasticity of substitution

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Abstract

This paper establishes a model to explore how all the countries in the world export to an importer. The trade frictions of an exporter include both a variable cost and a fixed cost, and the exporter’s bilateral trade relationship is represented by the exported value, the number of exporting firms, and the average price of exported products. The model relaxes the usual assumption that the price index in the market is exogenous (see, e.g., Chaney, 2008), and this leads to the following main results. First, the trade frictions negatively affect its own bilateral trade relationship and positively affect all its cross bilateral trade relationships. Nonetheless, the variable cost has positive effect on its own average product price. The sizes of all the effects are related to the market share of the exporter. Second, the trade frictions have impact on the total number of firms in the market and the bilateral trade relationship is influenced by the Elasticity of Substitution (ES) of the market. Both effects increase with the fixed cost. Finally, the price index is positively affected by both the trade frictions and the ES. Raising the ES can reduce the total number of exporting firms in the market.

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1 Introduction

Chaney (2008) establishes a model for the trade system that includes an importer and all the countries exporting to the importer. The model is restricted in a sector that consists of a group of products homogenous at some level. The market price index of the importer is endogenous and depends on the prices of all the goods sold in the market. In turn, the index will influence the market demand for the goods available in the market and will further affect the export from all the exporters to the importer. At the equilibrium level of model, Chaney (2008) solves the price index and then by doing comparative statics derives the effect of an exporter’s trade friction on the value of products exported from the exporter to the importer. In the derivation, Chaney (2008) assumes that the exporter’s share in the market of the importer approaches zero. This assumption implies that price variations of the exported products from the exporter, which are caused by changing the trade friction, do not impact the market price index. Thus, in the derivation, the price index is actually exogenous under this assumption. However, in reality the market share of an exporter in an importer may be much larger than zero. For example, Waugh (2010) shows that, for purchase of the goods from the US, Mexico uses around 30 percent of its total expenditure and this figure in Canada reaches more than 40 percent. To have a more general analysis on trade, this paper sets up a trade model parallel to the one of Chaney (2008) and relaxes the assumption of zero market share. In the following two paragraphs, we introduce the endogenous variables of interest in the model.

For a country exporting to the importer, we adopt three endogenous variables to indicate the bilateral trade relationship which are the exported value, the number of exported product varieties and the average price of the exported products. Moreover, the exported value is decomposed into the intensive margin and the extensive margin.\(^1\) In addition, the average exported value per firm will also be considered in the bilateral unidirectional export. In the export, the exporter will benefit not only from the higher exported value but also from the more exported product varieties. Haddad, Lim and Saborowski (2010), Stanley (1999) and Herzer and Nowak-Lehmann (2006) explain how the diversification of the export can favour the exporter. Also in the export, lower average product price from the exporter will benefit the consumers in the importer.

\(^1\)In the model section, we explain the decomposition methods.
For the importer, we use both the market price index and the total number of product varieties in the market to represent the structure of the market. In the market, we will also calculate the shares of the total expenditure which are used respectively for the variable cost of all the exporters, for the fixed cost of them and for the profit of them. Of the importer, both more imported product varieties and lower market price index can contribute to the welfare. Broda and Weinstein (2011) show that the welfare of the importer increases with its import diversification. According to Arkolakis, Costinot and Rodríguez-Clare (ACR, 2012), the welfare of the importer is inversely related to the market price index of the importer.\(^2\)

Next, we list the issues this paper considers at equilibrium level of the model. First, we will explore how the trade friction of an exporter makes its own bilateral trade relationship different to the others. Then, we will do comparative statics to evaluate the trade friction’s effects on its own bilateral trade relationship, on its cross bilateral trade relationships and also on the market structure. It is worth to notice that the second effect reveals the multilateral resistance caused by changing the trade friction. If we assumed that the share of the exporter in the importer’s market approached zero, the market price index would be exogenous in response to the change of the trade friction and then the friction would have no effect on its cross bilateral trade relationships. However, this assumption is not made in this paper.

The Elasticity of Substitution (ES), which is between varieties in the market of the importer, plays an important role to characterize the market. The larger the ES, the more competitive the market. In another direction, when the ES is small the market is more likely to be divided into monopolies. The export of an exporter to the market depends on the demand structure shaped by the ES, given the market size and the exporter’s properties. Therefore, some previous papers have worked on empirical estimation of the ES (e.g., Feenstra, 1994; Yilmazkuday, 2012; Bernard, Eaton, Jensen, and Kortum, 2003). Even though, little work has theoretically evaluated the direct influence of the ES on the exporter’s export.\(^3\) Therefore, this paper will investigate how the ES influences all the bilateral trade relationships and affects the market structure.

\(^2\)This relationship is mathematically presented in Model section where we will also show that the import diversification is negatively related to the market price index.

\(^3\)The trade friction of the exporter affects its own exported value to the market. Chaney (2008) discovers the role of the ES in this effect.
The remainder of the paper will be arranged as follows. The related previous literature will be reviewed in the next section. In Section 3, we will establish the trade model and also will solve the model at its equilibrium level. At the equilibrium level of the model, we will in section 4 discover how the trade friction of a bilateral export affects all the endogenous variables introduced in Introduction section and will in Section 5 derive the influences of the ES on these endogenous variables. Then, all the results in this paper will be discussed and summarized in Section 6. Finally, Section 7 concludes this paper.

2 Literature Review

In this section, we will first review some of the previous models closely related to the model in this paper. Dixit and Stiglitz (1977; hereafter DS) construct a monopolistic competitive model for trade analysis. The main components of their model include for the importer a Constant Elasticity of Substitution (CES) utility and the two-stage budgeting procedure, and also include for an exporter a linear cost function with both a variable cost and a fixed cost. In the importer, the two-stage budgeting procedure implies that a share of the total expenditure is used for consumption in a sector’s market and the share is a function of the market’s price index. The sector includes some products grouped by a particular categorization. All the models discussed in this paper take the skeleton of their model. Chaney (2008) and Manova (2013) let the share be a constant, which simplifies the trade analysis within the sector and will be adopted by this paper. The model of Helpman, Melitz and Rubinstein (2008; hereafter HMR) do not group products into sectors, neither do Meltiz (2003) and Anderson and van Wincoop (2003).

In the model of DS (1977), the homogeneous firms freely enter the market until the last firm receives zero profit. In the model of Melitz (2003), a firm in a country can be established after randomly choosing a productivity level from a given distribution. Then the firm will be run if it can acquire a positive profit from the market, and will be closed otherwise. The total number of potential firms in the country is restricted by the total labor supply of the country. Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008) set up their model similar to Melitz (2003). In contrast, HMR (2008) assume that the number of total firms in an exporter is exogenous and that the firm productivity levels follow the bounded Pareto distribution. In the model of HMR (2008), the productivity distribution is identical for all exporters. The model of Chaney (2008) is close to that of HMR (2008), except that, in the exporter, the total number of firms depends on the
exogenous total labor supply and that the Pareto distribution of firm productivity is unbounded. This unbounded distribution form allows Chaney to manipulate his model in achieving some theoretical results. HMR (2008) introduce the country-level productivity for an exporter which serves as the price of cost in exporting of the exporter. Instead, Chaney (2008) uses the wage rate of the exporter to represent the exporter’s country-level productivity.

The recent development of the DS (1977) type model is on the cost function, and is made by Arkolakis (2010) who replaces the exporting fixed-cost by the expenditure on advertisement. A firm will enter a larger market, if it pays more on advertisement. Therefore, the firm needs to choose the optimal value of the advertising expenditure to maximize its profit. Eaton, Kortum and Kramarz (EKK, 2011) also use the endogenous exporting fixed-cost in their model. Differently, Anderson and van Wincoop (2003) modify the DS (1977) model by dropping the exporting fixed-cost. Similar to DS (1977), they also do not differentiate all the goods from one exporter. Therefore, in the model of Anderson and van Wincoop (2003), an exporting country can be deemed a large exporting firm.

Then, we will review some of the findings on basis of these theoretical trade models. In the model of Anderson and van Wincoop (2003), the exporting cost is symmetric between the two exporting directions in a bilateral trade. With this assumption, their model results in a gravity function for the trade value in a bilateral unidirectional export. The arguments of the function include the importer output, the exporter output, the importer price index and the exporter price index. Instead of the price indices, Chaney’s (2008) gravity function includes the costs in the bilateral export. Moreover, HMR (2008) showed that, conditional on some additional assumptions, their model can also be used to derive the gravity function in Anderson and van Wincoop (2003).

In a bilateral unidirectional export, the ES plays an important role. To check the role, Chaney (2008) depicts the way through which the ES influences the effect of the bilateral trade friction on the exported value. In comparison, this paper will do comparative statics to explore how the ES directly affects both the market structure of the importer and the bilateral trade relationship. Anderson and van Wincoop (2003) suggest that the unidirectional bilateral export is affected by its own trade cost, by other exporting costs of the exporter and by other importing costs of the importer. The last two effects represent the multilateral resistance to this bilateral
trade. To obtain the multilateral resistance, they first generate the output shares of both the exporter and the importer and the shares are of the output of the entire world. Then they explore the roles the two shares play in the marginal effect of the exporting cost on the bilateral exports.\textsuperscript{4} Differently, this paper proposes that the bilateral export is not affected by the costs of the exporter in exporting to other countries. Instead, we will obtain the mechanism through which the trade friction of the unidirectional bilateral export affects the export from other exporters to the importer.

3 The Model

All countries in the world are in set $\Omega$, and they can export to sector $s$ of country $j$. Following Chaney (2008), country $j$ with output $Y_j$ has a two-stage budgeting procedure. First, country $j$ treats all the products as being homogenous within a sector and heterogeneous across sectors. Then it maximizes its country-level Cobb-Douglas utility. As a result of the maximization, country $j$'s expenditure in sector $s$ is $\delta_{js} Y_j$ and $\delta_{js}$ is a proportion parameter. On the second stage, $j$ maximizes its utility in sector $s$ which is

$$U_{js} = \left\{ \int_{l \in B_{js}} [x_{js}(l)]^\frac{\sigma-1}{\sigma} dl \right\}^{\frac{\sigma}{\sigma-1}} \quad \text{s.t.} \quad \delta_{js} Y_j = \int_{l \in B_{js}} p_{js}(l)x_{js}(l) dl. \quad (1)$$

Product varieties available in market $s$ of country $j$ are gathered into set $B_{js}$ and form a continuum. Therefore, the continuum is decomposable by these product varieties. The measure of the product varieties is $O_B$. For Product $l$, the price and the consumption are $p_{js}(l)$ and $x_{js}(l)$ respectively. Parameter $\sigma$ is the ES between product varieties in $B_{js}$ and the restriction on $\sigma$ is $\sigma > 1$. Positive $(\frac{\sigma-1}{\sigma})$ can accommodate zero consumption of product variety in the utility (Dixit and Stiglitz, 1977), and the ratio in unit interval ensures concavity of the utility function. The reason to take the CES form for the utility function is its broad implications on the consumer preferences in the market.\textsuperscript{5} When $\sigma$ descends from infinity to one, the utility form transforms from linearity to Cobb-Douglas.\textsuperscript{6} Accordingly, the market of sector $s$ in $j$ is altered

\textsuperscript{4}In this analysis, Anderson and van Wincoop (2003) assume that the exporting variable cost is invariant among all country pairs. The bilateral exports are the exported value of the bilateral trade which is adjusted by the output levels of both the exporter and the importer.

\textsuperscript{5}Recently, some research (e.g., Mrázová and Neary, 2014, and Zhelobodko, Kokovin, Parenti and Thisse, 2012) tries to avoid the CES utility and adopts a more general utility structure that is not restricted by the constant elasticity of substitution. Notwithstanding, this paper needs to check the $\sigma$’s effects on some of the endogenous variables and so will sticks on the CES utility.

\textsuperscript{6}L’Hospital’s rule is needed to prove that the CES utility with $\sigma \rightarrow 1$ is in Cobb-Douglas form.
from a non-differentiated market to a market fairly divided into monopolies. The monopolistic powers are granted by the consumers’ extreme love of variety.

After the maximization, the Marshallian demand for good \( l \) is

\[
x^*_j s(l) = \left[ \frac{p_{js}(l)}{P_j} \right]^{-\sigma} \delta_j s Y_j\] and \( P_j s = \left\{ \int_{l \in B_{js}} \left[ \frac{p_{js}(l)}{P_j} \right]^{1-\sigma} dl \right\}^{\frac{1}{1-\sigma}}.\] \( (2) \)

The price index of market \( s \) in \( j \) is \( P_{js} \), and it will rise if some product varieties in \( B_{js} \) increase in price and or if \( O_B \) shrinks. If we assume \( P_{js} \) is not affected by \( p_{js}(l) \), according to demand function (2) the own-price elasticity of demand for each product variety in the market is

\[
\frac{\partial \ln x^*_j s(l)}{\partial \ln p_{js}(l)} \bigg|_{p_{js}(l) = 0} = -\sigma.\] \( (3) \)

This result echoes the aforementioned role of \( \sigma \) in characterising the market.

A firm is assumed to produce only one distinct variety of product. Firm \( l \) is randomly drawn from sector \( s \) of country \( i \), and \( i \) is randomly picked up from in \( \Omega \). As \( \Omega \) includes all countries in the world, \( i = j \) is possible. The firm’s cost to export to \( j \) is

\[
C_{ijs}(l) = \frac{c_{is}}{\varphi_s} x_{js}(l) + c_{is} f_{ijs}.\] \( (4) \)

The melting-iceberg parameter for the variable cost is \( \tau_{ijs} \) and larger than one. \( f_{ijs} \) is the volume of the fixed costs. Hence, frictions of the export depend on \( \tau_{ijs} \) and \( f_{ijs} \). The unit price for both the variable and the fixed costs is \( c_{is} \), and its inverse represents the productivity level of country \( i \). \( \gamma \) The productivity among firms in sector \( s \) is \( \varphi_s \) and follows a Pareto distribution whose density and support are

\[
g(\varphi_s) = \gamma_s \frac{\varphi_s^{\gamma_s}}{\varphi_s^{\gamma_s+1}} \quad \text{and} \quad \varphi_s \in [\varphi_s L, +\infty).\] \( (5) \)

respectively. \( \gamma \) is an important but exogenous parameter. The distribution is invariant across countries. According to the cost function of (4), the productivity level of sector \( s \) in a country

\[\text{Precisely, } \tau_{ijs} \text{ is the variable-cost rate. But for simplicity this paper names it variable cost.} \]

\[\text{This is the aforementioned country-level productivity proposed by HMR (2008).} \]

\[\text{Arkolakis, Costinot and Rodriguez-Clare (2012) list numerous papers who adopt this productivity distribution.}\]
which is proportionally different to $\varphi_s$ is captured by the variable cost of the country.\textsuperscript{10} The cost to establish a firm is sunk and therefore is not involved in the firm’s profit maximization. This assumption allows the firm always to have non-negative profit and hence the firm will not quit. Moreover, for all the exporters new firm will not be founded in the period considered by the model. Consequently, the continuum of firms in sector $s$ of $i$ is exogenous and has measure $N_{is}$. Actually, this model is a short-run model in market $s$ of $j$.

Additionally, the output from sector $s$ in country $j$ is small that it has no impact on $Y_j$. Also, parameter restriction $\gamma_s > \sigma - 1$ needs to be imposed.\textsuperscript{11} Both linear cost function (4) and the sunk-cost assumption allow firm $l$’s profit maximization in exporting to country $j$ to be independent to the firm’s export to other importers. So, as illustrated in Figure 1 the export from $i$’s sector $s$ to $j$ is not affected by $i$’s export to market $s$ in another country like $j_0$ or $j_2$.

### 3.1 The Bilateral Equilibrium

First, we assume $P_{js}$ is exogenous that it is not impacted by a firm’s price decision or by the change of an exporter’s general price level. Under this assumption and in market $s$, the Marshallian demand function in (2) tells us that the $j$’s demand for one product is not affected by its demand for any other products. Then, $i$’s export to market $s$ in $j$ is independent of the export from other exporters to the market. Thus, shown in Figure 1 the bilateral equilibrium between exporter $i$ and the market in $j$ is represented by the thick solid-line arrow and can be identified regardless of the other bilateral trade relations indicated by the thin solid-line arrows.

\textsuperscript{10}This statement can be interpreted by the following example. For sector $s$ in $i$, the productivity is $2\varphi_s$ and the variable cost in exporting to $j$ is $\tau_{isj}^0$. Given the cost function of (4), we can let the productivity be $\varphi_s$ and, accordingly, let the variable cost be $\tau_{isj} = \frac{\varphi_s}{\gamma_s + 1}$.\textsuperscript{11}The restriction guarantees $\varphi_s^{\gamma_s - \gamma_s + 1} = 0$ for $\varphi_s \to \infty$ which will be used in the following derivation. Chaney (2008) and EKK (2011) also adopt similar restriction in their models.
Firm $l$ will apply the mark-up equation to set its price in exporting to $j$, and the price is

$$ \tilde{p}_{js}(l) = \frac{\sigma}{\sigma - 1} \frac{c_{is} r_{ijs}}{\varphi_s}. $$

(6)

Then, given the demand in (2), price (6) and cost (4), the revenue and profit of firm $l$ are respectively

$$ \tilde{r}_{js}(l) = \left[ \frac{\tilde{p}_{js}(l)}{P_{js}} \right]^{1/\sigma} \delta_{js} Y_j \quad \text{and} \quad \tilde{\pi}_{js}(l) = \frac{1}{\sigma} \tilde{r}_{js}(l) - c_{is} f_{ijs}. $$

(7)

The profit function is akin to the one of Melitz (2003). From the function, the ratio between the firm’s variable cost and its revenue can be derived as

$$ \tilde{\kappa}_{js}^{(\tau)}(l) = \frac{\sigma - 1}{\sigma}. $$

(8)

It is invariant for all firms in the market, for which the variable-cost ratio of each exporter in the market is also $\frac{\sigma - 1}{\sigma}$.

