Banks, Credibility, and Macroeconomic Evolution after a Production Shock

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Abstract

One of the important functions of financial intermediation is intertemporal risk smoothing. This paper studies the effects of a production shock in a closed economy and compares the abilities of market-based and bank-based financial systems in processing the shock. The analysis of the shock propagation indicates that a competitive banking system may collapse in absence of a proper regulation. Paradoxically, it is the credibility of banks that makes bank-based economies fragile. A necessary and sufficient condition for successfull bailout schemes is proposed.

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1 Introduction

The recent financial crisis revived the attention towards financial systems. The development of events required coordinated liquidity injections by the Fed, the European Central Bank and some other central banks in 2007-08. Government interventions ranged from bailouts of individual banks (e.g. Northern Rock in the UK) to the federal takeover of Fannie Mae and Freddie Mac in the U.S. Usually, a credit crunch is seen as a consequence of an economic slow down. However, the 2007-08 crisis arose "... in what the consensus termed as a "Goldilocks economy"."[1] Among several types of crisis triggering mechanisms (such as external shock, political shock, self-fulfilling panics, or exhaustion of borrowing resources, see Sachs, 1998), neither suits to explain the start of the crisis, except the external shock in a broad sense. The current paper focuses on shock-triggered, non-recession driven banking crises and their relation to the economic slow down. To do this, the paper studies a system of intermediaries, which experience insufficient repayment from their borrowers who suffer from a production shock.

A "classical" example of a shock-driven crisis may be found back in the seventies, when the oil shock led to different consequences in market-based and bank-based financial systems. Allen and Gale (1997) used the example of the oil shock to explain the motivation behind their study of intertemporal risk smoothing by financial intermediation. Their welfare analysis shows that in an economy with stochastic shocks, there exists a feasible intertemporal allocation, which is a Pareto-improvement compared to the one achieved through financial markets. This allocation can be achieved through a financial intermediary with monopoly power. The inefficiency of the market economy is due to the limited participation constraint in the overlapping generations world. The monopoly power allows intermediary to create reserves and to smooth stochastic shocks. If the individuals, who used to invest through the banking system, obtain an access to markets, competition between markets and intermediaries destroys Pareto-superiority of the intermediated economy: the resulting allocation is not better than the one achieved in a market economy.

Still, there is a question, whether an intermediated economy performs better or worse than a market economy after a single unrepeated shock, like the oil shock in the seventies

[1] As Professor Roubini, NYU, said in his interview to the *Daily Telegraph*, ("US mortgage crisis goes into meltdown" by A. Evans-Pritchard, online version from 24.02.2007 at www.telegraph.co.uk).

or the Wall Street Crash of 1929, or the default of the Russian Government of 1998, or the mortgage borrowers' default in the US in 2007. If the shock is unpredictable, or if expected losses are zero, the intermediary of Allen and Gale (1997) would not create reserves against it. If intergenerational risk sharing through transfers from young generations to old generations is possible, the intermediary can use such transfers to provide a better intertemporal allocation than the market. The current paper shows that an intermediated economy with competitive financial intermediaries may collapse after a single shock in a finite number of periods, after which no intergenerational transfers are possible. Therefore, the intermediation fails to provide a Pareto-improvement in the long run and an appropriate regulation is needed to correct this failure.

The role of banking systems in processing stochastic shocks is also addressed by Gersbach and Wenzelburger (2008), who study the stability of the banking system in a closed economy in presence of production shocks. In particular, they show that risk premia in a competitive banking sector are not high enough to prevent the default of the banking system after a series of sufficiently many negative production shocks. Whilst Allen and Gale (1997) focus on the welfare and efficiency issues, Gersbach and Wenzelburger (2008) focus on the prevention of macroeconomic collapse in the intermediated economy. These two papers are the closest to the current one; in both of them repeated shocks are given by exogenous stochastic processes.

The current paper presents a close-up of the impact of a single production shock, in order to study the intertemporal propagation of shocks and the influence of individual shocks on different groups of agents. To study the propagation of a single shock over many periods, the current paper reduces the stochastic component to a single temporary negative production shock, which is a special case of a shock distribution function. Such degenerated shock distribution function can also be seen as a metaphor for a long enough sequence of negative shocks in a stochastic process with zero mean. This approach is distinct from an analysis of one single period out of a sequence of them in both models above, since it not only highlights the impact of the shock, but also studies the subsequent effects. It is shown that the degree of the shock is important for its propagation, and small shocks can fail to reveal the difference between markets and banking system. With more severe shocks, market economy concentrates the impact of the shock in one period, whereas the banking system magnifies the impact of the shock and transfers it from period to period.

Another difference of the current paper from the two above is the multimarket structure of the model. This allows one to show that the shock has several transmission channels. First, the shock can change the general equilibrium, which leads to a change in the subsequent equilibrium path. In the current model, it is the equilibrium wage rate, which determines the wealth endowment of the agents in the next period, and hence the equilibrium in the next period. The market channel allows the propagation of a severe shock in both market and intermediated economies. Second, the shock can create deficits in the banking system, which are financed through newly acquired deposits from the next generation of agents. Paradoxically enough, it is the credibility of banks that makes bank based economy fragile. In absence of credibility banking system replicates financial markets.

Banking regulation is shown to be able to prevent some of the negative consequences of the shock. In particular, allowing banks to make profits prevents the default of the banking system.[2] A particular example of such a regulation would be deposit rate ceilings like in Gersbach and Wenzelburger (2008) and Mavrotas and Vinogradov (2007). In the current paper a more general rule is formulated to encompass various regulatory policies and determine the sufficient level of banks' profits to prevent the collapse.[3] With regards to the welfare, the final judgment upon the efficiency of a regulated banking sector depends on how one evaluates the burden of the shock in the first period compared to the distribution of the burden over several periods.

In a broader context, the current paper contributes to the literature on macroeconomics of banking (see e.g. Bernanke and Gertler ,1987; Bencivenga and Smith, 1991; Banerji et al., 2004), functions of banks (Benston and Smith, 1975; Diamond and Dybvig, 1983; Diamond, 1984; Chemmanur and Fulghieri ,1994), and the efficiency of banking regulation (see e.g. Demirgüc-Kunt and Detragiache, 2000; Barth et al., 2004).

The paper proceeds as follows. Section 2 describes the general macroeconomic environment and discusses the nature of the shock. Section 3 studies the market-based economy that, in line with the literature, is taken as a benchmark. If the inability of the population to access markets is the only reason for banks to exist, banking system should at least replicate the market economy. Section 4 demonstrates that it is not the case. Section 5 discusses reg-

[2] The inability of risk premia to prevent banking collapse in Gersbach and Wenzelburger (2007) is due to the fact that competitive banks cannot set risk premia high enough to create a cushion against a sequence of shocks. Mavrotas and Vinogradov (2007) apply the model of the current paper to the case of non-technological shocks, and consider some particular examples of regulation.

[3] There is also another strand of the literature that studies positive effects of policy measures aimed at generating extra profits of banks. For example Hellman et al. (2000) show that deposit rate ceilings create a franchise value and through this make banks' investment more prudent. The focus of the current paper is on the ability of banking systems to withstand a systemic shock, as opposed to prudential issues.

ulatory measures, which could improve the performance of the banking system. The paper concludes with a discussion of results.

2 Macroeconomic Environment

The description will follow as close as possible the notation of Diamond (1965), whose model is a good departure for the analysis. Though in the current paper productive firms are not assumed to exist infinitely long, the problem of intergenerational lending does not arise: any debtor-creditor relationships only appear between the members of one generation. This is an important issue, since it underlines that although banks (that will be introduced later) are long-living institutions, they are not critical for the existence and functioning of the economy.

2.1 Agents, Preferences and Technologies

The economy consists of overlapping generations. Each generation is distributed over the interval $[0, 1]$ and divided into two groups: workers and entrepreneurs with η - the share of workers in each generation. All agents live for two periods and are endowed with one unit of labor in the beginning of their lives. Entrepreneurs are distinct from workers in that they have access to a production technology in the second period of their lives. The whole young generation works, consumes and saves. The old generation consumes (if workers) or produces and consumes (if entrepreneurs).

Generation $t \geq 1$ is born in the beginning of period t, becomes old in the beginning of period $t+1$ and lives until the end of period $t+1$. All generations are identical, except the old generation of period $t = 1$, which lives only for one period and is initially endowed with some amount of savings used for production. This generation will be neglected in the analysis.

All agents of each generation t have identical intertemporal utility functions $u_t\left(c^0, c^1\right)$ with c^0 = consumption of an agent of generation t when young, and c^1 = his consumption when old. The utility function is continuous, twice differentiable, strictly increasing, quasi-concave and satisfies

$$\lim_{c^0 \to 0} \frac{\frac{\partial u_t}{\partial c^0}}{\frac{\partial u_t}{\partial c^1}} = \infty; \lim_{c^1 \to 0} \frac{\frac{\partial u_t}{\partial c^0}}{\frac{\partial u_t}{\partial c^1}} = 0$$

Utility functions are identical among generations. The utility level of an agent born in period t will hereinafter be represented through $u\left(c_t^0, c_{t+1}^1\right)$.