To earn positive profit from the export, the productivity of firm $l$ ought to be higher than the breakeven productivity,

$$ \tilde{\varphi}_{ijs} = \frac{\sigma}{\sigma - 1} \frac{c_{is} r_{ijs}}{P_{js}} \left( \frac{\sigma c_{is} f_{ijs}}{\delta_{js} Y_j} \right)^{\frac{1}{\sigma - 1}}, $$

(9)

which is solved from letting $\pi_{ijs}(l)$ be zero. In the export that is from $i$ to market $s$ of $j$, if firm $l_0$ is at the breakeven point the firm is a marginal firm. Firm $l_0$’s product price in the market can be calculated by bringing (9) back to (6), and with the price its revenue in the market can be derived by using the revenue function in (7). Then, they are

$$ \tilde{p}_{js}(l_0) = P_{js} \left( \frac{\sigma c_{is} f_{ijs}}{\delta_{js} Y_j} \right)^{-\frac{1}{\sigma - 1}} \quad \text{and} \quad \tilde{r}_{js}(l_0) = \sigma c_{is} f_{ijs}. $$

(10)

respectively. On the another extreme, the distribution of productivity $\varphi_s$ is unbounded from above. Additionally with the continuum of firms in sector $s$ of $i$, the exported value that is from $i$ to market $s$ in $j$ is positive. This statement can be generalized to $t$ for all $t \in \Omega$. In the real world, the firms in sector $s$ of $i$ do not form a continuum and the productivities of the firms are the realizations of $\varphi_{is}$ in the sector. Thus, the highest firm productivity in sector $s$ of $i$ is not infinitely large and may be smaller than $\tilde{\varphi}_{ijs}$. So, country $i$ may not export to market $s$ of
This fact explains why the unidirectional bilateral exports are zero for some country pairs in the real world. Nevertheless, the continuum assumption will simplify the analysis of the model in this paper and, on the other hand, it does not mis-specify the model dramatically.

In the unidirectional bilateral export, the exported value and the exporting scale are calculated respectively by

\[
\tilde{r}_{ijs} = \int_{\tilde{\varphi}_{ijs}}^{\infty} \tilde{v}_{ijs} \, d\varphi_s \quad \text{and} \quad \tilde{n}_{ijs} = \int_{\tilde{\varphi}_{ijs}}^{\infty} N_{is} g(\varphi_s) \, d\varphi_s, \quad \text{where} \quad \tilde{v}_{ijs} = N_{is} \tilde{r}_{js}(l) g(\varphi_s). \tag{11}
\]

As defined earlier, \(N_{is}\) is the measure of firms in sector \(s\) of \(i\). The exporting scale actually is the measure of exporting firms in the export. The result in (A1)\(^{12}\) gives\(^{13}\)

\[
\tilde{r}_{ijs} = N_{is} \delta_{js} Y_j M_{ijs} P_{js}^{\gamma_s} \quad \text{and} \quad \tilde{h}_{ijs} = \frac{\tilde{r}_{ijs}}{\delta_{js} Y_j}.
\]

Term \(M_{ijs}\) is defined in (A2). In the form of exported value \(\tilde{r}_{ijs}\), if we link \(N_{is}\) to exporter \(i\)’s output the form can be modified to a gravity equation similar to Chaney (2008). The share of the exported value in the market is \(\tilde{h}_{ijs}\). In this model, the share determines the exported value because the expenditure in market \(s\) of \(j\) is exogenous. Similarly, the average price of the exported products in the export is

\[
\tilde{p}_{ijs} = \int_{\tilde{\varphi}_{ijs}}^{\infty} \tilde{p}_{js}(l) \frac{g(\varphi_s)}{\tilde{\varphi}_{ijs}} \, d\varphi_s = \frac{\gamma_s P_{js}}{\gamma + 1} \left( \frac{\sigma c_{is} f_{ijs}}{\delta_{js} Y_j} \right)^{-\frac{1}{\gamma + 1}} = \frac{\gamma_s}{\gamma + 1} \tilde{p}_{js}(l_0). \tag{12}
\]

The calculation of (12) is in (A6). \(\tilde{p}_{js}(l_0)\) is the cut-off price in the export and defined in (10). Given (A1), (A5) and (A6), we can simply derive the price-index elasticities of these three variables and they are

\[
\frac{\partial \ln \tilde{r}_{ijs}}{\partial \ln P_{js}} = \frac{\partial \ln \tilde{n}_{ijs}}{\partial \ln P_{js}} = \gamma_s \quad \text{and} \quad \frac{\partial \ln \tilde{p}_{ijs}}{\partial \ln P_{js}} = 1. \tag{13}
\]

Therefore, in the export higher \(P_{js}\) can raise the mean price of the exporter and boost both the exporter’s exported value and its exporting scale.

Within the export that is from country \(i\) to market \(s\) in \(j\), we will discover, at the equilibrium

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\(^{12}\)The labels with A are for the expressions in Appendices.
\(^{13}\)The expression of \(\tilde{n}_{ijs}\) is given by (A5). We do not display the expression here, as little information is given by the expression.
how the behaviours of the firms vary with own productivity. Based on (6), the very productive firms set very low product prices close to zero. Among the firms, as productivity decline, product price rises due to (6) and both revenue of firm and profit of firm fall due to (7). Larger variable cost $\tau_{ijs}$ can accelerate the rise of price. At the end, the highest product price is at the breakeven point where the firms are defined as marginal firms and earn both least revenue and no profit. Therefore, given that a firm is either on exporting state or on non-exporting state, only the marginal firms will change their states of export after an external shock onto the equilibrium. We conclude that, in a market at the equilibrium level, if an exporter has larger fixed cost both its highest product price and its average product price are comparatively lower regardless of the exporter’s variable cost. This conclusion is drawn from (10) and (12).

3.2 The Multilateral Equilibrium

In the previous discussion, market price index $P_{js}$ is exogenous. It is reasonable to assume that measure $\#B_{js}$ is very large and that firm $l$’s decision on its own price has no impact on $P_{js}$, such that mark-up price (6) remains unchanged. Notwithstanding, according to the form of $P_{js}$ in (2) exporter $i$’s contribution to price index $P_{js}$ is

$$p_{ijs}^{1-\sigma} = N_{is} \int_{\tilde{\phi}_{ijs}}^{\infty} \tilde{p}_{ijs}(l)[\tilde{p}_{ijs}(l)]^{1-\sigma} g(\tilde{\varphi}_s)d\tilde{\varphi}_s,$$

and it is not reasonable to neglect the $p_{ijs}^{1-\sigma}$’s impact on $P_{js}$ especially when a large proportion of the firms exporting to market $s$ of $j$ are from exporter $i$. Thus, $P_{js}$ is endogenous at country level. To solve the optimal value of the endogenous $P_{js}$ we derive, for all countries in the world, the contributions on $P_{js}$ and then bring them into the price-index expression in (2). (A13) gives the solved price index and it is

$$\bar{P}_{js} = \left[ \sum_{l \in \Omega} (N_{is}M_{ij}) \right]^{-\frac{1}{\gamma_s}},$$

where $M_{ij}$ is based on (A2). The form of $\bar{P}_{js}$ is close to the one in Chaney (2008). Further, $\bar{P}_{js}$ will reversely influence the export from each exporter to market $s$ in $j$. Consequently, if $P_{js}$
is endogenous all the bilateral exporting relations directed toward this market are involved in
determining the new equilibrium that therefore is multilateral. These relations are represented
by all the solid-line arrows in Figure 1, they together form the trade system.

At the multilateral equilibrium level, the revenue of exporter $i$ can be derived by using $\tilde{P}_{js}$
to substitute for $P_{js}$ in $\tilde{r}_{ijs}$ and it is

$$\tilde{r}_{ijs} = \tilde{h}_{ijs} \delta_j Y_j,$$

where $\tilde{h}_{ijs} = \frac{\sum_{t \in \Omega} (N_{ts} M_{ijs})}{\sum_{t \in \Omega} \left[ N_{ts} (c_{ts} \tau_{tjs}) - \gamma_s (c_{ts} f_{tjs})^{1 - \frac{\gamma_s}{\sigma_s - 1}} \right]}.$$

These results are from (A14). By ACR (2012), the welfare of sector $s$ in $j$ is defined as

$$\bar{w}_{js} = \frac{\delta_j Y_j}{\tilde{P}_{js}}.$$

Melitz and Redding (2014) also apply this type of welfare definition. Then we use the first
expression of $\tilde{h}_{ijs}$ in (16) to derive the elasticity of welfare $\bar{w}_{js}$ with respect to market share $\tilde{h}_{ijs}$
and find

$$\frac{\partial \ln \bar{w}_{js}}{\partial \ln \tilde{h}_{ijs}} \bigg|_{M_{ijs}=\text{constant}} = \frac{-1}{\gamma_s}$$

that is given by (A17). This elasticity is consistent with the corresponding one in the paper of
ACR (2012), except for the condition of constant $M_{ijs}$. To make the elasticity form valid, this
precondition requires that the change of $\tilde{h}_{ijs}$ originates only from change of the trade frictions
of other exporters excluding $i$. ACR (2012) prove the elasticity form according to Melitz’s (2003)
model. $\tilde{h}_{ijs}$ is an endogenous variable in the multilateral equilibrium. Thus, in the equilibrium
its change will cause change of the market shares of some other exporters which will also affect
the welfare. So the final effect of $\tilde{h}_{ijs}$ on welfare $\bar{w}_{js}$ may not be $-\frac{1}{\gamma_s}$. Instead, this paper
will investigate how, in the equilibrium, the exogenous variables affect the welfare through their
effects on endogenous price index $\tilde{P}_{js}$. The second expression of $\tilde{h}_{ijs}$ in (16) is similar to the
market share forms respectively in Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008)
and in Chaney (2008).

Also, at the multilateral equilibrium level exporter $i$’s average product price, its exporting

\footnote{If possible, controlling all the variables in the second expression in (16) for statistically estimating $\tilde{r}_{ijs}$ will no longer suffer from the problem of omitted variable.}
scale and its breakeven productivity are \( \bar{p}_{ijs} \), \( \bar{n}_{ijs} \) and \( \bar{\varphi}_{ijs} \) respectively. They can be calculated by bringing \( \bar{P}_{js} \) back to \( \bar{p}_{ijs} \), \( \bar{n}_{ijs} \) and \( \bar{\varphi}_{ijs} \). From (A4) and (A19), we have

\[
\bar{n}_{ijs} = \bar{\varphi}_{ijs} \bar{n}_{is} \frac{\gamma_s}{\sigma} \frac{\bar{h}_{ijs} \delta_{js} Y_j}{c_{is} f_{ijs}}. \tag{18}
\]

The first equation gives the relationship between exporting scale \( \bar{n}_{ijs} \) and breakeven productivity \( \bar{\varphi}_{ijs} \), and indicates that, in the equilibrium, more firms in sector \( s \) of \( i \) are swept out from the export if the breakeven productivity for the sector in \( i \) is higher. This relationship holds even in the bilateral equilibrium. From the second expression of \( \bar{n}_{ijs} \) in (18), we find two properties of the multilateral equilibrium which are also in EKK (2011). First, conditional on \( c_{ts} f_{tjs} = c_{is} f_{ijs} \) for all \( t \in \Omega \), market share \( \bar{h}_{ijs} \) equals \( \frac{\bar{n}_{ijs}}{\sum_{t \in \Omega} \bar{n}_{tjs}} \), that is the \( i \)'s share in the total exporting firms of the market. Second, the ratio of \( i \)'s total fixed costs to its exported value can be re-arranged to

\[
\bar{\kappa}_{js}^{(f)} = \frac{\bar{n}_{ijs} c_{is} f_{ijs}}{\bar{h}_{ijs} \delta_{js} Y_j} = \frac{1}{\gamma_s} \left( \frac{\gamma_s + 1}{\sigma} - 1 \right). \tag{19}
\]

Accordingly,

\[
\bar{\kappa}_{js}^{(f)} = \frac{\sigma - 1}{\sigma} \quad \text{and} \quad \bar{\kappa}_{js}^{(\pi)} = \frac{1}{\gamma_s} \left( \frac{\sigma - 1}{\sigma} \right) \tag{20}
\]

are the variable-cost ratio and the profit ratio respectively. \( \bar{\kappa}_{js}^{(f)} \) is from (8) and \( \bar{\kappa}_{js}^{(\pi)} \) is equal to \( 1 - \bar{\kappa}_{js}^{(\tau)} - \bar{\kappa}_{js}^{(f)} \). More generally, these three ratios are also for the total exported value to the market, as no exporter specified variable appears in the expressions of the ratios. Consequently, either for the whole market or for any of the exporters to the market, the exported value at the equilibrium level is divided into variable cost, fixed cost and profit. The division depends only on the market’s structure characterised by the ES of the market.

According to (18), \( \bar{n}_{kjs} \) can be symmetrically written out for both \( k \neq j \) and \( k \in \Omega \). Then, at the multilateral equilibrium level we summarize all the exporting scales of exporter in market \( s \) of \( j \) and then get the measure of all the firms participating in the export to the market which is

\[
\bar{n}_{js} = \bar{\kappa}_{js}^{(f)} \delta_{js} Y_j, \quad \text{where} \quad \bar{F}_{js} = \left[ \mathbb{E}_t \left( \frac{1}{c_{ts} f_{tjs}} \right) \right]^{-1} \quad \text{and} \quad \mathbb{E}_t(\xi_t) = \sum_{t \in \Omega} (\bar{h}_{tjs} \xi_t). \tag{21}
\]

They are from (A20). In the equilibrium, as \( \bar{\kappa}_{js}^{(f)} \delta_{js} Y_j \) is the market’s expenditure on fixed cost,
\( \tilde{F}_{js} \) is defined as the fixed-cost index of the market. To better understand the fixed-cost index, we introduce a random variable of fixed cost which is \( c_s f_{js} \). In the market sample, \( c_{ts} f_{tjs} \) for \( t \in \Omega \) is an observation of the random fixed-cost and probability \( \mathbb{P}(c_s f_{js} = c_{ts} f_{tjs}) \) is equal to \( \tilde{h}_{tjs} \). Function \( \mathbb{E}_t(\frac{1}{c_{ts} f_{tjs}}) \) actually gives the mean of the inversed random fixed-cost in the market sample. Moreover, the inverse of the mean is the fixed-cost index in the market. Based on the expression of \( \tilde{n}_{js} \) in (21), we propose that, at the equilibrium level, the total number of firms exporting to a market relies on the fixed-cost index of the market if the properties of the market are given.

3.3 The Intensive and the Extensive Margins

For the export that is from sector \( s \) of \( i \) to \( j \), three different popular approaches have been applied to define the intensive and the extensive margins. They will be introduced in the following. If the product varieties included in the export is \( B_{ijs} \in B_{js} \), calculating the two margins defined by Hummels and Klenow (2005) needs the exports of \( B_{ijs} \) which are from all other exporters to \( j \). Unexpectedly, that cannot be achieved because the model of this paper adopts the assumption of heterogeneous firms. The second approach is adopted by EKK (2004). They treat, in the export, the exported value as the product of the exporting scale and the average exported value per firm which are the extensive and the intensive margins respectively.\(^{18}\)

The former is \( \tilde{n}_{ijs} \), and the latter is derived from the second expression of (18) and takes the form of

\[
q_{ijs} = \frac{\tilde{h}_{ijs} \delta_{js} Y_j}{\tilde{n}_{ijs}} = \frac{\sigma \gamma_s c_{is} f_{ijs}}{\gamma_s - (\sigma - 1)}
\]

that is not affected by variable cost \( \tau_{ijs} \). It is also not a function of \( P_{js} \) because presented in (13) the price-index elasticities of \( \tilde{r}_{ijs} \) and of \( \tilde{n}_{ijs} \) are equal.\(^{19}\) So, the form of the average exported value per firm is unchanged even if the \( P_{js} \) is exogenous. This form is identical to that in Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008).

Now, we will introduce the third approach. At the multilateral equilibrium level, the exported value from \( i \) to \( j \) is \( \tilde{r}_{ijs} \) and can be calculated in a different way. By referring to (11)

---

\(^{18}\)EKK (2004) introduce the two margins, but they do not assign a specific term to each of the margins.