The production technology produces a consumption/capital good. The technology is identical among entrepreneurs and among periods and is given by $f(k,l)$, where $k =$ physical capital and $l =$ the amount of labor used for production. The full production cycle takes exactly one period, after which new capital and new labor should be employed for the next production cycle. This could be seen as though capital is fully depreciated after one period.[4] The production function is continuous, twice differentiable, increasing, concave, satisfies $f(0,0) = 0$ and

$$\lim_{l\to 0}\frac{\frac{\partial f}{\partial l}}{\frac{\partial f}{\partial k}} = \infty;\ \lim_{k\to 0}\frac{\frac{\partial f}{\partial l}}{\frac{\partial f}{\partial k}} = 0$$

2.2 Shock

The economy may suffer from a production shock. Often, economics deals with technology shocks, which are events that change a production function in macroeconomic models. Technology shocks are permanent and mostly considered to be positive (see e.g. Galí, 2004, for some discussion). In contrast to technology shocks, production (or productivity) shocks can be negative. Another common type of shock in economics is a supply shock, which can be a consequence of a technology shock (and then the supply shock is mostly positive) or not (most negative supply shocks are not technology-driven and are not necessarily productivity-driven). In a dynamic framework, the literature distinguishes between permanent and non-permanent shocks (see e.g. Hall, 1988). Temporary (non-permanent) shocks change the production technology only for one period, whereas permanent shocks leave the production technology changed for the subsequent periods until infinity or until the next shock. It is also necessary to distinguish between the shock impact (instantaneous effects of the shock) and the subsequent effects (some discussion can be found in de Jong and Penzer, 1998). The shock in the current paper is taken to be an unexpected temporary change in output.

Assume that an entrepreneur of generation t employs k_{t+1} units of capital and l_{t+1} units of labor. The production technology produces $f(k_{t+1}, l_{t+1})$ units of consumption/capital good in absence of shocks. In presence of shocks, the actual output y_{t+1} in period $t+1$ is determined by the shock parameter q_{t+1}:

$$y_{t+1} = q_{t+1} f(k_{t+1}, l_{t+1}) \tag{1}$$

The analysis here focuses on a negative shock, therefore $q_{t+1} \in [0,1]$. Furthermore, the shock is assumed to be unpredictable and temporary. If the shock has its impact in period

[4] The assumption of the full depreciation can be relaxed if one assumes that the production function captures total supply of goods, including both newly produced goods and the rest of the capital stock which remains undepreciated.

$\tau + 1$, the distribution of the shock parameter in time can be written as

$$q_{t+1} = \begin{cases} 1 & \text{if } t \neq \tau \\ q^* < 1 & \text{if } t = \tau \end{cases} \tag{2}$$

The shock may equally happen in any period, therefore if we denote with T the total number of periods the economy will go through, the probability of the shock is given by $\Pr(q_{t+1} = q^*) = \frac{1}{T} \underset{T\to\infty}{\to} 0$. This captures the unpredictability of the shock.

2.3 Decision-making and Priority of Payments

Consider a typical generation $t \geq 1$. Each member of this generation may be employed by some old entrepreneur, who offers the wage rate of w_t. Since the production facilities of these entrepreneurs are affected by shock q_t, actual wage payment per unit of labor $\widehat{w}_t$ may differ from w_t. The value of $\widehat{w}_t$ is determined below.

Potential entrepreneurs of generation t solve, when young, their intertemporal utility maximization problem, which determines their consumption c_t^0 and savings s_t^E in period t, as well as their consumption c_{t+1}^1 in period $t+1$. They face the following first-period budget constraint:

$$c_t^0 + s_t^E = \widehat{w}_t l_t^E$$

Here $l_t^E \in [0,1]$ is the part of the unit labor endowment of an agent, which he wishes to be employed. Since unemployed labor delivers no utility to the agent, but the employed labor strictly increases his consumption, it is optimal for him to supply $l_t^E = 1$ units of labor.[5]

The second-period budget constraint of the entrepreneur restricts his second-period consumption to the profit of the firm. The entrepreneur uses his savings s_t^E of the first period of his life to acquire a part of capital stock k_{t+1} used in production. The rest $\left(k_{t+1} - s_t^E\right)$ is financed through credit.[6] When old, the entrepreneur employs l_{t+1} units of labor for production in period $t+1$. Given the price system with the price of goods normalized to unity, the real wage rate in period $t+1$ equals w_{t+1}, and the real gross interest rate r_{t+1}, which applies to credit granted to entrepreneurs in period t and repaid in period $t+1$, the entrepreneur pays $w_{t+1}l_{t+1}$ for the labor, and $r_{t+1}\left(k_{t+1} - s_t^E\right)$ for the capital employed in the production. It will be assumed that entrepreneurs have perfect foresight regarding the future wage rate w_{t+1}. The entrepreneur enjoys limited liability, and his expected profit is

$$E_{t+1} = \max\left[q_{t+1} f\left(k_{t+1}, l_{t+1}\right) - r_{t+1}\left(k_{t+1} - s_t^E\right) - w_{t+1}l_{t+1}, 0\right] \tag{3}$$

[5] If $\widehat{w}_t = 0$, the agent is indifferent with regards of how much labor $l_t^E \in [0,1]$ he supplies.