\(^{19}\)\( \partial \left( \frac{f_{ijs}}{\tilde{n}_{ijs}} \right) / \partial P_{js} = \left( \frac{\partial \ln f_{ijs}}{\partial \ln P_{js}} \right) - \left( \frac{\partial \ln \tilde{n}_{ijs}}{\partial \ln P_{js}} \right) = 0 \). So \( P_{js} \) has no effect on the average exported value per firm. Thus, \( \frac{f_{ijs}}{\tilde{n}_{ijs}} = \frac{\tilde{r}_{ijs}}{\tilde{n}_{ijs}} \) is true.
that shows the calculation of the exported value at the bilateral equilibrium level, \( \bar{r}_{ijs} \) is

\[
\bar{r}_{ijs} = \int_{\bar{\varphi}_{ijs}}^{\infty} \bar{v}_{ijs} \, d\varphi \Rightarrow d\bar{r}_{ijs} = \left( \int_{\bar{\varphi}_{ijs}}^{\infty} \frac{\partial \bar{v}_{ijs}}{\partial o_{js}} \, d\varphi \right) \, do_{js} - \left( \bar{v}_{ijs} \bigg|_{\varphi_s = \bar{\varphi}_{ijs}} \frac{\partial \bar{\varphi}_{ijs}}{\partial o_{js}} \right) \, do_{js}.
\]

(23)

Evaluating \( \bar{v}_{ijs} \) at \( P_{ijs} = \bar{P}_{ijs} \) gives \( \bar{v}_{ijs} \). The Leibniz integral rule is applied to decompose the differential of \( \bar{r}_{ijs} \) in (23), where \( o_{js} \) is an exogenous variable belonging to \( \{\sigma, \tau_{tjs}, f_{tjs}\} \) for \( t \in \Omega \). According to Chaney (2008), on the right hand side of the decomposition equation the first and the second parts are the intensive and the extensive margins respectively. Both of the margins are shown in Figure 2. In the equilibrium, the movement of breakeven productivity \( \bar{\varphi}_{ijs} \) will let some of the firms change their states of export in the export and will further generate the EM area of the extensive margin in the figure. Analogously, shifting integrand \( \bar{v}_{ijs} \) changes the exported value of the non state-altering firms in the export and then create the IN area of the intensive margin.

In (23) \( \bar{r}_{ijs} \) is separated into \( \bar{r}^{(in)}_{ijs} \) and \( \bar{r}^{(ex)}_{ijs} \). For the bilateral export in the equilibrium, the first part can be regarded as the exported value stuck on the non state-altering firms and the second part can be thought of the exported value brought by the state-altering firms. However, the two parts cannot be expressed exactly because we will never know, in the export, which firms will change their states of export before an effect. The extensive marginal effect of \( o_{js} \) on
\( \bar{r}_{ij} \) can be re-arranged to (A24) that is
\[
\frac{\partial \bar{r}_{ij}^{(ex)}}{\partial o_{js}} = \sigma c_{is} f_{ijs} \frac{\partial \bar{n}_{ij}}{\partial o_{js}}.
\] (24)

Implied by this equation, raising the change of \( \bar{n}_{ij} \) enlarges, in the same direction, extensive margin \( d\bar{r}_{ij}^{(ex)} \) by the rate of \( \sigma c_{is} f_{ijs} \). This rate is a marginal firm’s exported value in the bilateral export at the equilibrium level and is presented in (10). Therefore, this implication verify the previous statement that only the marginal firms will change their states of export after \( o_{js} \)’s effect on the equilibrium. Equation (24) actually reveals the relationship between the two extensive margins that are \( \bar{n}_{ij} \) and \( \frac{\partial \bar{r}_{ij}^{(ex)}}{\partial o_{js}} \) and are respectively applied by EKK (2004) and by Chaney (2008). In the rest of the paper, the intensive and the extensive margins are Chaney’s (2008).

In the next couple of sections, we will explore how, in the multilateral equilibrium, \( o_{js} \) affects an endogenous variable. In the equilibrium, the elasticity of the endogenous variable with respect to \( o_{js} \) can be divided into a direct and an indirect sub-elasticities. The direct sub-elasticity represents the effect of \( o_{js} \) on the endogenous variable, given that \( P_{js} \) is assumed to be exogenous and thus not to be impacted by \( o_{js} \). In comparison, the indirect sub-elasticity reveals how \( o_{js} \) affects the endogenous variable only through impacting endogenous price index \( \bar{P}_{js} \). The direct sub-elasticity can be calculated by setting \( \frac{\partial \bar{P}_{js}}{\partial o_{js}} = 0 \) and then deriving the \( o_{js} \) elasticity of the endogenous variable. From the original \( o_{js} \) elasticity of the endogenous variable, excluding the direct sub-elasticity gives the indirect sub-elasticity.

4 Trade Frictions

The trade fictions on the export that is from \( i \) to market \( s \) of \( j \) include both variable cost \( \tau_{ijs} \) and fixed cost \( f_{ijs} \). In this section, we will find out how the two costs impact the trade system described in Figure 1. Each the impact is through a different mechanism, so they will be discussed separately.

4.1 Effects of Variable Cost \( \tau_{ijs} \)

We first assume market price index \( P_{js} \) is exogenous. So the export from \( i \) to \( j \)’s market \( s \) is in a bilateral equilibrium and is irrelevant to the export from other exporters to the market. Thus,
the impact of variable cost $\tau_{ijs}$ is restricted in the bilateral export from $i$ and is indicated by step $\textcircled{1}$ in Figure 3. According to the mark-up price in (6), when $\tau_{ijs}$ rises every exporting firm in the unidirectional bilateral export will increase product price to keep its profit maximized. Given (7), higher product price of such a firm will make the firm earn, from the export, less profit and less revenue. As reported in Section 3.1, in the export the marginal firms set the highest product price and earn no profit. Therefore, the marginal firms are no longer profitable in the export, after the growth of their prices, and then they will quit from the market. Every other exporting firm will reduce its exported value to the market, also due to the growth of its own price. Generally, conditional on (A1), (A5) and (A6) the impact of $\tau_{ijs}$ on the bilateral equilibrium can be expressed as

$$
\frac{\partial \ln \tilde{r}_{ijs}}{\partial \ln \tau_{ijs}} = \frac{\partial \ln \tilde{n}_{ijs}}{\partial \ln \tau_{ijs}} = -\gamma_s < 0 \quad \text{and} \quad \frac{\partial \ln \tilde{p}_{ijs}}{\partial \ln \tau_{ijs}} = 0.
$$

The first two effects can be understood directly from the discussion. With the rise of $\tau_{ijs}$, in the export the firms with the highest product price quit and the other exporting firms raise their product prices. Given that the two opposite effects cancel each other out, average price $\tilde{p}_{ijs}$ is not affected by $\tau_{ijs}$. Actually, each elasticity in (25) will be the direct $\tau_{ijs}$ sub-elasticity of the corresponding endogenous variable at the multilateral equilibrium level.

The exported value that are from other exporters, excluding $i$, to market $s$ of $j$ are not impacted by $\tau_{ijs}$ if market price index $P_{js}$ is exogenous. Given the first expression in (25), $i$'s exported value to market $s$ in $j$ is reduced by raising $\tau_{ijs}$. Hence, after rise of $\tau_{ijs}$ the total
exported value from all the exporters to the market declines and therefore is less than total expenditure $\delta_{j}Y_{j}$ in the market. As a result, the market in the bilateral equilibrium is not cleared after the rise of $\tau_{ij}$. So the assumption of exogenous $P_{js}$ ought to be relaxed. For exporter $i$ affected by the rise of $\tau_{ij}$, both the drop in the exporting scale and the rise in the product prices should impact $P_{js}$ because of the form of $P_{js}$ in (2). These impacts are in step 2 of Figure 3. When all the exporters, including exporter $i$, are aware of the shift of $P_{js}$, they will adjust their behaviours in exporting to the market. This is why Anderson and van Wincoop (2003) define $P_{js}$ as the multilateral resistance variable. These adjustments are reflected by the effects of step 3 in Figure 3, and will further reversely affect $P_{js}$. Finally, the recursive reactions between all the exporters in the market and $P_{js}$ will continue until the multilateral Nash equilibrium reaches a new state. These reactions are represented by the arrows of step 4 in Figure 3.

In the multilateral equilibrium, the elasticity of the price index with respect to variable cost $\tau_{ij}$ is exactly the market share of exporter $i$ and is given by (A28).

$$\frac{\partial \ln \bar{P}_{js}}{\partial \ln \tau_{ij}} = \bar{h}_{ij} > 0.$$  \hspace{1cm} (26)

The effect is positive and the size of the effect relies on $\bar{h}_{ij}$. $\bar{P}_{js}$ is unit elastic with respect to $\tau_{ij}$ when country $i$ is the only exporter in market $s$ of $j$, and is not impacted by $\tau_{ij}$ when $i$’s share in the market approaches zero. According to (26), the $\tau_{ij}$ elasticity of an endogenous variable in the equilibrium equals the direct sub-elasticity of the elasticity, conditional on $\bar{h}_{ij} = 0$.

In the multilateral equilibrium and for exporter $i$, the own variable-cost elasticities of the exported value, of the exporting scale and of the average product price are in (A33), (A35) and (A29) respectively. They are display as

$$\frac{\partial \ln \bar{r}_{ij}}{\partial \ln \tau_{ij}} = \frac{\partial \ln \bar{n}_{ij}}{\partial \ln \tau_{ij}} = -\gamma_{s}(1 - \bar{h}_{ij}) < 0 \quad \text{and} \quad \frac{\partial \ln \bar{p}_{ij}}{\partial \ln \tau_{ij}} = \bar{h}_{ij} > 0.$$  \hspace{1cm} (27)

Each of the first two elasticities consists of two parts that are $-\gamma_{s}$ and $\gamma_{s}\bar{h}_{ij}$. The first part is the direct sub-elasticity that equals the corresponding elasticity in (25), and the second part is the indirect sub-elasticity. The two sub-elasticities are opposite in sign, and the direct sub-

\[\text{footnote}{Unlike Chaney (2008), we are not able to investigate the }\sigma\text{’s effects on the elasticities because }\sigma\text{ impacts }\bar{h}_{ij}\text{ in a very complicated way. The impact will be explored in the next section.}\]
elasticity dominates the sign of the elasticity. Therefore, in each of the first two elasticities from (27), market share $\bar{h}_{ijs}$ plays a attenuation role. Generally, in the equilibrium an exporter’s exported value and its exporting scale will decline with the rise of its own variable cost. When $P_{js}$ is exogenous, $\tau_{ijs}$ does not affect average product price $\bar{p}_{ijs}$ because of $\frac{\partial \ln \bar{p}_{ijs}}{\partial \ln \tau_{ijs}} = 0$ in (25). So, in (27) the $\tau_{ijs}$ elasticity of $\bar{p}_{ijs}$ only includes a positive indirect sub-elasticity.

Following Chaney (2008), after the decomposition of $\frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \tau_{ijs}}$, the intensive and the extensive elasticities of $\bar{r}_{ijs}$ with respect to $\tau_{ijs}$ are

$$\frac{\partial \bar{r}_{ijs}^{(in)}}{\partial \tau_{ijs} \bar{r}_{ijs}} = -(\sigma - 1) (1 - \bar{h}_{ijs}) < 0 \quad \text{and} \quad \frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial \tau_{ijs} \bar{r}_{ijs}} = -[\gamma_s - (\sigma - 1)] (1 - \bar{h}_{ijs}) < 0 \quad (28)$$

respectively. They are shown in both (A37) and (A36). Again, any one of the two elasticities includes a positive indirect sub-elasticity with factor $\bar{h}_{ijs}$ and also includes a negative direct sub-elasticity not related to that factor. Market share $h_{ijs}$ also has attenuation effect in both of the two elasticities that are negative. It follows that, for an exporter in the multilateral equilibrium, both the exported value of the non state-altering firms and the exported value brought by the state-altering firms drop when the variable cost of the exporter rises.

Then, we check the cross effects of $\tau_{ijs}$. Given $k \in \Omega$ and $k \neq i$, in the multilateral equilibrium the $\tau_{ijs}$’s effect on the export of $k$ can be expressed as

$$\frac{\partial \ln \bar{r}_{kjs}}{\partial \ln \tau_{ijs}} = \frac{\partial \ln \bar{n}_{kjs}}{\partial \ln \tau_{ijs}} = \gamma_s \bar{h}_{ijs} > 0 \quad \text{and} \quad \frac{\partial \ln \bar{p}_{kjs}}{\partial \ln \tau_{ijs}} = \bar{h}_{ijs} > 0 \quad (29)$$

which are from (A40), (A44) and (A39) respectively. As $\tau_{ijs}$ has no effect on $k$’s export to $j$’s market $s$ when market price index $P_{js}$ is exogenous, the elasticities in (29) do not include direct sub-elasticity. Therefore, each of the elasticities in (29) equals the indirect sub-elasticity in the relevant elasticity in (27). The first two elasticities in (29) show that, in the equilibrium, cross variable-cost elasticities of an exporter’s exported value and of its exporting scale are positive and only depend on the cross variable-cost originated market share. The last equations in (27) and in (29) jointly give, in the equilibrium, that the elasticity of an exporter’s average product price with respect to an variable cost is exactly the variable-cost originated market share.

The intensive and the extensive $\tau_{ijs}$ elasticities of revenue $\bar{r}_{kjs}$ are which are given by (A46)
and (A45) respectively.

\[
\frac{\partial r^{(in)}_{kjs}}{\partial \tau_{ijs}} \tau_{kjs} = (\sigma - 1) \bar{h}_{ijs} > 0 \quad \text{and} \quad \frac{\partial r^{(ex)}_{kjs}}{\partial \tau_{ijs}} \tau_{kjs} = [\gamma_s - (\sigma - 1)] \bar{h}_{ijs} > 0.
\]

(30)

They indicate that, in the multilateral equilibrium, the intensive and the extensive margins of an exporter which are generated by rise of a cross variable cost are positive. The two elasticities in (30) also only include indirect sub-elasticity, and so they are with slope of \( \bar{h}_{ijs} \) and do not bear any variable specific to \( k \). Therefore, they are respectively equal to the indirect sub-elasticities in (28). From both (29) and (30) and by comparing them to both (27) and (28), we find that, in the equilibrium, the indirect \( \tau_{ijs} \) sub-elasticity of an exporter-specific endogenous variable is invariant across all the exporters because the indirect sub-elasticity works by \( \tau_{ijs} \)'s effect on \( \bar{P}_{js} \) and also because \( \bar{P}_{js} \) enters the exporter-specific variable symmetrically over all the exporters. The elasticities in both (29) and (30) exhibit the multilateral resistance on \( k \)'s export in the equilibrium, in reaction to the fall of \( \tau_{ijs} \). These resistance elasticities with respect to \( \tau_{ijs} \) decrease as \( \bar{h}_{ijs} \) shrinks, and vanish with \( \bar{h}_{ijs} \rightarrow 0 \).

Finally, across all the exporters in the multilateral equilibrium we accumulate the \( \tau_{ijs} \)'s effects on exporter’s exporting scale and also accumulate the its effects on exporter's exported value. So, in the market of the equilibrium, both the \( \tau_{ijs} \) elasticity of the measure of all the exporting firms and the effect of \( \tau_{ijs} \) on the total exported value are

\[
\frac{\partial \ln \bar{n}_{ijs}}{\partial \ln \tau_{ijs}} = \gamma_s \bar{h}_{ijs} \left( 1 - \frac{\bar{F}_{js}}{c_{is} f_{ijs}} \right) \quad \text{and} \quad \sum_{t \in \Omega} \frac{\partial r_{tjs}}{\partial \tau_{ijs}} = 0
\]

(31)

respectively. They are proved in (A47) and (A48). Fixed-cost index \( \bar{F}_{js} \) are defined in (21). It can be inferred from the first equation in (31) that, in the equilibrium, a variable cost will has positive impact on the total number of exporting firms in the market if and only if the variable-cost originated fixed cost is larger than the fixed-cost index of the market. The mathematical expression of the condition is \( c_{is} f_{ijs} > \bar{F}_{js} \). For the second expression in (31), in the multilateral equilibrium the total exported value in the market is not impacted by the variable cost of any exporter, and therefore, after change of the variable cost, is still equal to the total expenditure from the market. Thus, with the endogenous market price index in the trade system of Figure 1, the market clearing condition after the impact of a variable cost is satisfied.
4.2 Effects of Fixed Cost $f_{ijs}$

Analogously to variable cost $\tau_{ijs}$, we will analyze the impact of fixed cost $f_{ijs}$ on the trade system illustrated in Figure 1. First, we also assume exogenous market price index $P_{js}$. Change of $f_{ijs}$ will only impact the export that is from $i$ to market $s$ of $j$, which is indicated by step $\mathbb{1}$ in Figure 3. With the rise of $f_{ijs}$, in the export the firms from $i$ will not change their prices given the mark-up price function in (6) and also will have their revenues unaffected as the revenue function in (7). However, according to the profit function in (7), larger $f_{ijs}$ will reduce the profit of each firm in the export. Further, the marginal firms in the export will be crowded out because their profits are zero before the profit reduction. So, when $f_{ijs}$ rises, in the export the exported value of exporter $i$ declines and the average product price of the exporter drops. The reason for the drop in the average price is that the marginal firms withdrawn from the market were setting the highest price in the unidirectional bilateral export. In general, all these variations are reflected in the following elasticities:\footnote{Here, Chaney (2008) also finds the expression for $\frac{\partial \ln \tilde{r}_{ijs}}{\partial \ln f_{ijs}}$.}

$$
\frac{\partial \ln \tilde{r}_{ijs}}{\partial \ln f_{ijs}} = -\frac{\gamma_{s} - \sigma + 1}{\sigma - 1} < 0, \quad \frac{\partial \ln \tilde{n}_{ijs}}{\partial \ln f_{ijs}} = -\frac{\gamma_{s}}{\sigma - 1} < 0 \quad \text{and} \quad \frac{\partial \ln \tilde{p}_{ijs}}{\partial \ln f_{ijs}} = -\frac{1}{\sigma - 1} < 0,
$$

which are directly derived from (A1), (A5) and (A6). As $P_{js}$ is held constant, the elasticities in (32) will be the direct sub-elasticities that are respectively in the $f_{ijs}$ elasticities of $\tilde{r}_{ijs}$, of $\tilde{n}_{ijs}$ and of $\tilde{p}_{ijs}$. In comparison between the last expressions in (25) and in (32), fixed cost $f_{ijs}$ has negative effect on average price $\tilde{p}_{ijs}$ but variable cost $\tau_{ijs}$ has no effect on it.