[6] In general, the difference $k_{t+1} - s_t^E$ might be negative. In an equilibrium (see section 3), this is impossible, otherwise the demand for credit is zero, but the supply of loanable funds is strictly positive.

For the case his revenue is not high enough to cover the expenditures, there exists a *priority of payments*: workers have the highest priority, the creditors have lower priority, and the entrepreneur himself has the lowest priority. Therefore, period's $t+1$ total wage expenditures of the entrepreneur e_{t+1} are either wage payoffs at the rate w_{t+1} per unit of labor, or the entire production if it does not exceed the total wage payoff due:

$$e_{t+1} = \min\left[w_{t+1}l_{t+1}, q_{t+1}f\left(k_{t+1}, l_{t+1}\right)\right] \tag{4}$$

The rest is used to repay the creditors, and hence the payment to creditors b_{t+1} amounts to:

$$b_{t+1} = \min\left[r_{t+1}\left(k_{t+1} - s_t^E\right), q_{t+1}f\left(k_{t+1}, l_{t+1}\right) - e_{t+1}\right] \tag{5}$$

Equation (4) determines the actual wage payment per unit of labor by each individual entrepreneur:[7]

$$\widehat{w}_{t+1} = \min\left[w_{t+1}, \frac{q_{t+1}f\left(k_{t+1}, l_{t+1}\right)}{l_{t+1}}\right] \tag{6}$$

Summarizing and substituting for $l_t^E = 1$, one obtains the expected utility maximization problem of entrepreneurs[8] in the form

$$\begin{aligned} \max_{c_t^0, c_{t+1}^1, s_t^E} \quad & \mathbf{E}_t\left[u\left(c_t^0, c_{t+1}^1\right)\right] \\ \text{subject to} \quad & c_t^0 = \widehat{w}_t - s_t^E \\ & c_{t+1}^1 = \max\left[q_{t+1}f\left(k_{t+1}, l_{t+1}\right) - r_{t+1}\left(k_{t+1} - s_t^E\right) - w_{t+1}l_{t+1}, 0\right] \end{aligned} \tag{7}$$

Separately from the utility maximization (due to Fisher's separation theorem), entrepreneurs solve the expected profit maximization problem of the firm. Since the shock is effectively unanticipated, the problem reduces to

$$\max_{k_{t+1}, l_{t+1}} f\left(k_{t+1}, l_{t+1}\right) - r_{t+1}\left(k_{t+1} - s_t^E\right) - w_{t+1}l_{t+1} \tag{8}$$

The properties of the production function guarantee that there are no corner solutions to the problem. The internal solution produces the demand functions for capital $k_{t+1} = k\left(r_{t+1}, w_{t+1}\right)$ and for labor $l_{t+1} = l\left(r_{t+1}, w_{t+1}\right)$. The solution of the expected utility maximization problem determines the savings function of entrepreneurs $s_t^E = s^E\left(\widehat{w}_t, w_{t+1}, r_{t+1}\right)$. As in Diamond (1965), $0 < \frac{\partial s^E}{\partial \widehat{w}_t} < 1$ (one cannot save more than one unit from a one unit

[7] If entrepreneurs are unable to meet their credit or wage obligations, they have to go through a bankruptcy procedure which effectively means that all the property of the entrepreneurs is sold to (partially) cover their obligations. Therefore, entrepreneurs have no incentives to cheat by claiming that they have experienced a bad shock.

[8] Entrepreneurs might choose whether they invest their savings s_t^E into their firms or act as creditors in the credit market. If the entrepreneurs opt for not running firms, their optimization problem is identical to that of the workers. However, this case is irrelevant for the analysis. The equilibrium outcome would guarantee that the credit interest rate is below the expected profitability of the firms. Otherwise, all entrepreneurs avoid running firms and the demand for credit is zero whilst the credit supply is positive.

increase in endowment); additionally, it can be shown that $\frac{\partial s^E}{\partial w_{t+1}} < 0$ and $\frac{\partial s^E}{\partial r_{t+1}} > 0$.

Workers of generation t solve, when young, the intertemporal utility maximization problem similar to that of entrepreneurs. This determines their consumption c_t^0 and savings s_t^W in period t, as well as the consumption c_{t+1}^1 in period $t+1$.

The budget constraint of a typical worker of generation t for the first period of his life is

$$c_t^0 + s_t^W = \widehat{w}_t l_t^W$$

Similarly to entrepreneurs, $l_t^W = 1$ in the worker's individual optimum.