In the trade system depicted in Figure 1 and with exogenous $P_{js}$, the market clearing condition is not satisfied in reaction to change of exporter’s fixed cost because that change will affect its own exports, for the first elasticity in (32), and will have no influence on the exports of other exporters. When the assumption of exogenous $P_{js}$ is relaxed, the effects of fixed cost $f_{ijs}$ on the export that is from $i$ to market $s$ in $j$ will further go over from step $\mathbb{2}$ to $\mathbb{4}$ in Figure 3. Finally, a new state of the multilateral Nash equilibrium is established which includes both market $s$ in $j$ and all the exporters to the market. In the equilibrium, $f_{ijs}$ impacts price index $\bar{P}_{js}$ in the way of

$$
\frac{\partial \ln \bar{P}_{js}}{\partial \ln f_{ijs}} = \left( \frac{1}{\sigma - 1} - \frac{1}{\gamma_{s}} \right) \tilde{h}_{ijs} > 0
$$

given by (A50). The effect is positive and its size depends on $\tilde{h}_{ijs}$. Together with (26), we
propose, in the multilateral equilibrium, that a trade cost has positive impact on the market price index and further has negative effect on the sector’s welfare in the importer and that the sizes of the two elasticities related to the two effects depend on the market share of the exporter bearing the trade cost. In the \( f_{ijs} \) elasticity of an endogenous variable in the equilibrium, setting \( \bar{h}_{ijs} = 0 \) is equivalent to holding \( P_{js} \) exogenous and will degenerate the \( f_{ijs} \) elasticity into the direct sub-elasticity of the \( f_{ijs} \) elasticity.

To view \( f_{ijs} \)'s effect on \( i \)'s export in the multilateral equilibrium, the \( f_{ijs} \) elasticities of \( \bar{r}_{ijs} \), \( \bar{n}_{ijs} \) and of \( \bar{p}_{ijs} \) are

\[
\frac{\partial \ln \bar{r}_{ijs}}{\partial \ln f_{ijs}} = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \left( 1 - \bar{h}_{ijs} \right) < 0, \quad \frac{\partial \ln \bar{n}_{ijs}}{\partial \ln f_{ijs}} = -\frac{\gamma_s}{\sigma - 1} \left( 1 - \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ijs} \right) < 0
\]

and

\[
\frac{\partial \ln \bar{p}_{ijs}}{\partial \ln f_{ijs}} = -\frac{1}{\sigma - 1} \left( 1 - \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ijs} \right) < 0
\]

respectively and are from (A54), (A56) and (A51). With the findings in (27), it is worth to notice that average product price \( \bar{p}_{ijs} \) is reduced by increasing fixed cost \( f_{ijs} \) but is raised by increasing variable cost \( \tau_{ijs} \). In each the elasticity from (34), the indirect sub-elasticity is with factor \( \bar{h}_{ijs} \) and the rest is the direct sub-elasticity that equals the corresponding elasticity in (32). Moreover, the direct sub-elasticity is negative and overcomes the indirect positive sub-elasticity in scale. Thus, \( \bar{h}_{ijs} \) plays attenuation role in the elasticities from (34) and these elasticities are negative. Consequently, in the equilibrium, larger own fixed cost can decrease exporter’s exported value, exporter’s exporting scale and exporter’s average product price but can bring up exporter’s average exported value per firm. The second result is proved by \( \frac{\partial \ln \bar{q}_{ijs}}{\partial \ln f_{ijs}} = 1 \) from (22).

To view more closely the \( f_{ijs} \)'s effect on exported value \( \bar{r}_{ijs} \), we decompose elasticity \( \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln f_{ijs}} \) into the intensive and the extensive elasticities that are derived into (A58) and (A57).

\[
\frac{\partial \bar{r}_{ijs}^{(in)}}{\partial f_{ijs}} \bar{r}_{ijs} = \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ijs} > 0, \quad \frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial f_{ijs}} \bar{r}_{ijs} = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \left( 1 - \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ijs} \right) < 0.
\]

The extensive elasticity is negative and can also be divided into both the indirect extensive and the direct extensive sub-elasticities. Thus, \( \bar{h}_{ijs} \) plays a role of attenuation in the extensive elas-

\[23\]The welfare of sector \( s \) in \( j \) is \( \bar{w}_{js} \) and equal to \( \frac{\delta_{js} Y_j}{P_{js}} \) defined in (17). We can prove \( \frac{\partial \ln \bar{w}_{js}}{\partial \ln \bar{P}_{js}} = -\frac{\partial \ln P_{js}}{\partial \ln \bar{a}_{js}}. \) Thus, the elasticity of \( \bar{w}_{js} \) is a function only of the elasticity of \( \bar{P}_{js} \).
ticity. In comparison, the intensive elasticity only includes the indirect intensive sub-elasticity, and so the direct intensive sub-elasticity is zero. This fact implies, in the trade system of Figure 1, if $P_{js}$ is exogenous the non state-altering firms in country $i$ will not change their exported value in response to the change of $f_{ij}$. This implication echoes the discussion early in this sub-section and also echoes the corresponding result from Chaney (2008). The information brought by (35) is that, to an exporter in the equilibrium, growth of own fixed cost raises the exported value of the non state-altering firms but reduces the exported value provided by the state-altering firms. Retrospectively looking at (27), (28), (34) and (35), we find that, in the equilibrium, if the direct sub-elasticity in a trade cost elasticity of an endogenous variable is not zero its sign will dominate the sign of the trade cost elasticity.

As a cross fixed cost, the $f_{ij}$'s effects on other exporters' exports in the multilateral equilibrium can also be evaluated. Given $k \neq i$ and $k \in \Omega$, we get

$$\frac{\partial \ln r_{kjs}}{\partial \ln f_{ij}} = \frac{\gamma_s - \sigma + 1}{\bar{h}_{ij}} > 0 \quad \text{and} \quad \frac{\partial \ln p_{kjs}}{\partial \ln f_{ij}} = \frac{\gamma_s - \sigma + 1}{\gamma_s (\sigma - 1)} \bar{h}_{ij} > 0. \quad (36)$$

They are from (A61) (A63) and (A59). Given the results in (36), in the equilibrium, the $f_{ij}$'s effect on export of $k$ is only through $P_{js}$, the elasticities from (36) include only indirect sub-elasticity and increase with market share $\bar{h}_{ij}$. These properties are also for both the intensive and the extensive $f_{ij}$ elasticities of $\bar{r}_{kjs}$ which are

$$\frac{\partial r_{kjs}^{(in)}}{\partial f_{ij}} \frac{f_{ij}}{\bar{r}_{kjs}} = \frac{\gamma_s - \sigma + 1}{\bar{h}_{ij}} > 0 \quad \text{and} \quad \frac{\partial r_{kjs}^{(ex)}}{\partial f_{ij}} \frac{f_{ij}}{\bar{r}_{kjs}} = \frac{(\gamma_s - \sigma + 1)^2}{\gamma_s (\sigma - 1)} \bar{h}_{ij} > 0 \quad (37)$$

by (A65) and (A64). The elasticities in (36) and (37) represent the level of multilateral resistance caused by drop of $f_{ij}$, and they are positive and proportional to market share $\bar{h}_{ij}$. Then we can propose that, in the equilibrium, raising a cross fixed-cost of an exporter can push up the exporter’s revenue, its exporting scale and its average product price, and also can create a positive intensive and a positive extensive margins for the exporter’s exported value. Each the cross fixed-cost elasticity in both (36) and (37) does not change for all $t \in \{\Omega \setminus i\}$, and it equals the indirect sub-elasticities in the related own fixed-cost elasticity in either (34) or (35). From this comparison, we find that, similar to $\tau_{ij}$, the indirect sub-elasticity of an endogenous variable with respect to $f_{ij}$ is unchanged across exporters in the equilibrium.
To evaluate, in the equilibrium, how \( f_{ijs} \) impacts the measure of all the exporting firms in the market and impacts the total exported value to the market, we derive

\[
\frac{\partial \ln \bar{n}_{js}}{\partial \ln f_{ijs}} = \frac{\gamma_s h_{ijs}}{\sigma - 1} \left( \frac{\gamma_s - \sigma + 1}{\gamma_s} - \frac{\bar{F}_{js}}{c_{is} f_{ijs}} \right) \text{ and } \sum_{t \in \Omega} \frac{\partial r_{tjjs}}{\partial f_{ijs}} = 0 \tag{38}
\]

in (A66) and (A67). Market fixed-cost index \( \bar{F}_{js} \) is introduced in (21). The necessary and sufficient condition for

\[
\frac{\partial \ln \bar{n}_{js}}{\partial \ln f_{ijs}} > 0 \text{ is } c_{is} f_{ijs} > \frac{\gamma_s}{\gamma_s - \sigma + 1} \bar{F}_{js}, \text{ where } \frac{\gamma_s}{\gamma_s - \sigma + 1} > 1.
\]

In comparison to the finding from (31), the relationship between \( \bar{n}_{js} \) effects of \( f_{ijs} \) and of \( \tau_{ijs} \) can be built. If \( f_{ijs} \) has positive effect on \( n_{js} \), so does \( \tau_{ijs} \); and if \( \tau_{ijs} \) has negative effect on \( n_{js} \), so does \( f_{ijs} \). Generally, in the multilateral equilibrium increasing a trade cost of an exporter is more likely to enlarge the total number of exporting firms in the market if the fixed cost of the exporter is comparatively larger. To find the reason for this result, we go back to expression (10). In the equilibrium, a marginal firm with a higher fixed cost exports comparatively more and therefore, when the firm quits from the export due to rise of its own trade frictions, the market expenditure released by this firm can be absorbed by the more than one marginal firm which are from the other exporters with lower fixed cost. Both with the early discussion in this section and the second expression in (38), we propose that, after change of trade frictions in the trade system of Figure 1, the market clearing condition is not satisfied in the bilateral equilibrium but is satisfied in the multilateral equilibrium.

5 Influence from ES \( \sigma \)

As explained in the model section, larger ES \( \sigma \) represents less heterogeneity in consumer preferences within \( j \)'s market \( s \). Thus, the market with a larger ES is more competitive. Given the demand function in (2), when price index \( P_{js} \) is exogenous, shift of \( \sigma \) will change demand on every product available in market \( s \) of \( j \) and will further make all the exporting firms in the market simultaneously adjust their behaviours in exporting to the market. In the trade system with exogenous \( P_{js} \), it is difficult to predict the firm behaviours after change of \( \sigma \) and so we directly derive the \( \sigma \) elasticities of exporter \( i \)'s exported value, of its exporting scale and of its

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average product price which respectively are

\[
\frac{\partial \ln \tilde{r}_{ijs}}{\partial \ln \sigma} = - \left[ \frac{\sigma \gamma_s}{(\sigma - 1)^2} \ln \left( \frac{\eta_{ijs}}{\sigma} \right) - \frac{\gamma_s + 1}{\gamma_s - \sigma + 1} \right], \quad \frac{\partial \ln \tilde{n}_{ijs}}{\partial \ln \sigma} = - \frac{\sigma \gamma_s}{(\sigma - 1)^2} \ln \left( \frac{\eta_{ijs}}{\sigma} \right)
\]

and

\[
\frac{\partial \ln \tilde{p}_{ijs}}{\partial \ln \sigma} = - \left[ \frac{\sigma}{(\sigma - 1)^2} \ln \left( \frac{\eta_{ijs}}{\sigma} \right) + \frac{1}{\sigma - 1} \right], \quad \text{where } \eta_{ijs} = \frac{\delta_{ij} Y_j}{c_{is} f_{ijs}}.
\]

They are from (A7), (A9) and (A10).

According to (A8), (A9) and (A11), the conditions that let the elasticities in (39) be negative are

\[
\eta_{ijs} > \sigma \exp \left[ \frac{(\gamma_s + 1)(\sigma - 1)^2}{(\gamma_s - \sigma + 1)\sigma \gamma_s} \right], \quad \eta_{ijs} > \frac{\sigma}{\eta^{(\sigma)}}, \quad \text{and } \eta_{ijs} > \sigma \exp \left[ \frac{1 - \sigma}{\eta^{(p)}} \right],
\]

respectively. Here, \( \exp(\cdot) \) is the exponential function with the base of the Euler's number. \( \eta_{ijs} \) is shown in (A3), and is the ratio of the total expenditure from market \( s \) of \( j \) to a firm's fixed cost from sector \( s \) in \( i \). Thus, \( \eta_{ijs} \) should be a very large number. Now, let us see an unreasonable inference based on small \( \eta_{ijs} \). If \( \eta_{ijs} \) were small, the exporting scale of \( i \) in exporting to market \( s \) of \( j \) would be small. Thus, changing the mark-up price of a firm in the bilateral export could affect \( i \)'s contribution to price index \( \bar{P}_{js} \) and could further impact \( \bar{P}_{js} \). This inference is contrary to the previous assumption that the mark-up price of the firm has no impact on \( \bar{P}_{js} \). Therefore, \( \eta_{ijs} \) needs to be very large. In contrast, the criteria in (40) should be fairly small. To have some empirical evidence, we set \( \sigma = 2.98 \) and \( \gamma_{is} = 4.87 \), which are from EKK (2011), to evaluate the criteria and then we get \( \eta^{(r)} = 5.16 \), \( \eta^{(\sigma)} = 2.98 \) and \( \eta^{(p)} = 1.53 \). It can be almost surely concluded that \( \eta_{ijs} \) is larger than any of the criteria and that the elasticities in (39) are negative.

Consequently, the negative elasticities in (39) indicate that, after a growth of the ES in a market, each exporter to the market will have its exported value, its exporting scale and its average product price decline before all the firms in the market realize the change of the market price index. Nevertheless, the story does not end up here. Although the reactions of the exporters make normal sense in the more competitive market that is caused by the larger ES in the market, the total exported value to the market is much less than the market demand that is exogenous. To clear the market, the assumption of exogenous market price index needs to be relaxed. In the trade system and with endogenous \( \bar{P}_{js} \), the exporter reactions to change of \( \sigma \) will affect \( \bar{P}_{js} \) and then the recursive interactive procedure similar to step 4 in Figure 3.
will proceed between $\bar{P}_{js}$ and the export of all the exporters. The procedure will end up at a
new state of the multilateral Nash equilibrium of the trade system. In the equilibrium, $\bar{P}_{js}$ is
affected by $\sigma$ in form of

$$\frac{\partial \ln \bar{P}_{js}}{\partial \ln \sigma} = \frac{\bar{P}_{js}^\gamma_s}{\gamma_s} \sum_{t \in \Omega} \left[ N_{ts} M_{tjs} \left( \frac{\partial \ln \bar{r}_{tjs}}{\partial \ln \sigma} \bigg|_{\bar{P}_{js}=\text{constant}} \right) \right].$$

(41)

shown in (A78). Similar to the proof of $\frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \sigma} < 0$ from both (39) and (40), $\frac{\partial \ln \bar{r}_{tjs}}{\partial \ln \sigma} < 0$ given constant $\bar{P}_{js}$ can also be proved generally for all $t \in \Omega$. Thus, in the multilateral equilibrium, increasing the ES of the market will raise the level of the market price index, and further can lower the importer’s welfare in the sector which is defined in (17). Later in this section, we will find a reason for this proposition.

Next, we will explore, in the multilateral equilibrium, how exporter $i$’s exported value, its exporting scale and its average product price are influenced by $\sigma$. So the $\sigma$ elasticities of the three variables are

$$\frac{\partial \ln \bar{p}_{ijs}}{\partial \ln \sigma} = \frac{\sigma}{(\sigma - 1)^2} d_{ijs} - \frac{\gamma_s^2 + \sigma - 1}{\gamma_s (\sigma - 1) (\gamma_s - \sigma + 1)}; \quad \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \sigma} = \frac{\gamma_s \sigma}{(\sigma - 1)^2} d_{ijs} \quad \text{and}$$

$$\frac{\partial \ln \bar{n}_{ijs}}{\partial \ln \sigma} = \frac{\gamma_s \sigma}{(\sigma - 1)^2} d_{ijs} - \frac{\gamma_s + 1}{\gamma_s - \sigma + 1}, \quad \text{where} \quad d_{ijs} = \ln (c_{is} f_{ijs}) - E_t [\ln (c_{is} f_{ijs})].$$

(42)

They can be found in (A71), (A73) and (A74). Market mean function $E_t(.)$ is introduced in (21). $d_{ijs}$ is the deviance of the $i$’s fixed-cost which is from the mean of the random log fixed-cost in the market. The signs of the elasticities in (42) are ambiguous, and determined by the value of log fixed-cost deviance $d_{ijs}$. For each elasticity in (42) we solve the criterion of $d_{ijs}$. Thus, the elasticity is positive if $d_{ijs}$ is larger than the criterion, and is non positive otherwise. So, the criteria for the three elasticities in (42) are

$$d^{(p)} = \frac{(\gamma_s^2 + \sigma - 1)(\sigma - 1)}{\sigma \gamma_s (\gamma_s - \sigma + 1)}, \quad d^{(r)} = 0 \quad \text{and} \quad d^{(n)} = \frac{(\gamma_s + 1)(\sigma - 1)^2}{\sigma \gamma_s (\gamma_s - \sigma + 1)}$$

(43)

respectively. Generally, in the equilibrium, when the ES in the market rises an exporter bearing a larger fixed cost is more likely to raise its average product price and export more both in value and in number of product varieties. Clearly, the rise of ES $\sigma$ can bring up the average exported value of firm for an exporter in the equilibrium. By using (22), this statement can be verified
by \( \frac{\partial \ln \bar{q}_{ijs}}{\partial \ln \sigma} = \frac{\gamma_s + 1}{\gamma_s + 1 - \sigma} > 1 \). So \( \bar{q}_{ijs} \) is elastic with respect to \( \sigma \).