Workers lend their savings s_t^W to young entrepreneurs at rate r_{t+1}. If after the realization of shock q_{t+1} in period $t+1$ the actual credit payoff to an individual worker is less than $r_{t+1}s_t^W$, the worker (creditor) experiences a deficit and his second period budget constraint should be adjusted for this underpayment. The actual credit payoff from each individual entrepreneur b_{t+1} is given by (5), resulting in an aggregate payoff of $(1-\eta)\, b_{t+1}$ and hence an average creditor obtains $\frac{1-\eta}{\eta} b_{t+1}$.

Definition 2.1 ***Deficit*** *of an individual creditor in period* $t+1$ *is*

$$d_{t+1}^W = \frac{1-\eta}{\eta} b_{t+1} - r_{t+1} s_t^W \tag{9}$$

The second-period budget constraint of the worker restricts the second-period consumption to be equal to the return on his savings adjusted with a possible deficit:

$$c_{t+1}^1 = r_{t+1} s_t^W + d_{t+1}^W$$

Since d_{t+1}^W is conditioned on q_{t+1}, consumption in the second period is uncertain. Substituting for $l_t^W = 1$ and summarizing, one can write the expected utility maximization problem of workers of this generation as follows:[9]

$$\begin{aligned} \max_{c_t^0, c_{t+1}^1, s_t^W} \quad & \mathbf{E}_t \left[u\left(c_t^0, c_{t+1}^1\right)\right] \\ \text{subject to} \quad & c_t^0 = \widehat{w}_t - s_t^W \\ & c_{t+1}^1 = r_{t+1} s_t^W + d_{t+1}^W \end{aligned} \tag{10}$$

This problem determines the savings function of workers $s_t^W = s^W(\widehat{w}_t, r_{t+1})$. As in the entrepreneurs' case above, $0 < \frac{\partial s^W}{\partial \widehat{w}_t} < 1$ (an increase in income leads to an increase in

[9] Formally, there are two stochastic components in the budget constraints: first, it is $\widehat{w}_t$, which is determined by the realization of the shock in period t, and second, it is d_{t+1}^W, determined by the realization of the shock in period $t+1$. The model describes the world with (almost) safe production technology and no alternative assets. It could be extended for the case with a safe asset. Particularly, this would imply strictly positive real interest rates in equilibrium.

savings but each unit of income generates less than one unit of savings) and $\frac{\partial s^W}{\partial r_{t+1}} > 0$ (savings increase in the interest rate). Contrast to entrepreneurs, workers' savings do not depend on the future wage rate.

2.4 Degrees of Shock

In any period $t+1$, one can determine two critical values of the shock parameter. First, $\overline{q}_{t+1}$ such that for any $q_{t+1} \geq \overline{q}_{t+1}$ total production of an individual entrepreneur covers all his production expenses:

$$\overline{q}_{t+1} = \frac{r_{t+1}\left(k_{t+1} - s_t^E\right) + w_{t+1}l_{t+1}}{f\left(k_{t+1}, l_{t+1}\right)}$$

Second, $\underline{q}_{t+1}$ such that for any $q_{t+1} > \underline{q}_{t+1}$ total production covers at least wage expenses:

$$\underline{q}_{t+1} = \frac{w_{t+1}l_{t+1}}{f\left(k_{t+1}, l_{t+1}\right)}$$

Further, it will be assumed that the shock takes place after the economy settles in stationary equilibrium. In stationary equilibrium, the values of $\overline{q}_{t+1}$ and $\underline{q}_{t+1}$ are constant in time, and will be hereinafter denoted with $\overline{q}$ and $\underline{q}$ respectively. Given the priority of payments, and the two critical values above, one can distinguish between four degrees of shock:

1. *Small shock:* $q^* \in [\overline{q}, 1]$. Both employees and creditors are repaid in full.

2. *Moderate shock:* $q^* \in \left[\underline{q}, \overline{q}\right)$. Entrepreneurs are bankrupts, employees are repaid in full, and creditors obtain the residual. Payoff to workers from each entrepreneur is $e_{t+1} = w_{t+1}l_{t+1}$, debt repayment is $b_{t+1} = q^* f\left(k_{t+1}, l_{t+1}\right) - w_{t+1}l_{t+1}$.

3. *Severe shock:* $q^* \in \left(0, \underline{q}\right)$. Entrepreneurs are bankrupt, the value of production does not suffice to repay workers in full. Debt repayments are zero, $b_{t+1} = 0$, the wage payment is $e_{t+1} = q^* f\left(k_{t+1}, l_{t+1}\right)$

4. *Extreme shock* $q^* = 0$ corresponds to a complete destruction of production facilities. Entrepreneurs have zero revenue, wage payment and credit repayment is zero.