Moreover, the intensive and the extensive \( \sigma \) elasticities of \( \bar{r}_{ijs} \) are from (A76) and (A75) and are

\[
\frac{\partial \bar{r}^{(in)}_{ijs}}{\partial \sigma} \bar{r}_{ijs} = \frac{\sigma}{\sigma - 1} d_{ijs} + \frac{\gamma_s + 1}{\gamma_s} \quad \text{and} \quad \frac{\partial \bar{r}^{(ex)}_{ijs}}{\partial \sigma} \bar{r}_{ijs} = \frac{\sigma}{(\sigma - 1)^2} \frac{\gamma_s + 1}{\gamma_s} d_{ijs} - \frac{\gamma_s + 1}{\gamma_s}. \tag{44} \]

\[
d^{(in)} = -\frac{(\gamma_s + 1)(\sigma - 1)}{\gamma_s \sigma} \quad \text{and} \quad d^{(ex)} = d^{(n)} \tag{45}.
\]

are the \( d_{ijs} \) criteria to determine the signs of the two elasticities in (44), which is analogous to what the \( d_{ijs} \) criteria in (43) are used for. Hence, for an exporter in the equilibrium, if the exporter’s fixed cost is comparatively large there is a high probability that a positive intensive margin and a positive extensive margin are generated by growth of ES \( \sigma \). Discussed in Section 3, in the multilateral equilibrium the average product price of an exporter with larger fixed cost is comparatively lower. Therefore, when the market is more competitive due to higher ES of the market, the exporter has an advantage in attracting consumption. This reason may explain the role of \( d_{ijs} \) in determining the signs of the elasticities in both (42) and (44). It is interesting that the criteria in both (43) and (45) can be ranked as

\[
d^{(b)} > d^{(n)} = d^{(ex)} > d^{(r)} > d^{(in)}. \]

In the ranked sequence, if \( d_{ijs} \) is larger than a specific criterion the \( \sigma \) elasticity corresponding to each the criterion on the right of the specific criterion is positive, and vice versa.

Finally, we investigate how the market structure in the trade system of Figure 1 is shaped by ES \( \sigma \) when the market price index is endogenous. In the multilateral equilibrium, the \( \sigma \) elasticity of the total exporting firms in the market and the effect of \( \sigma \) on the total exported value to the market are

\[
\frac{\partial \ln \bar{n}_{ijs}}{\partial \ln \sigma} = \frac{\gamma_s \sigma}{(\sigma - 1)^2} \bar{F}_{ijs} \text{Cov}_t \left[ \ln (c_{ijs} f_{ijs}), \frac{1}{c_{ijs} f_{ijs}} \right] - \frac{\gamma_s + 1}{\gamma_s - \sigma + 1} \quad \text{and} \quad \sum_{t \in \Omega} \frac{\partial \bar{n}_{ijs}}{\partial \sigma} = 0. \tag{46}
\]

where \( \text{Cov}_t (\xi_t, \zeta_t) = E_t (\xi_t \zeta_t) - E_t (\xi_t) E_t (\zeta_t) \).

The two expressions in (46) are from both (A79) and (A81). Fixed-cost \( c_{ijs} f_{ijs} \) is a random variable in the market sample from the trade system and is defined in Sub-section 3.2. The correlation
term in \( \frac{\partial \ln \bar{n}_{js}}{\partial \ln \sigma} \) from (46) actually is the multilateral-level covariance in the market sample and between \( \ln(c_s f_{js}) \) and \( \frac{1}{c_s f_{js}} \). First, \( c_s f_{js} \) is positive quadrant dependent to itself. Second, \( \ln(c_s f_{js}) \) is a monotonically increasing function of \( c_s f_{js} \) and \( \frac{1}{c_s f_{js}} \) monotonically decreases in \( c_s f_{js} \). According to Egozcue, Fuentes Garcia and Wong (2009), the covariance can prove to be negative with these two sufficient conditions. In result, \( \frac{\partial \ln \bar{n}_{js}}{\partial \ln \sigma} \) is negative. The negative effect is a reason for the positive effect of \( \frac{\partial \ln P_{js}}{\partial \ln \sigma} \) due to the \( P_{js} \)'s expression in (2). In the equilibrium, the total number of exporting firms in the market is reduced when the ES of the market is larger. As higher \( \sigma \) indicates that the consumers in the market have less love of variety, the market adjustment reflects the changing propensity of the consumers. Moreover, the value of elasticity \( \frac{\partial \ln \bar{n}_{js}}{\partial \ln \sigma} \) is negatively related to market fixed-cost index \( \bar{F}_{js} \). Based on the second equation in (46), the total exported value to the market in the equilibrium is not affected by the ES of the market. Therefore, incorporating the earlier discussion for (31) and for (38), we conclude that, after being impacted by an exogenous variable in the trade system, the system is still balanced when price index \( P_{js} \) is endogenous and, in contrast, is no longer balanced when \( P_{js} \) is exogenous.

In the equilibrium, the producer surplus ratio for each exporter is

\[
\tilde{\kappa}_{js}^{(\pi)} = \tilde{\kappa}_{js}^{(\pi)} + \tilde{\kappa}_{js}^{(f)} = \frac{1}{\sigma}. 
\]

Then we have

\[
\frac{\partial \ln \tilde{\kappa}_{js}^{(\pi)}}{\partial \ln \sigma} = \frac{1}{\sigma - 1} > 0 \quad \text{and} \quad \frac{\partial \ln \tilde{\kappa}_{js}^{(\pi f)}}{\partial \ln \sigma} = -1. \tag{48}
\]

Fixed-cost ratio \( \tilde{\kappa}_{js}^{(f)} \) and profit ratio \( \tilde{\kappa}_{js}^{(\pi)} \) are from (19) and (20) respectively. The results in (48) shows that, for all the exporters in the equilibrium, the producer surplus ratio decreases with \( \sigma \), though the profit ratio increases with \( \sigma \). Hence, it is not clear whether the exporters in the market can benefit from rise of \( \sigma \).

6 Discussion and Summary

Following Chaney (2008), this paper establishes a model to explore the trade system within a sector of products. The trade system is centered at an importer and also includes all the countries exporting to the importer. By analyzing the model, we find some interesting results which will be summarized in this section. The following discussion is within the trade system.

\footnote{Please refer to Gijbels and Sznajder (2013) for how two random variables can be positive quadrant dependent.}
and at its final equilibrium level.\footnote{The final equilibrium is the multilateral equilibrium mentioned in the previous sections.}

First, we present some behaviours of firm. In an exporter, the less productive firms will export less, earn less profit and set a higher price. If the fixed cost of one exporter is less than that of another, both the highest product price and the average product price are higher in the former exporter than in the latter exporter. In consequence, the highest price of product in the market of the importer is charged by the least productive firm in the exporter who bears the lowest exporting fixed cost among all the exporters. In an exporter, the exported value of the least productive firm depends only on both the exporter’s fixed cost and the importer’s ES. For an exporter in response to an external shock to the trade system, the ratio of the extensive margin to the margin of the number of exporting firms is the exported value of the least productive firm in the exporter. This ratio implies that the least productive firm in the exporter is the first to react to the external shock.

Second, the impact of an exporter’s trade friction on the market is discussed. The trade friction comprises both a variable cost and a fixed cost. The trade friction has positive effect on the market price index of the importer and the effect is proportional to the exporter’s market share. Thus, bringing up the trade friction can lower the welfare level of the importer. The total number of varieties in the market is inversely related to the fixed-cost index of the market. When the exporter’s fixed cost is larger than the fixed-cost index, the variable cost of the exporter has positive impact on the total number of varieties in the market. In contrast, the total variety number is positively impacted by the fixed cost of the exporter if the fixed cost is larger than a value that is higher than the fixed-cost index.

Third, we summarize how a trade friction affects a bilateral export. Raising a trade friction of an exporter can push up another exporter’s exported value, its number of exporting firms and the average price of its exported products, and also can generate a positive extensive and a positive intensive margins for another exporter. These effects reflect the multilateral resistance of this trade friction on other exporters’ export. In different direction, bringing up own trade friction can reduce an exporter’s exported value and its number of exporting firms, and also can create a negative extensive margin for the exporter. The variable cost of an exporter has positive effect on the average product price of the exporter, but the exporter’s fixed cost has
negative effect on its average product price. Another two opposite effects made by the two costs are that a positive intensive margin of an exporter may attribute to rise of its own fixed cost or to drop of its own variable cost. In each of these trade friction effects on a bilateral export, the market share of the effect-originated exporter plays a compensative role if the effect is negative or plays an enhancement role if the effect is positive. Also in each of these effects, the market share indicates the indirect effect of the trade friction on the corresponding endogenous variable and the indirect effect is through affecting the market price index. Consequently, the role of the market share implies that the trade friction of an exporter has positive indirect effect on all these endogenous variables. Another inference from all these effects is that if the trade friction of an exporter has a non-zero direct-effect on one of the endogenous variables the direct effect is negative and is, in scale, larger than the indirect effect.

Fourth, the influence of the ES on an exporter’s export depends on the exporter’s log fixed-cost deviated from the market mean of log fixed-cost. If the log fixed-cost deviance is positive and large, we find that the ES is more likely to have positive effect on the exporter’s exported value, on the number of its exporting firms and on the average price of its exported products and also find that, of the exporter, both the intensive marginal effect by the ES and the extensive marginal effect by the ES are also more likely to be positive. For an exporter, only increasing the ES in a rate will make the average exported value per firm rise in the same rate and this effect is not related to the deviance. The average value of exports per firm in the exporter is positively related to its own fixed cost and is unrelated to its own variable cost.

At last, we explain what the ES means to the market. Only raising the level of the ES will undermine the welfare of the importer, through increasing the market price index of the importer. If the ES is large, the consumers in the importer intend not to differentiate the products in the market. Reacting to the intention, the number of product varieties in the market can be reduced by the growth of the ES. In the total revenue of each the exporter, the ES positively affects the rate of profit but negatively affects the rate of producer surplus. Hence, it is plausible what influence the higher ES can exert to the exporter.
7 Conclusion

This paper studies, within a sector of products, how both the trade friction in a unidirectional bilateral export and the ES of the importer’s market affect all the export to the market which includes the bilateral export. The trade friction is decomposed into a variable cost and a fixed cost. The bilateral export is measured by the exported value, by the number of exporting firms and also by the average product price. The crux in the analysis is that when all the product prices in the bilateral export are affected we relax the assumption of exogeneity for the price index in the market. The main results are in the following.

Only raising the trade friction in the unidirectional bilateral export will make the bilateral export decline and will on the other hand boost the export from the other exporters to the market. As an exception, the variable cost of the bilateral export has positive effect on its own average product price. Among all the effects, the market share of the bilateral export attenuates the negative effects and strengthens the positive effects. The roles of the market share attribute to the fact that the effect of the bilateral trade friction on the market price index is positive and is an increasing function of the market share.

The trade friction of the bilateral export can impact the total number of exporting firms to the market. On another direction, the elasticity of substitution in the market has influence on the bilateral export. Both of the two effects increase with the fixed cost of the bilateral export. The elasticity of substitution in the market positively affects the market price index and negatively affects the total number of exporting firms in the market.

Finally, all the theoretical findings of the paper totally rely on the model setup. They reveal some relationships in international trade which have not been done by previous literature. We have not found any previous empirical research closely related to these theoretical findings. Some papers (e.g., Crozet and Koenig, 2010) have estimated the trade friction’s effect on its own bilateral trade. However, they assume the effect is constant and therefore have not investigated what factors determine size of the effect. So, for further study, empirically testing these findings should be valuable.
References


Appendix 1

In the analysis of this section, market price index $P_{js}$ is exogenous. For the export from country $i$ to sector $s$ of $j$, we will derive exported value $\tilde{r}_{ij}s$, exporting scale $\tilde{n}_{ij}s$ and average product price $\tilde{p}_{ij}s$. This section will also evaluate how the three variables are affected by ES $\sigma$.

From the first equation in (11) by both (5) and (7), the exported value is

$$\tilde{r}_{ij}s = \int_{\tilde{\varphi}_{ij}s}^{\infty} N_{is}\tilde{r}_{js}(l)g(\varphi_s) \, d\varphi_s = \frac{N_{is}\gamma_s\varphi_s^{\gamma_s}L}{\gamma_s - \sigma + 1} \left[ \frac{\sigma c_{is}\tilde{r}_{ij}s}{(\sigma - 1)P_{js}} \right]^{1-\sigma} \delta_{js}Y_j\tilde{\varphi}_{ij}s^{\gamma_s-\gamma_s-1},$$

$$\Rightarrow \tilde{r}_{ij}s = N_{is}\delta_{js}Y_j M_{ij}s P_{js}.$$  \hspace{1cm} (A1)

Expression (9) is also used for calculating $\tilde{r}_{ij}s$ in (A1). Here $M_{ij}s$ takes the form of

$$M_{ij}s = \gamma_s\varphi_s^{\gamma_s}L \left( \frac{\sigma c_{is}\tilde{r}_{ij}s}{(\sigma - 1)(\sigma - 1)} \right)^{\gamma_s} \left( \frac{\eta_{ij}s}{\sigma} \right)^{\frac{\gamma_s}{\sigma-1}}.$$  \hspace{1cm} (A2)

with $\eta_{ij}s = \delta_{js}Y_j c_{is}f_{ij}s$.  \hspace{1cm} (A3)

Next, the exporting scale can be derived from the second equation in (11) and is

$$\tilde{n}_{ij}s = \int_{\tilde{\varphi}_{ij}s}^{\infty} g(\varphi_s) \, d\varphi_s = \tilde{\varphi}_{ij}s N_{is}\varphi_s^{\gamma_s}L \text{ by (5)},$$

$$\Rightarrow \tilde{n}_{ij}s = N_{is}\gamma_s\varphi_s^{\gamma_s} \left[ \frac{\sigma c_{is}\tilde{r}_{ij}s}{(\sigma - 1)P_{js}} \right]^{\gamma_s} \left( \frac{\eta_{ij}s}{\sigma} \right)^{\frac{\gamma_s}{\sigma-1}} \text{ by both (9) and (A3).}$$  \hspace{1cm} (A4)

The last expression in (A4) presents the relationship between $\tilde{n}_{ij}s$ and $\tilde{\varphi}_{ij}s$. As this relationship is impervious to price index $P_{js}$, it is also the relationship between $\tilde{n}_{ij}s$ and $\tilde{\varphi}_{ij}s$ when $\tilde{P}_{js}$ is endogenous. Finally, the average product price based on the first equation in (12) is

$$\tilde{p}_{ij}s = \int_{\tilde{\varphi}_{ij}s}^{\infty} \tilde{p}_{js}(l)g(\varphi_s) \, d\varphi_s = \gamma_s\varphi_s^{\gamma_s}P_{js} \left[ \frac{\sigma c_{is}\tilde{r}_{ij}s}{(\sigma - 1)P_{js}} \right]^{\gamma_s} \left( \frac{\eta_{ij}s}{\sigma} \right)^{\frac{\gamma_s+1}{\sigma-1}},$$

$$\Rightarrow \tilde{p}_{ij}s = \frac{\gamma_s}{\gamma_s + 1} P_{js} \left( \frac{\eta_{ij}s}{\sigma} \right)^{\frac{1}{\sigma-1}} \text{ by both (5) and (9).}$$  \hspace{1cm} (A5)

In the derivation of the first step we use (5), (6), (9) and (A3).

To each of the three endogenous variables, its elasticities with respect to its own trade frictions and to price index $P_{js}$ can be simply derived from (A1), (A5) and (A6). So, these
elasticities will be directly displayed in the paper when they are needed. Here, we only present the derivation of each the endogenous variable’s elasticity with respect to ES $\sigma$.

From (A1) and (A2), we have

$$
\tilde{r}_{ijs} = \frac{1}{\gamma_s - \sigma + 1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\gamma_s} \left( \frac{\eta_{ijs}}{\sigma} \right) \frac{2^\gamma_s}{\gamma_s - 1} (c_{is} f_{ijs})^{-\gamma_s} \gamma_s \varphi_{sL}^s N_{is} \delta_{js} Y_j P_{js}^\gamma_s,
$$

$$
\leftarrow \frac{\partial \tilde{r}_{ijs}}{\partial \sigma} = \frac{\tilde{r}_{ijs}}{Z_1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\gamma_s} \left( \frac{\eta_{ijs}}{\sigma} \right) \frac{2^\gamma_s}{\gamma_s - 1} \frac{(\sigma - 1)^2 (\gamma_s + 1) - \sigma \gamma_s (\gamma_s - \sigma + 1) \ln \left( \frac{\eta_{ijs}}{\sigma} \right)}{\eta_{ijs} (\sigma - 1)^2 (\gamma_s - \sigma + 1)^2},
$$

$$
\Rightarrow \frac{\partial \ln \tilde{r}_{ijs}}{\partial \ln \sigma} = \frac{\partial \tilde{r}_{ijs}}{\partial \sigma} \tilde{r}_{ijs} - \frac{\sigma \gamma_s}{(\sigma - 1)^2} \ln \left( \frac{\eta_{ijs}}{\sigma} \right) = -\frac{\sigma \gamma_s}{(\sigma - 1)^2} \ln \left( \frac{\eta_{ijs}}{\sigma} \right),
$$

(A7)

$$
\Rightarrow \frac{\partial \ln \tilde{r}_{ijs}}{\partial \ln \sigma} < 0 \text{ conditional on } \eta_{ijs} > \sigma \exp \left( \frac{(\gamma_s + 1) (\sigma - 1)^2}{\sigma \gamma_s (\gamma_s - \sigma + 1)} \right). \tag{A8}
$$

(A5) gives

$$
\tilde{n}_{ijs} = \left( \frac{\sigma}{\sigma - 1} \right)^{-\gamma_s} \left( \frac{\eta_{ijs}}{\sigma} \right) \frac{2^\gamma_s}{\gamma_s - 1} (c_{is} f_{ijs})^{-\gamma_s} \varphi_{sL}^s N_{is} P_{js}^\gamma_s,
$$

$$
\Rightarrow \frac{\partial \tilde{n}_{ijs}}{\partial \sigma} = \frac{\tilde{n}_{ijs}}{Z_2} \left( -\left( \frac{\gamma_s}{\sigma - 1} \right)^2 \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{\eta_{ijs}}{\sigma} \right) \frac{2^\gamma_s}{\gamma_s - 1} \ln \left( \frac{\eta_{ijs}}{\sigma} \right) \right),
$$

$$
\Rightarrow \frac{\partial \ln \tilde{n}_{ijs}}{\partial \ln \sigma} = \frac{\partial \tilde{n}_{ijs}}{\partial \sigma} \tilde{n}_{ijs} = -\frac{\sigma \gamma_s}{(\sigma - 1)^2} \ln \left( \frac{\eta_{ijs}}{\sigma} \right) = \frac{\partial \ln \tilde{n}_{ijs}}{\partial \ln \sigma} < 0 \text{ given } \eta_{ijs} > \sigma. \tag{A9}
$$

Then from (A6), get

$$
\frac{\partial \tilde{p}_{ijs}}{\partial \sigma} = \frac{\gamma_s}{\gamma_s + 1} P_{js} \left[ \frac{\eta_{ijs}}{\sigma} \frac{2^\gamma_s}{\gamma_s - 1} \frac{\sigma - 1 + \sigma \ln \left( \frac{\eta_{ijs}}{\sigma} \right)}{\sigma (\sigma - 1)^2} \right],
$$

$$
\Rightarrow \frac{\partial \ln \tilde{p}_{ijs}}{\partial \ln \sigma} = \frac{\partial \tilde{p}_{ijs}}{\partial \sigma} \tilde{p}_{ijs} = -\left[ \frac{\sigma}{(\sigma - 1)^2} \ln \left( \frac{\eta_{ijs}}{\sigma} \right) + \frac{1}{\sigma - 1} \right], \tag{A10}
$$

$$
\Rightarrow \frac{\partial \ln \tilde{p}_{ijs}}{\partial \ln \sigma} < 0 \text{ for } \eta_{ijs} > \sigma \exp \left( \frac{1 - \sigma}{\sigma} \right). \tag{A11}
$$

**Appendix 2**

Now, the market price index is assumed to be endogenous in the trade system of Figure 1. At the multilateral equilibrium level of the trade system, we will find export $i$’s exported value, its exporting scale and its average product price. Accordingly, the three variables are $\tilde{r}_{ijs}$, $\tilde{n}_{ijs}$
and \( p_{ijs} \). To the whole market in the equilibrium, we will calculate endogenous market price index \( \bar{P}_{js} \) and \( n_{js} \) that is the measure of the total exporting firms. The inferences from these five endogenous variables will also be shown in this section.

Given (14) by (5) and (6), exporter \( i \) contributes to \( \bar{P}_{js} \) in form of

\[
\tilde{p}_{ijs}^{1-\sigma} = N_{is} \int_{\varphi_{ijs}}^{\infty} \tilde{p}_{jls} (l) [1-\sigma g(\varphi_{s}) \varphi_{ijs}]^1-\sigma \varphi_{ijs}^{-\gamma_s}^{-1},
\]

\[
\Rightarrow \tilde{p}_{ijs}^{1-\sigma} = N_{is} M_{ijs} P_{js}^{\gamma_s-1} \text{ by (9).} \tag{A12}
\]

Symmetrically, we can derive \( \tilde{p}_{kjs} \) for all \( k \in (\Omega \setminus i) \). Recall the assumption that the continuum of the product varieties available in market \( s \) of \( j \) is decomposable by these product varieties. Then, after the price-index contributions of all the exporters enter the expression of \( P_{js} \) in (2), have

\[
\bar{P}_{js}^{1-\sigma} = \sum_{t \in \Omega} \tilde{p}_{tjs}^{1-\sigma} = \bar{P}_{js}^{\gamma_s+1-\sigma} \sum_{t \in \Omega} (N_{ts} M_{tjs}) \text{ given the form of (A12),}
\]

\[
\Rightarrow \bar{P}_{js} = \left[ \sum_{t \in \Omega} (N_{ts} M_{tjs}) \right]^{-\frac{1}{\gamma_s}}. \tag{A13}
\]

When (A13) with both (A2) and (A3) enters (A1), we get the exported value of \( i \) at the multilateral equilibrium level which is

\[
\bar{r}_{ijs} = \frac{N_{is} M_{ijs}}{\sum_{t \in \Omega} (N_{ts} M_{tjs})} \delta_{jjs} Y_j = \frac{N_{is} (c_{ijs} \tau_{ijs})^{-\gamma_s} (c_{ijs} f_{ijs})^{1-\frac{\gamma_s}{\sigma-1}}}{\sum_{t \in \Omega} \left[ N_{ts} (c_{ts} \tau_{tjs})^{-\gamma_s} (c_{ts} f_{tjs})^{1-\frac{\gamma_s}{\sigma-1}} \right]} \delta_{jjs} Y_j. \tag{A14}
\]

So define \( \beta_{ijs} = N_{is} (c_{ijs} \tau_{ijs})^{-\gamma_s} (c_{ijs} f_{ijs})^{1-\frac{\gamma_s}{\sigma-1}}. \tag{A15} \)

Make use of the first expression of \( \tilde{h}_{ijs} \) from (A14) and associate it with (A13), and then get

\[
\tilde{h}_{ijs} = N_{is} M_{ijs} \bar{P}_{js}^{\gamma_s}. \tag{A16}
\]
With welfare \( \bar{w}_{js} = \frac{\delta_{js}Y_j}{P_{js}} \) in (17), the result in (A16) can be used to derive

\[
\bar{P}_{js} = \left( \frac{\bar{h}_{ij}}{N_{is}M_{ij}} \right)^{\frac{1}{\gamma_s}} \Rightarrow \frac{\partial \bar{P}_{js}}{\partial h_{ij}} = \frac{\bar{P}_{js}}{\gamma_s \bar{h}_{ij}} \Rightarrow \frac{\partial \bar{w}_{js}}{\partial h_{ij}} = -\frac{\delta_{js}Y_j}{\gamma_s P_{js} \bar{h}_{ij}} \Rightarrow \frac{\partial \ln \bar{w}_{js}}{\partial \ln h_{ij}} = -\frac{1}{\gamma_s}.
\]

In the consecutive steps of derivation, \( M_{ij} \) is held constant. So, more precisely, the result should be written as

\[
\frac{\partial \ln \bar{w}_{js}}{\partial \ln \bar{h}_{ij}} \bigg|_{M_{ij} = \text{constant}} = -\frac{1}{\gamma_s}.
\]  

(A17)

Plugging \( \bar{P}_{js} \) into (A5) gives the exporting scale of \( i \) in the equilibrium which is

\[
\bar{n}_{ij} = \frac{N_{is} \varphi_{is}^{\gamma_s}}{\bar{P}_{js} \bar{h}_{ij}} \left( \frac{\sigma c_{is} \bar{n}_{ijs}}{(\sigma - 1)} \right)^{-\gamma_s} \left( \frac{\delta_{js}Y_j}{\sigma c_{is} f_{tjs}} \right)^{\frac{\gamma_s}{\sigma - 1}} = \gamma_s - \frac{\sigma}{\gamma_s} - \frac{2}{\gamma_s} \gamma_s \left( \frac{\bar{h}_{tjs} \delta_{js}Y_j}{c_{is} f_{tjs}} \right) \bar{P}_{js} \text{ given (A3).} 
\]

(A18)

In (A18), utilize (A13) with both (A2) and (A3) and then find

\[
\bar{n}_{ij} = \frac{\gamma_s - \sigma}{\gamma_s c_{is} f_{tjs} \sum_{t \in \Omega} \beta_{tij} \bar{h}_{tjs}} \delta_{js}Y_j \bar{P}_{js} \text{ by both (A14) and (A15). (A19)}
\]

In general, aggregating \( \bar{n}_{tjs} \) for all \( t \in \Omega \) gives

\[
\bar{n}_{js} = \sum_{t \in \Omega} \left( \bar{n}_{tjs} \right) = \frac{\bar{h}_{ij} \varphi_{ij}^{\gamma_s} \gamma_s}{\sigma \gamma_s} \delta_{js}Y_j \sum_{t \in \Omega} \left( \bar{h}_{tjs} / c_{is} f_{tjs} \right) = \frac{\bar{n}_{tjs}^{(f)} \gamma_s \delta_{js}Y_j}{\gamma_s c_{is} f_{tjs}}.
\]

(A20)

Fixed-cost ratio \( \bar{\kappa}_{js}^{(f)} \) and fixed-cost index \( \bar{F}_{js} \) are respectively defined in (19) and (21).

The relationship between \( \varphi_{ij} \) and \( n_{ij} \) is \( n_{ij} = \varphi_{ij}^{\gamma_s} N_{is} \varphi_{is}^{\gamma_s} \) in (A4), and can be used to show

\[
\varphi_{ij} \gamma_s - 2 = \left( \frac{\bar{n}_{ij}}{\gamma_s N_{is} \varphi_{is}^{\gamma_s}} \right)^{\frac{\sigma - 2}{\gamma_s}} \text{ and } \frac{\partial \varphi_{ij}}{\partial o_{js}} = -\frac{1}{\gamma_s N_{is} \varphi_{is}^{\gamma_s}} \left( \frac{\bar{n}_{ij}}{\gamma_s N_{is} \varphi_{is}^{\gamma_s}} \right)^{\frac{\gamma_s + 1}{\gamma_s}} \frac{\partial \bar{n}_{ij}}{\partial o_{js}}.
\]

(A21)

\( o_{js} \in \{ \sigma, \tau_{ij}, f_{tjs} \} \) for all \( t \in \Omega \) is defined previously. Given by (23) incorporating (5), (6), (7)
and (11), the extensive marginal effect of $o_{js}$ on $\bar{r}_{ijs}$ is

$$
\frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial o_{js}} = -\bar{v}_{ijs} \frac{\partial \bar{\varphi}_{ijs}}{\partial o_{js}} = -N_{is} \delta_{js} Y_j \gamma_s \varphi_s L \left[ \frac{\sigma c_{is} r_{ijs}}{(\sigma - 1) P_{js}} \right]^{1 - \sigma} \varphi_{ijs}^{\sigma - \gamma_s - 2} \frac{\partial \bar{\varphi}_{ijs}}{\partial o_{js}}.
$$

(A22)

After the last two terms in (A22) are replaced respectively by their expressions in (A21), get

$$
\frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial o_{js}} = \delta_{js} Y_j \left[ \frac{\sigma c_{is} r_{ijs}}{(\sigma - 1) P_{js}} \right]^{1 - \sigma} \left( \frac{\bar{n}_{ijs}}{N_{is} \varphi_{sL}} \right)^{\gamma_s} \frac{\partial \bar{n}_{ijs}}{\partial o_{js}} = \sigma c_{is} f_{ijs} \frac{\partial \bar{n}_{ijs}}{\partial o_{js}}.
$$

(A23)

The second equality in (A23) needs (A18) and is the formula between the two extensive margins defined previously. Also directly from (23), we can acquire the relationship of

$$
\frac{\partial \ln \bar{r}_{ijs}}{\partial \ln o_{js}} = \frac{\partial \bar{r}_{ijs}^{(in)}}{\partial o_{js}} + \frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial o_{js}}.
$$

(A24)

Finally, the $i$'s average product price in the equilibrium can be found by substituting $\bar{P}_{js}$ for $P_{js}$ in (A6) and is

$$
\bar{p}_{ijs} = \frac{\gamma_s}{\gamma_s + 1} \bar{P}_{js} \left( \frac{\eta_{ijs}}{\sigma} \right)^{\frac{1}{\sigma - 1}} = \frac{\gamma_s}{\gamma_s + 1} \left[ \sum_{t \in \Omega} (N_{is} M_{ijs}) \left( \frac{\sigma}{\eta_{ijs}} \right)^{\frac{\gamma_s}{\sigma - 1}} \right]^{-\frac{1}{\gamma_s}} \text{ by (A13),}
$$

(A25)

$$
\Rightarrow \bar{p}_{ijs} = \frac{\gamma_s}{\gamma_s + 1} \left[ \frac{N_{is} \gamma_s \varphi_{sL}}{\gamma_s - \sigma + 1} \left( \frac{\sigma c_{is} r_{ijs}}{\sigma - 1} \right)^{-\gamma_s} \frac{\sigma c_{is} f_{ijs}}{\delta_{js} Y_j} \sum_{t \in \Omega} (\beta_{tjs}) \right]^{-\frac{1}{\gamma_s}},
$$

(A26)

Here, the second-step derivation applies (A2), (A3) and (A15).

**Appendix 3**

In the next section, we will mathematically investigate how the multilateral equilibrium of the trade system is impacted by some of the exogenous variables. The equilibrium level is represented by the three country specific endogenous variables that are $\bar{r}_{ijs}$, $\bar{n}_{ijs}$ and $\bar{p}_{ijs}$, and also by the two endogenous variables of the market which are $\bar{P}_{js}$ and $\bar{n}_{js}$. In the equilibrium, we will derive
the elasticities of each the country specific endogenous variables which are respectively to its own trade frictions, to its cross trade frictions and to the ES. Then, also in the equilibrium, the elasticities of each the market related endogenous variable which are respectively to an exporter’s trade frictions and to the ES will also be found.

Appendix 3A

First, let us check the impact of variable cost $\tau_{ijs}$ on the multilateral equilibrium. Given (A2), we have $\frac{\partial M_{ijs}}{\partial \tau_{ijs}} = -\frac{\gamma_s M_{ijs}}{\tau_{ijs}}$. Then bring it into (A13) and get

$$\frac{\partial P_{js}}{\partial \tau_{ijs}} = \frac{1}{\gamma_s} \left[ \sum_{t \in \Omega} (N_{ts} M_{ijs}) \right]^{-\frac{1}{\gamma_s}} N_{is} \frac{\partial M_{ijs}}{\partial \tau_{ijs}} = \frac{N_{is} M_{ijs} P_{js}}{\tau_{ijs}} \bar{h}_{ijs} \frac{\bar{P}_{js}}{\tau_{ijs}}. \quad (A27)$$

(A16) is adopted for the last equality. So the elasticity is

$$\frac{\partial \ln P_{js}}{\partial \ln \tau_{ijs}} = \frac{\partial P_{js}}{\partial \tau_{ijs}} \frac{\bar{P}_{js}}{\bar{P}_{ijs}} \bar{h}_{ijs}. \quad (A28)$$

With (A27), the first expression of $\bar{P}_{ijs}$ in (A25) gives

$$\frac{\partial \bar{P}_{ijs}}{\partial \tau_{ijs}} = \frac{\bar{P}_{ijs}}{P_{js}} \frac{\partial P_{js}}{\partial \tau_{ijs}} \bar{P}_{ijs} \frac{\bar{P}_{js}}{\tau_{ijs}} \Rightarrow \frac{\partial \ln \bar{P}_{ijs}}{\partial \ln \tau_{ijs}} = \bar{h}_{ijs}. \quad (A29)$$

Based on (A15), get

$$\frac{\partial \beta_{ijs}}{\partial \tau_{ijs}} = -\frac{\gamma_s \beta_{ijs}}{\tau_{ijs}}. \quad (A30)$$

(A14) and (A15) give

$$\bar{h}_{ijs} = \frac{\beta_{ijs}}{\sum_{t \in \Omega} (\beta_{ijs})}. \quad (A31)$$

Then we have

$$\frac{\partial \bar{h}_{ijs}}{\partial \tau_{ijs}} = \frac{\partial \beta_{ijs}}{\partial \tau_{ijs}} \left[ \sum_{t \in \Omega} (\beta_{ijs}) - \beta_{ijs} \right] \left[ \sum_{t \in \Omega} (\beta_{ijs}) \right]^{-2} = -\frac{\gamma_s \bar{h}_{ijs}}{\tau_{ijs}} (1 - \bar{h}_{ijs}) \text{ by (A30).} \quad (A32)$$