Note that the degrees of the shock are relative to economic conditions, which determine $\overline{q}$ and $\underline{q}$. In a capital intensive economy both $\overline{q}$ and $\underline{q}$ should be expected to be lower than in a labor intensive one, which makes capital intensive economies more prone toward production shocks. Appendix A provides further discussion of the degrees of the shock and relates them to the level of economic development.

3 Market Economy

This section describes the equilibrium before and after the production shock in an economy, in which the exchange of resources takes place through markets, and no intermediation is needed. We refer to this case as a market economy.

3.1 Market equilibrium

The following summarizes the life cycle of a typical generation $t \geq 1$. All agents of this generation exchange their unit labor endowment for $\widehat{w}_t$ units of goods. Out of this amount, workers and entrepreneurs create their savings s_t^W and s_t^E respectively. In the end of period t, entrepreneurs of generation t acquire an additional capital stock $I_t = k_{t+1} - s_t^E$. Investment in the production technology takes place in the end of period t. There exists a credit market, in which workers can trade their savings against promissory notes of entrepreneurs. Credit market clears in period t with interest rate r_{t+1}.

There also exists a labor market in each period t. Entrepreneurs of generation t employ members of generation $t+1$ for production in period $t+1$ at the wage rate w_{t+1}. Since the supply of labor is fixed at unity, the equilibrium wage rate only depends on the labor demand. Therefore, the labor market of period $t+1$ clears at the wage rate w_{t+1}, which is known already in period t.

Period $t+1$ starts and the shock parameter q_{t+1} is realized. Each old entrepreneur's wage expenditures are e_{t+1}, and each member of generation $t+1$ obtains $\widehat{w}_{t+1}$ per unit of labor. Capital payoffs from entrepreneurs of generation t to workers of the same generation take place within period $t+1$ and amount to b_{t+1} from each individual entrepreneur. Workers realize deficit of $d_{t+1}^W = \frac{1-\eta}{\eta} b_{t+1} - r_{t+1} s_t^W$.

The analysis focuses on temporary equilibria in each period t conditioned on the realization of the shock parameter q_t (Markov equilibria). Each period's t temporary equilibrium is parametrized on $\widehat{w}_t$ inherited from the previous period according to (6). In the very first period, $\widehat{w}_t$ is given by the initial condition w_1.

Definition 3.1 *A (Markov) equilibrium in the shock-exposed market economy in period $t \geq 1$ under the parameter $\widehat{w}_t$ is an array of the price vector $\left\{r_{t+1}^*, w_{t+1}^*\right\}$ and of the allocation vector $\left\{k_{t+1}^*, l_{t+1}^*, s_t^{E*}, s_t^{W*}\right\}$, which for a given q_t provides that the credit and labor markets clear:*

1. $(1-\eta)\left(k_{t+1} - s_t^E\right) = \eta s_t^W$

2. $(1-\eta)\, l_{t+1} = 1$

The left-hand side in the first equilibrium condition represents the aggregate demand of entrepreneurs for credit resources and the right-hand side is the aggregate supply of credit resources by workers. In the second equilibrium condition the left-hand side represents the aggregate demand of entrepreneurs for labor, whereas the supply of labor is fixed at 1. Knowing the equilibrium of period t, and the realization of the shock q_{t+1}, one can determine realized deficit in period $t+1$: $d_{t+1}^{W} = \frac{1-\eta}{\eta} b_{t+1} - r_{t+1} s_t^W$.

Note that the equilibrium of period t is not conditioned on the level of deficits d_t^W. This is the distinctive property of the market economy (compared to the intermediated economy that will be discussed later in the paper). The level of deficits d_t^W is only relevant for the level of consumption of old workers in period t, but not for the future equilibria.[10]

Proposition 3.1 *The equilibrium exists and is unique for any period $t \geq 1$ if $\widehat{w}_t > 0$.*

The proof of the proposition is based on Arrow and Debreu (1954), see Appendix C. Note that an extreme shock ($q_t = 0$) implies $\widehat{w}_t = 0$ and hence violates the existence of the equilibrium.

3.2 Graphic representation

The equilibrium may be represented in terms of two lines in the (w_{t+1}, r_{t+1})-plane depicting equilibria in the credit market (CM-line) and in the labour market (LM-line) correspondingly (see Fig. 1. Formally, using the notation that follows (8) and (10), the CM-line is the locus of (w_{t+1}, r_{t+1}) to solve

$$(1-\eta)\left(k\left(r_{t+1}, w_{t+1}\right) - s^E\left(\widehat{w}_t, w_{t+1}, r_{t+1}\right)\right) = \eta s_t^W\left(\widehat{w}_t, r_{t+1}\right) \quad \text{(CM)}$$

The LM-line is the locus of (w_{t+1}, r_{t+1}) to solve

$$(1-\eta)\, l\left(r_{t+1}, w_{t+1}\right) = 1 \quad \text{(LM)}$$

[10] It is easy to check that in each period the aggregate consumption by both old and young generations together with aggregate savings by the young generation sum up to the aggregate output, see Appendix B.