Moreover, by $\bar{r}_{ijs} = \bar{h}_{ijs} \delta_{js} Y_j$ in (A14) we get

$$\frac{\partial \bar{r}_{ijs}}{\partial \tau_{ijs}} = -\frac{\gamma_s}{\tau_{ijs}} (1 - \bar{h}_{ijs}) \bar{r}_{ijs} \Rightarrow \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \tau_{ijs}} = -\gamma_s (1 - \bar{h}_{ijs}). \quad (A33)$$
With the second expression of \( \bar{n}_{ijs} \) in (A19), we find

\[
\frac{\partial \bar{n}_{ijs}}{\partial \tau_{ijs}} \frac{\partial \bar{h}_{ijs}}{\partial \tau_{ijs}} = \frac{\bar{n}_{ijs}}{h_{ijs}} \frac{\gamma_s \bar{h}_{ijs}}{\tau_{ijs}} (1 - \bar{h}_{ijs}) = -\frac{\gamma_s \bar{n}_{ijs}}{\tau_{ijs}} (1 - \bar{h}_{ijs}) \text{ given (A32).} \tag{A34}
\]

Then the elasticity is

\[
\frac{\partial \ln \bar{n}_{ijs}}{\partial \ln \tau_{ijs}} \tau_{ijs} = \frac{\partial \bar{n}_{ijs}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{n}_{ijs}} = -\gamma_s (1 - \bar{h}_{ijs}). \tag{A35}
\]

(A34) and formula (A23) jointly give

\[
\frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{r}_{ijs}} = \frac{\partial \bar{n}_{ijs}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{n}_{ijs}} = -\left( \gamma_s - \sigma + 1 \right) \frac{\bar{h}_{ijs} \delta_{js} Y_j}{\tau_{ijs}} (1 - \bar{h}_{ijs}). \tag{A36}
\]

In achieving the second equation in this expression, the second form of \( \bar{n}_{ijs} \) in (A19) is used. This result conditional on \( \bar{r}_{ijs} = \bar{h}_{ijs} \delta_{js} Y_j \) in (A14) can be used to reach

\[
\frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{r}_{ijs}} = -\left( \gamma_s - \sigma + 1 \right) \left( 1 - \bar{h}_{ijs} \right). \tag{A36}
\]

Moreover, according to the equation of (A24), we derive

\[
\frac{\partial \bar{r}_{ijs}^{(in)}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{r}_{ijs}} = \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \tau_{ijs}} - \frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{r}_{ijs}} = -(\sigma - 1) \left( 1 - \bar{h}_{ijs} \right) \text{ with (A33).} \tag{A37}
\]

\( k \neq i \) and \( k \in \Omega \) are defined previously. Symmetrically based on (A14), (A19) and (A25), we can write out \( \bar{r}_{kjs}, \bar{n}_{kjs} \) and \( \bar{p}_{kjs} \) respectively. For the first form of \( \bar{p}_{ijs} \) in (A25), we get

\[
\bar{p}_{kjs} = \frac{\gamma_s}{\gamma_s + 1} \frac{\bar{p}_{js}}{P_{js}} \left( \eta_{kjs} \frac{\sigma}{\sigma - 1} \right)^{\frac{1}{\sigma - 1}}, \tag{A38}
\]

\[
\Rightarrow \frac{\partial \bar{p}_{kjs}}{\partial \tau_{ijs}} = \frac{\bar{p}_{kjs}}{P_{js}} \frac{\partial \bar{p}_{js}}{\partial \tau_{ijs}} = \frac{\bar{p}_{kjs} \bar{h}_{ijs}}{\tau_{ijs}} \text{ given (A27),}
\]

\[
\Rightarrow \frac{\partial \ln \bar{p}_{kjs}}{\partial \ln \tau_{ijs}} = \bar{h}_{ijs}. \tag{A39}
\]

Conditional on the forms of (A14) and of (A15), we have

\[
\bar{r}_{kjs} = \bar{h}_{kjs} \delta_{js} Y_j \text{ and } \bar{h}_{kjs} = \frac{\beta_{kjs}}{\sum_{t \in \Omega} (\beta_{tjs})}. \tag{A40}
\]
Then, from them and (A14), get
\[ \frac{\partial \bar{h}_{kjs}}{\partial \tau_{ijs}} = -\frac{\beta_{kjs}}{\left[ \sum_{t \in \Omega} (\beta_{tjs}) \right]^2} \frac{\partial \beta_{ijs}}{\partial \tau_{ijs}} = \frac{\gamma_s h_{kjs} \beta_{ijs}}{\tau_{ijs} \left[ \sum_{t \in \Omega} (\beta_{tjs}) \right]^2} = \frac{\gamma_s h_{kjs} h_{ijs}}{\tau_{ijs}} \text{ given (A30).} \] (A41)

It can be used to derive the related elasticity that is
\[ \frac{\partial \bar{r}_{kjs}}{\partial \tau_{ijs}} = \frac{\gamma_s}{\tau_{ijs}} \bar{h}_{kjs} h_{ijs} \delta_{js} Y_j \Rightarrow \frac{\partial \ln \bar{r}_{kjs}}{\partial \ln \tau_{ijs}} = \gamma_s \delta_{ijs} \text{ by the expression of } \bar{r}_{kjs} \text{ in (A40).} \] (A42)

From the second equality in (A19), acquire
\[ \bar{n}_{kjs} = \frac{\gamma_s - \sigma + 1}{\sigma \gamma_s} \bar{h}_{kjs} \frac{\delta_{js} Y_j}{c_{ks} f_{kjs}}. \] (A43)

Then, further with (A41) we get
\[ \frac{\partial \bar{n}_{kjs}}{\partial \tau_{ijs}} = \frac{\bar{n}_{kjs}}{\bar{h}_{kjs}} \frac{\partial \bar{h}_{kjs}}{\partial \tau_{ijs}} = \frac{\gamma_s \bar{n}_{kjs}}{\tau_{ijs}} ar{h}_{ijs} \Rightarrow \frac{\partial \ln \bar{n}_{kjs}}{\partial \ln \tau_{ijs}} = \gamma_s \bar{h}_{ijs}. \] (A44)

Adopting the formula of (A23), we use (A43) and (A44) to get
\[ \frac{\partial \bar{r}_{kjs}^{(ex)}}{\partial \tau_{ijs}} = \frac{\sigma c_{ks} f_{kjs}}{\tau_{ijs}} \frac{\partial \bar{n}_{kjs}}{\partial \tau_{ijs}} = \frac{\sigma c_{ks} f_{kjs}}{\tau_{ijs}} \gamma_s \bar{n}_{kjs} \frac{\delta_{js} Y_j}{\tau_{ijs}} = \frac{\bar{h}_{kjs} \delta_{js} Y_j}{\tau_{ijs}} (\gamma_s - \sigma + 1) \bar{h}_{ijs}, \] (A45)
\[ \Rightarrow \frac{\partial \bar{r}_{kjs}^{(ex)}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{r}_{kjs}^{(ex)}} = (\gamma_s - \sigma + 1) \bar{h}_{ijs} \text{ given the form of } \bar{r}_{kjs} \text{ in (A40).} \]

For the equation in (A24), find
\[ \frac{\partial \bar{r}_{kjs}^{(in)}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{r}_{kjs}^{(in)}} = \frac{\partial \ln \bar{r}_{kjs}}{\partial \ln \tau_{ijs}} - \frac{\partial \bar{r}_{kjs}^{(ex)}}{\partial \tau_{ijs}} \frac{\tau_{ijs}}{\bar{r}_{kjs}^{(ex)}} = (\sigma - 1) \bar{h}_{ijs} \text{ with both (A42) and (A45).} \] (A46)

By the second expression of \( \bar{n}_{ijs} \) in (A20) also with both (A32) and (A41), we have
\[ \frac{\partial \bar{n}_{ijs}}{\partial \tau_{ijs}} = \frac{\gamma_s - \sigma + 1}{\sigma \gamma_s} \frac{\delta_{js} Y_j}{\tau_{ijs}} \left[ \frac{\partial \bar{h}_{ijs}}{\partial \tau_{ijs}} \frac{1}{c_{is} f_{ijs}} + \sum_{t \in (\Omega \setminus i)} \left( \frac{\partial \bar{h}_{tjs}}{\partial \tau_{ijs}} \frac{1}{c_{ts} f_{tjs}} \right) \right], \]
\[ = -\frac{\gamma_s - \sigma + 1}{\sigma} \frac{\delta_{js} Y_j}{\tau_{ijs}} \left[ \frac{1}{c_{is} f_{ijs}} - \sum_{t \in \Omega} \left( \frac{\bar{h}_{tjs}}{c_{ts} f_{tjs}} \right) \right]. \]
So the elasticity is
\[
\frac{\partial \ln \bar{n}_{js}}{\partial \ln \tau_{ij}} = \frac{\partial \bar{n}_{js}}{\partial \tau_{ij}} \frac{\tau_{ij}}{\bar{n}_{js}} = \gamma_s \bar{h}_{ij} \left(1 - \frac{\bar{F}_{js}}{c_{is} \bar{f}_{ij}}\right). \tag{A47}
\]

\(\bar{F}_{js}\) is defined in (21). Also with (A32) and (A41), we have
\[
\sum_{t \in \Omega} \left( \frac{\partial \bar{h}_{tjs}}{\partial \tau_{ij}} \right) = \delta_{js} \sum_{t \in \Omega} \left( \frac{\partial \bar{h}_{tjs}}{\partial \tau_{ij}} \right) = \frac{\gamma_s}{\tau_{ij}} \left(\sum_{t \in \Omega} (\bar{h}_{tjs}) - 1\right) \delta_{js} Y_j = 0. \tag{A48}
\]

**Appendix 3B**

Next, we investigate how fixed-cost \(f_{ij}\) impacts the multilateral equilibrium. Given (A2) and (A3), we derive
\[
\frac{\partial M_{ijs}}{\partial f_{ij}} = \left(1 - \frac{\gamma_s}{\sigma - 1}\right) M_{ijs} f_{ij}. \tag{A13}
\]

With this result gives
\[
\frac{\partial \bar{P}_{js}}{\partial f_{ij}} = \frac{\gamma_s - \sigma + 1}{(\sigma - 1) \gamma_s} \bar{P}_{js} f_{ij}. \tag{A49}
\]

Reaching (A49) needs (A16). Then the corresponding elasticity is
\[
\frac{\partial \ln \bar{P}_{js}}{\partial \ln f_{ij}} = \frac{\partial \bar{P}_{js}}{\partial f_{ij}} \frac{f_{ij}}{P_{js}} \bar{P}_{js} = \left(\frac{1}{\sigma - 1} - \frac{1}{\gamma_s}\right) \bar{h}_{ij}. \tag{A50}
\]

On basis of (A49), use both (A3) and the first equation in (A25) to derive
\[
\frac{\partial \bar{p}_{ij}}{\partial f_{ij}} = \bar{p}_{ij} \left(1 - \frac{1}{\sigma - 1}\right) \left(\frac{\partial \bar{P}_{js}}{\partial f_{ij}} \bar{p}_{ij} f_{ij} - \frac{1}{\sigma - 1} \frac{1}{\bar{P}_{js}} \bar{P}_{js} f_{ij}\right) = -\frac{1}{\sigma - 1} \bar{p}_{ij} \left(1 - \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ij}\right),
\]

\[
\Rightarrow \frac{\partial \ln \bar{p}_{ij}}{\partial \ln f_{ij}} = \frac{\partial \bar{p}_{ij}}{\partial f_{ij}} \frac{f_{ij}}{\bar{p}_{ij}} = -\frac{1}{\sigma - 1} \left(1 - \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ij}\right). \tag{A51}
\]

From (A15), find
\[
\frac{\partial \beta_{ij}}{\partial f_{ij}} = \left(1 - \frac{\gamma_s}{\sigma - 1}\right) \frac{\beta_{ij}}{f_{ij}}. \tag{A52}
\]
Then, according to (A31) with (A52)'s form, derive
\[
\frac{\partial \bar{h}_{ijs}}{\partial f_{ijs}} = \frac{\partial \beta_{ijs}}{\partial f_{ijs}} \left[ \sum_{t \in \Omega} \left( \beta_{tjs} \right) - \beta_{ijs} \right] \left[ \sum_{t \in \Omega} (\beta_{tjs}) \right]^{-2} = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \frac{\bar{h}_{ijs}}{f_{ijs}} \left( 1 - \bar{h}_{ijs} \right). \tag{A53}
\]

Using (A53) with (A14), we get
\[
\frac{\partial \bar{r}_{ijs}}{\partial f_{ijs}} = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \frac{\bar{r}_{ijs}}{f_{ijs}} \left( 1 - \bar{h}_{ijs} \right) \Rightarrow \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln f_{ijs}} - \frac{\partial \bar{r}_{ijs}}{\partial f_{ijs}} \frac{f_{ijs}}{\bar{r}_{ijs}} = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \left( 1 - \bar{h}_{ijs} \right). \tag{A54}
\]

Given (A53), we can use the second expression of \( \bar{n}_{ijs} \) in (A19) to find
\[
\frac{\partial \bar{n}_{ijs}}{\partial f_{ijs}} = \frac{\bar{n}_{ijs}}{\bar{h}_{ijs}} f_{ijs} \left( \frac{\partial \bar{h}_{ijs}}{\partial f_{ijs}} \frac{1}{f_{ijs}} - \frac{\bar{h}_{ijs}}{f_{ijs}^2} \right) = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \frac{\bar{n}_{ijs}}{f_{ijs}} \left( 1 - \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ijs} \right). \tag{A55}
\]

Then the related elasticity is
\[
\frac{\partial \ln \bar{n}_{ijs}}{\partial \ln f_{ijs}} - \frac{\partial \bar{n}_{ijs}}{\partial f_{ijs}} \frac{f_{ijs}}{\bar{n}_{ijs}} = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \left( 1 - \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ijs} \right). \tag{A56}
\]

With (A19) and (A55), start from formula (A23) to get
\[
\frac{\partial r_{ijs}^{(ex)}}{\partial f_{ijs}} = \sigma c_{ijs} f_{ijs} \frac{\partial \bar{n}_{ijs}}{\partial f_{ijs}} = -\sigma c_{ijs} \bar{n}_{ijs} \left( \frac{\gamma_s}{\sigma - 1} - \frac{\gamma_s - \sigma + 1}{\sigma - 1} \bar{h}_{ijs} \right).
\]

Then, given the second expression of \( \bar{n}_{ijs} \) in (A19), we get
\[
\frac{\partial r_{ijs}^{(ex)}}{\partial f_{ijs}} = \left( -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \frac{1}{\sigma - 1} \frac{r_{ijs}^{(ex)}}{f_{ijs}} \right) \frac{r_{ijs}^{(ex)}}{f_{ijs}} \Rightarrow \frac{\partial r_{ijs}^{(ex)}}{\partial f_{ijs}} \frac{f_{ijs}}{r_{ijs}^{(ex)}} = Z_4. \tag{A57}
\]

From (A24) with both (A54) and (A57), find
\[
\frac{\partial r_{ijs}^{(in)}}{\partial f_{ijs}} \frac{f_{ijs}}{r_{ijs}} = \frac{\partial \ln r_{ijs}}{\partial f_{ijs}} - \frac{\partial r_{ijs}^{(ex)}}{\partial f_{ijs}} \frac{f_{ijs}}{r_{ijs}} = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \left( 1 - \bar{h}_{ijs} \right) Z_4 = \frac{\gamma_s - \sigma + 1}{\gamma_s} \bar{h}_{ijs}. \tag{A58}
\]

Now, we evaluate how \( f_{ijs} \) affects the endogenous variables from country \( k \). Given (A49),
using (A38) gives

\[
\frac{\partial \tilde{p}_{kjs}}{\partial f_{ijs}} = \frac{\partial \tilde{P}_{j}}{\partial f_{ijs}} \frac{\partial f_{ijs}}{\partial f_{ijs}} = \tilde{p}_{kjs} \gamma_{s} - \sigma + 1 \tilde{h}_{ijs} \Rightarrow \frac{\partial \ln \tilde{p}_{kjs}}{\partial \ln f_{ijs}} = \frac{\gamma_{s} - \sigma + 1 \tilde{h}_{ijs}}{(\sigma - 1) \gamma_{s}}. \tag{A59}
\]

With both (A14) and (A52)'s form, applying the expression of \( \tilde{h}_{kjs} \) in (A40) has

\[
\frac{\partial \tilde{h}_{kjs}}{\partial f_{ijs}} = -\beta_{kjs} \left( \sum_{t \in \Omega} (\beta_{tjs}) \right)^{2} \frac{\partial f_{ijs}}{\partial f_{ijs}} = \frac{\gamma_{s} - \sigma + 1}{\sigma - 1} \tilde{h}_{ijs} \Rightarrow \frac{\partial \ln \tilde{h}_{kjs}}{\partial \ln f_{ijs}} = \frac{\gamma_{s} - \sigma + 1}{\sigma - 1} \tilde{h}_{ijs}. \tag{A60}
\]

Then, given the expression of \( \tilde{r}_{kjs} \) in (A40), find

\[
\frac{\partial \tilde{r}_{kjs}}{\partial f_{ijs}} = \frac{\gamma_{s} - \sigma + 1}{\sigma - 1} \tilde{r}_{kjs} \tilde{h}_{ijs} \Rightarrow \frac{\partial \ln \tilde{r}_{kjs}}{\partial \ln f_{ijs}} = \frac{\gamma_{s} - \sigma + 1}{\sigma - 1} \tilde{h}_{ijs}. \tag{A61}
\]

From (A43), acquire

\[
\frac{\partial \tilde{n}_{kjs}}{\partial f_{ijs}} = \frac{\partial \tilde{h}_{kjs}}{\partial f_{ijs}} \tilde{h}_{ijs} \Rightarrow \frac{\partial \ln \tilde{n}_{kjs}}{\partial \ln f_{ijs}} = \frac{\gamma_{s} - \sigma + 1}{\sigma - 1} \tilde{h}_{ijs}. \tag{A62}
\]

Apply formula (A23) with (A40), (A43) and (A62), and then derive

\[
\frac{\partial \tilde{r}_{kjs}}{\partial f_{ijs}} = \sigma c_{ks} f_{kjs} \frac{\partial \tilde{n}_{kjs}}{\partial f_{ijs}} \Rightarrow \frac{\partial \ln \tilde{r}_{kjs}}{\partial \ln f_{ijs}} = \frac{\gamma_{s} - \sigma + 1}{\sigma - 1} \tilde{h}_{ijs}. \tag{A64}
\]

Further, use (A24) to get

\[
\frac{\partial \tilde{r}_{kjs}}{\partial f_{ijs}} \tilde{r}_{kjs} = \frac{\gamma_{s} - \sigma + 1}{\gamma_{s} (\sigma - 1)} \tilde{h}_{ijs}. \tag{A65}
\]

From (A20), get

\[
\frac{\partial \tilde{n}_{js}}{\partial f_{ijs}} = \frac{\gamma_{s} - \sigma + 1}{\sigma \gamma_{s}} \delta_{js} Y_{j} \left[ \sum_{t \in \Omega} \left( \frac{\partial \hat{h}_{tjs}}{\partial f_{ijs}} \right) \frac{1}{c_{ts} f_{tjs}} - \frac{\hat{h}_{ijs}}{c_{is} f_{ijs}^{2}} \right].
\]
In this form, making use of both (A53) and (A60) gives

\[
\frac{\partial \bar{n}_{js}}{\partial f_{ijs}} = Z_5 \left[ -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \bar{h}_{ijs} \left( 1 - \bar{h}_{ijs} \right) + \sum_{t \in (\Omega \setminus i)} \left( \frac{\gamma_s - \sigma + 1}{\sigma - 1} \frac{\bar{h}_{tjs}}{c_{is}f_{tjs}} \right) - \bar{h}_{ijs} \frac{\bar{h}_{ijs}}{c_{is}f_{ijs}} \right],
\]

\[
= -\frac{\gamma_s - \sigma + 1}{\sigma (\sigma - 1)} \bar{h}_{ijs} \delta_{js} Y_j \left[ \frac{1}{c_{is}f_{ijs}} - \frac{\gamma_s - \sigma + 1}{\gamma_s} \sum_{t \in \Omega} \left( \frac{\bar{h}_{tjs}}{c_{ts}f_{tjs}} \right) \right].
\]

Therefore, the \( f_{ijs} \) elasticity of \( \bar{n}_{js} \) is

\[
\frac{\partial \ln \bar{n}_{js}}{\partial \ln f_{ijs}} = \frac{\partial \bar{n}_{js}}{\partial f_{ijs}} \frac{f_{ijs}}{\bar{n}_{js}} = \frac{\gamma_s h_{ijs}}{\sigma - 1} \left( \frac{\gamma_s - \sigma + 1}{\gamma_s} - \bar{F}_{js} \right).
\]

\( F_{js} \) is given in (21). Use both (A53) and (A60) again, and then achieve

\[
\sum_{t \in \Omega} \left( \frac{\partial \bar{h}_{tjs}}{\partial f_{ijs}} \right) = \frac{\partial \bar{h}_{ijs}}{\partial f_{ijs}} + \sum_{t \in (\Omega \setminus i)} \left( \frac{\partial \bar{h}_{tjs}}{\partial f_{ijs}} \right),
\]

\[
= -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \bar{h}_{ijs} \left( 1 - \bar{h}_{ijs} \right) + \sum_{t \in (\Omega \setminus i)} \left( \frac{\gamma_s - \sigma + 1}{\sigma - 1} \frac{\bar{h}_{tjs}}{f_{ijs}} \right).
\]

By the form of (A14), the result is

\[
\sum_{t \in \Omega} \left( \frac{\partial \bar{h}_{tjs}}{\partial f_{ijs}} \right) = \delta_{js} Y_j \sum_{t \in \Omega} \left( \frac{\partial \bar{h}_{tjs}}{\partial f_{ijs}} \right) = -\frac{\gamma_s - \sigma + 1}{\sigma - 1} \bar{h}_{ijs} \left[ 1 - \sum_{t \in \Omega} (\bar{h}_{tjs}) \right] = 0.
\]

Appendix 3C

In the last subsection, we will evaluate how \( \sigma \) influences the trade system depicted in Figure 1 when the system is at the multilateral equilibrium level. From (A15), we have

\[
\frac{\partial \beta_{ijs}}{\partial \sigma} = \frac{\gamma_s}{(\sigma - 1)^2} \ln \left( c_{is}f_{ijs} \right) \beta_{ijs}.
\]

So, by (A31) get

\[
\frac{\partial \bar{h}_{ijs}}{\partial \sigma} = \frac{1}{\sum_{t \in \Omega} (\beta_{tjs})} \frac{\partial \beta_{ijs}}{\partial \sigma} - \frac{\beta_{ijs}}{\left( \sum_{t \in \Omega} (\beta_{tjs}) \right)^2} \sum_{t \in \Omega} \left( \frac{\partial \beta_{tjs}}{\partial \sigma} \right),
\]

\[
= \frac{\gamma_s}{(\sigma - 1)^2} \sum_{t \in \Omega} (\beta_{tjs}) \left\{ \ln \left( c_{is}f_{ijs} \right) - \frac{\sum_{t \in \Omega} \ln \left( c_{ts}f_{tjs} \right) \beta_{tjs}}{\sum_{t \in \Omega} (\beta_{tjs})} \right\}.
\]
Identical to letter $t$, $a$ also generally indicates an exporter in $\Omega$. In any summation over all exporters in $\Omega$, $t$ can be replaced by $a$ and vice versa. Thus, we re-arrange the last term in the braces of the previous expression and find

$$
\sum_{t \in \Omega} \left[ \ln \left( c_{tsf_tjs} \right) \beta_{tjs} \right] = \sum_{a \in \Omega} \left[ \ln \left( c_{asf_aajs} \right) \beta_{aajs} \right] = \mathbb{E}_t \left[ \ln \left( c_{tsf_tjs} \right) \right] \text{ given the form of (A31)}.
$$

Function form of $\mathbb{E}_t(.)$ is in (21). So, with the form of (A31), we can further derive

$$
\frac{\partial \hat{h}_{tjs}}{\partial \sigma} = \frac{\gamma_s \hat{h}_{tjs}}{(\sigma - 1)^2} d_{tjs} \text{ for } d_{tjs} \text{ defined in (42), (A69)}
$$

$$
\Rightarrow \frac{\partial \hat{r}_{tjs}}{\partial \sigma} = \frac{\gamma_s \hat{h}_{tjs} \delta_{js} Y_j}{(\sigma - 1)^2} d_{tjs} \text{ given (A14), (A70)}
$$

$$
\Rightarrow \frac{\partial \ln \hat{r}_{tjs}}{\partial \ln \sigma} = \frac{\partial \hat{r}_{tjs}}{\partial \sigma} \frac{\sigma}{\hat{r}_{tjs}} = \frac{\sigma \gamma_s}{(\sigma - 1)^2} d_{tjs}.
$$

Next, from the second expression of $\hat{n}_{tjs}$ in (A19), we find

$$
\hat{n}_{tjs} = \frac{\delta_{js} Y_j}{\gamma_s c_{tsf_tjs}} \left( \frac{\gamma_s - \sigma + 1}{\sigma} \hat{h}_{tjs} \right),
$$

$$
\Rightarrow \frac{\partial \hat{n}_{tjs}}{\partial \sigma} = \frac{\delta_{js} Y_j}{\gamma_s c_{tsf_tjs}} \left[ \frac{\gamma_s + 1}{\sigma^2} \hat{h}_{tjs} + \frac{\gamma_s - \sigma + 1}{\sigma} \frac{\partial \hat{h}_{tjs}}{\partial \sigma} \right],
$$

$$
\Rightarrow \frac{\partial \hat{n}_{tjs}}{\partial \sigma} = \frac{\hat{h}_{tjs} \delta_{js} Y_j}{\sigma \gamma_s c_{tsf_tjs}} \left[ \frac{\gamma_s (\gamma_s - \sigma + 1)}{(\sigma - 1)^2} d_{tjs} - \frac{\gamma_s + 1}{\sigma} \right] \text{ by (A69), (A71)}
$$

$$
\Rightarrow \frac{\partial \ln \hat{n}_{tjs}}{\partial \ln \sigma} = \frac{\sigma \gamma_s}{(\sigma - 1)^2} d_{tjs} - \frac{\gamma_s + 1}{\gamma_s - \sigma + 1}.
$$

From (A26), we have

$$
\bar{p}_{tjs} = \frac{\gamma_s}{\gamma_s + 1} c_{tsf_tjs} \left( N_{tjs} \varphi^{\gamma_s}_{sL} \right)^{-\frac{1}{\gamma_s}} \left( \frac{\sigma}{\sigma - 1} \right)^{-\frac{1}{\gamma_s}} \left( \frac{1}{\hat{n}_{tjs}} \right)
$$

Then we derive

$$
\frac{\partial \bar{p}_{tjs}}{\partial \sigma} = \bar{p}_{tjs} \frac{\sigma - 1}{\sigma} \frac{1}{\hat{n}_{tjs}} \left\{ \left[ \frac{1}{\sigma - 1} - \frac{\sigma}{(\sigma - 1)^2} \right] \frac{1}{\hat{n}_{tjs}} + \frac{1}{\sigma - 1} \frac{1}{\hat{n}_{tjs}} \frac{\partial \hat{n}_{tjs}}{\partial \sigma} \right\},
$$

$$
= \bar{p}_{tjs} \frac{\hat{h}_{tjs} \delta_{js} Y_j}{\sigma \gamma_s^2 \hat{n}_{tjs} c_{tsf_tjs}} \left[ \frac{\gamma_s (\gamma_s - \sigma + 1)}{(\sigma - 1)^2} d_{tjs} - \frac{\gamma_s + 1}{\sigma} \right] - \bar{p}_{tjs} \frac{\gamma_s}{\sigma (\sigma - 1)}
$$

by (A72).

It is worth to notice that $\frac{\hat{h}_{tjs} \delta_{js} Y_j}{\hat{n}_{tjs} c_{tsf_tjs}} = \bar{k}_{tjs}^{(f)}$. So, the previous expression can be further simplified
by the form of \( \tilde{k}_{js}^{(f)} \) which is in (19). Thus we get

\[
\frac{\partial p_{ijs}}{\partial \sigma} = \bar{p}_{ijs} \left[ \frac{\sigma}{(\sigma - 1)^2} d_{ijs} - \frac{\gamma_s^2 + \sigma - 1}{\gamma_s (\sigma - 1)(\gamma_s - \sigma + 1)} \right] \Rightarrow \frac{\partial \ln \bar{p}_{ijs}}{\partial \ln \sigma} = Z_0. \tag{A74}
\]

Use (A72) in formula (A23) and get

\[
\frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial \sigma} = \sigma c_{is} f_{ij} \frac{\partial \bar{n}_{ijs}}{\partial \sigma} = \frac{\bar{k}_{ijs} \delta_{js} Y_j}{\gamma_s} \left[ \frac{\gamma_s (\gamma_s - \sigma + 1)}{(\sigma - 1)^2} d_{ijs} - \frac{\gamma_s + 1}{\gamma_s} \right].
\]

Given the expression of (A14), further derive

\[
\frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial \sigma} \bar{r}_{ijs} = \frac{\sigma (\gamma_s - \sigma + 1)}{(\sigma - 1)^2} d_{ijs} - \frac{\gamma_s + 1}{\gamma_s}. \tag{A75}
\]

Then, applying equation (A24) with both (A70) and (A75) gives

\[
\frac{\partial \bar{r}_{ijs}^{(in)}}{\partial \sigma} \bar{r}_{ijs} = \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \sigma} - \frac{\partial \bar{r}_{ijs}^{(ex)}}{\partial \sigma} \bar{r}_{ijs} = \frac{\sigma}{\sigma - 1} d_{ijs} + \frac{\gamma_s + 1}{\gamma_s}. \tag{A76}
\]

After substituting \( \bar{P}_{js} \) for \( P_{js} \) in (A1) and keeping \( \bar{P}_{js} \) unaffected by \( \sigma \), we get

\[
\frac{\partial \bar{r}_{ijs}}{\partial \sigma} \frac{\partial M_{ijs}}{\partial \sigma} \Rightarrow \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \sigma} = \frac{\sigma}{M_{ijs}} \frac{\partial M_{ijs}}{\partial \sigma} \Rightarrow \frac{\partial M_{ijs}}{\partial \sigma} = \frac{\partial \ln r_{ijs}}{\partial \ln \sigma} \frac{M_{ijs}}{\partial \sigma}. \]

More precisely, the result is

\[
\frac{\partial M_{ijs}}{\partial \sigma} = \frac{M_{ijs}}{\sigma} \left( \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \sigma} \bar{P}_{js=constant} \right). \tag{A77}
\]

From (A13), have

\[
\frac{\partial \bar{P}_{js}}{\partial \sigma} = -\frac{1}{\gamma_s} \bar{P}_{js}^{\gamma_s + 1} \sum_{t \in \Omega} \left( N_{ts} \frac{\partial M_{ijs}}{\partial \sigma} \right) \Rightarrow \frac{\partial \ln \bar{P}_{js}}{\partial \ln \sigma} = -\frac{\sigma}{\gamma_s} \bar{P}_{js}^{\gamma_s} \sum_{t \in \Omega} \left( N_{ts} \frac{\partial M_{ijs}}{\partial \sigma} \right). \]

Given the form of (A77), the elasticity can be re-written as

\[
\frac{\partial \ln \bar{P}_{js}}{\partial \ln \sigma} = -\frac{\bar{P}_{js}^{\gamma_s}}{\gamma_s} \sum_{t \in \Omega} \left( N_{ts} M_{ijs} \left( \frac{\partial \ln \bar{r}_{ijs}}{\partial \ln \sigma} \bar{P}_{js=constant} \right) \right). \tag{A78}
\]

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Following both the form of \( \frac{\partial \bar{n}_{tjs}}{\partial \sigma} \) in (A72) and the form of \( d_{tjs} \) in (42), we can write out \( \frac{\partial \bar{n}_{tjs}}{\partial \sigma} \) for all \( t \in \Omega \) and it is

\[
\frac{\partial \bar{n}_{tjs}}{\partial \sigma} = \frac{\delta_{js} Y_j \gamma_s (\gamma_s - \sigma + 1)}{\sigma \gamma_s (\sigma - 1)^2} \frac{\bar{h}_{tjs}}{c_{tfs_j}} \left\{ \ln (c_{tfs_j}) - E_t [\ln (c_{tfs_j})] \right\} - \frac{\delta_{js} Y_j \gamma_s + 1}{\sigma \gamma_s \sigma} \frac{\bar{h}_{tjs}}{c_{tfs_j}}.
\]

Then accumulating all the partial effects gives

\[
\sum_{t \in \Omega} \left( \frac{\partial \bar{n}_{tjs}}{\partial \sigma} \right) = Z_7 \left\{ E_t \left[ \frac{\ln (c_{tfs_j})}{c_{tfs_j}} - E_t \left( \frac{1}{c_{tfs_j}} \right) E_t [\ln (c_{tfs_j})] \right] \right\} - Z_8 E_t \left( \frac{1}{c_{tfs_j}} \right),
\]

\[
= Z_7 \text{Cov}_t \left[ 1_{c_{tfs_j}}, \ln (c_{tfs_j}) \right] - Z_8 \bar{F}_{js}.
\]

Function \( E_t(\cdot) \) and fixed-cost index \( \bar{F}_{js} \) are defined in (21), and function \( \text{Cov}_t(\cdot, \cdot) \) is given in (47). According to \( \frac{\partial n_{js}}{\partial \sigma} = \sum_{t \in \Omega} \left( \frac{\partial \bar{n}_{tjs}}{\partial \sigma} \right) \), we get

\[
\frac{\partial \ln \bar{n}_{js}}{\partial \ln \sigma} = \frac{\partial \bar{n}_{js}}{\partial \sigma} \frac{\sigma}{\bar{n}_{js}} = \frac{\sigma \gamma_s \bar{F}_{js}}{(\sigma - 1)^2} \text{Cov}_t \left[ \frac{1}{c_{tfs_j}}, \ln (c_{tfs_j}) \right] - \frac{\gamma_s + 1}{\gamma_s - \sigma + 1}.
\]  

(A79)

In this derivation, the second expression of \( \bar{n}_{js} \) in (A20) is used. Make use of both the form of \( \frac{\partial h_{tjs}}{\partial \sigma} \) in (A69) and the form of \( d_{tjs} \) in (42), and find

\[
\sum_{t \in \Omega} \left( \frac{\partial \bar{h}_{tjs}}{\partial \sigma} \right) = \frac{\gamma_s}{(\sigma - 1)^2} \sum_{t \in \Omega} \left( \bar{h}_{tjds_{tjs}} \right) = Z_9 \sum_{t \in \Omega} \left\{ \bar{h}_{tjs} \ln (c_{tfs_j}) - E_t [\ln (c_{tfs_j})] \right\},
\]

(A80)

\[
\Rightarrow \sum_{t \in \Omega} \left( \frac{\partial \bar{h}_{tjs}}{\partial \sigma} \right) = Z_9 \left\{ E_t [\ln (c_{tfs_j})] - E_t [\ln (c_{tfs_j})] \right\} = 0.
\]

(A81)

The last summation in (A80) actually is the first central moment of random variable \( \ln (c_{s,fs_j}) \) in the market sample.