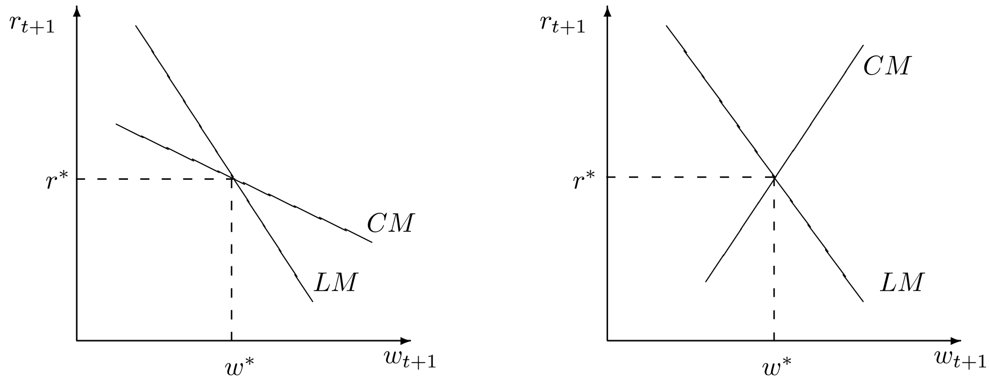

Figure 1. Market Equilibrium

Since the slope of the CM-line can be either negative or positive (but never smaller than the slope of the LM-line, see Lemma 6.2 in Appendix C), both cases are presented in the diagram. Since both cases lead to identical results, only one of them is shown in Fig. 2 which is a comparative statics representation of a change in the equilibrium after the shock. If the shock leads to a contraction of resources available in the credit market, the CM-line shifts upwards (for any given wage level equilibrium in the credit market will be achieved at a higher interest rate). A more detailed analysis of the effects of the shock is provided below.

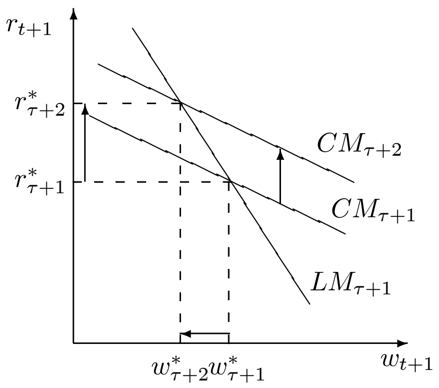

Figure 2. Changes in the market equilibrium due to a reduced availability of credit resources after the shock

3.3 Evolution after a shock

First, consider the market economy without shocks with an initial condition $w_1 > 0$.

Assume there exists such a path of equilibrium price systems $\{w_{t+1}, r_{t+1}\}_{t=0}^{\infty}$ that $w_{t+1} = w_t$ at least for all $t \geq \tau$. In the absence of shocks ($q_{t+1} = 1$) we obtain $\widehat{w}_{t+1} = w_{t+1}$. If the wage level stays unchanged, so does the actual wage payment $\widehat{w}_{t+1}$, and the interest rate level r_{t+1}. The existence of a single stable steady state is an assumption in Diamond (1965). The objective of the current paper is to study the difference between the ways the market and the intermediated economies cope with production shocks. It is easier done, if the market economy possesses a single stable steady state in absence of shocks. This assumption may be relaxed, in which case however it would not be obvious, what drives the instability of the steady state in the intermediated economy below. The instability might in that case be either a specific property of the intermediated economy or the heritage from the basic market economy model. To exclude the latter, it is convenient to deal with a market economy which possesses a single stable stationary equilibrium.

Consider now the market economy in its stationary equilibrium in some period τ and assume it is hit by the shock in period $\tau + 1$: $q_{\tau+1} = q^* < 1$.

If the shock is small, $q^* \in [\overline{q}, 1]$, entrepreneurs are able to fully pay both wages and debts in period $\tau + 1$. Next period starts with the same equilibrium as before the shock. The only population that suffers from the shock, are entrepreneurs of generation τ.

If the shock is moderate, $q^* \in \left[\underline{q}, \overline{q}\right)$, entrepreneurs are able to pay wages in full, but are unable to meet in full their debt obligations. Old workers experience in this case deficits

$$d_{\tau+1}^W = \frac{1-\eta}{\eta}\left(q^* f\left(k_{\tau+1}, l_{\tau+1}\right) - w_{\tau+1} l_{\tau+1}\right) - r_{\tau+1} s_\tau^W < 0$$

The fall in production causes deficits to change (d^W falls from $d_\tau^W = 0$ to some $d_{\tau+1}^W < 0$). Still, this does not change anything in the equilibrium path, since the old workers do not participate in the clearing of the new credit market. The only generation, which suffers from the shock, is the old generation. Young agents obtain the endowment of $\widehat{w}_{\tau+1} = w_{\tau+1} = w_\tau$, which allows them to clear credit and labor markets with the same prices and allocations as in the stationary equilibrium before.

The case of a severe shock, $q^* \in \left(0, \underline{q}\right)$, differs from the above in that the initial change in deficits is larger (since creditors receive nothing from entrepreneurs), and the young generation experiences wage underpayment. Generation $\tau + 1$ obtains the endowment of $\widehat{w}_{\tau+1} < w_{\tau+1} = w_\tau$. As a result, savings of the young generation, $s_{\tau+1}^W$, are smaller than those of the previous generation s_τ^W. This causes CM-line to shift upwards (for any new wage level, credit market clears with a higher interest rate, see Fig. 2). The resulting equilibrium wage level $w_{\tau+2}^*$ is lower than $w_{\tau+1}^* = w_\tau^*$. Along with that, the equilibrium interest rate increases from $r^*_{\tau+1}$ to $r^*_{\tau+2}$.

Since the clearing of the credit market does not involve old workers, the deficit is not transferred to the next period:

$$d^W_{\tau+n+1} = \frac{1-\eta}{\eta} r_{\tau+n+1} \left(k_{\tau+n+1} - s^E_{\tau+n} \right) - r_{\tau+n+1} s^W_{\tau+n} = 0 \tag{11}$$

which is valid for any $n \in \mathbb{N}$. Since there are no new shocks, the economy recovers to the stationary steady state, as soon as $q^* > 0$.[11]

This can be summarized in the following result.

Proposition 3.2 *Assume there exists a single stable stationary equilibrium in absence of shocks. The evolution of the market economy in presence of a shock depends on the degree of the latter:*

1. *If $q_{\tau+1} \geq \underline{q}$, then the market economy does not deviate from the steady state equilibrium path.*
2. *If $0 < q_{\tau+1} < \underline{q}$, then the market economy recovers to the steady state.*

Proposition 3.2 shows that the concept of stability in an economy without shocks may be extended to the case of the economy exposed to shocks. Note that small and moderate shocks only influence the consumption of the generation that is old in the shock period. If the shock is severe, this generation suffers from zero consumption, whereas the young generation of that period experiences wage payoffs below those in the steady state.

It is important that the old generation cannot smooth the burden of the shock through borrowings from the young generation: the old generation physically cannot pay back in the next period, since it dies in the end of the current period. This incomplete participation problem could be solved with help of a long-lived financial intermediary.

4 Intermediated Economy

Financial intermediation is present in the economy through banks, which collect savings from workers in form of deposits, and offer credit to entrepreneurs. The capital of financial intermediaries is assumed to be zero. It might be seen, e.g., as though financial intermediaries possess negligibly small capital and belong to old workers in each period t. The ownership is then transferred from one generation to another through bequests and no market for banks'

[11] Otherwise, the economy collapses in the shock period. The existence of the equilibrium is violated: $q_{\tau+1} = 0$ implies $\widehat{w}_{\tau+1} = 0$, and hence $s^E_{\tau+1} = s^W_{\tau+1} = 0$, though the credit demand is strictly positive.

stocks is needed. The ownership could change budget constraints in (10) through dividend payments, but due to the exogeneity of dividends for workers, the consumption-savings decision of the latter is unchanged. The banking system is assumed to be homogeneous and is further considered as a whole.

The sequence of events is the same as in the market economy, except for the credit market, which is now split into two parts: the deposit market and the credit market per se.

The collection of deposits starts in period t, when workers of generation t create their savings s_t^W. In the end of period t, entrepreneurs apply for credit to start their businesses. Payoffs of entrepreneurs to banks take place within period $t+1$. The value of deposits made with the banks is equal to the value of aggregate savings of workers ηs_t^W. In period $t+1$ banks have to pay the total of $\eta r_{t+1}^D s_t^W$ back to depositors.

It is assumed that no credit rationing takes place, and therefore no credit application is rejected. The amount of credit granted totals $(1-\eta)\left(k_{t+1}-s_t^E\right)$. Within period $t+1$ all entrepreneurs pay back to banks the total of $B_{t+1}=(1-\eta)\,b_{t+1}$, where b_{t+1} is defined as above with the only exception that r_{t+1} should be replaced with the credit rate r_{t+1}^C.

For the following analysis we can use the savings functions and the demand for production factors derived above for the market economy. The only changes concern the distinction between the credit and deposit markets. The savings function of entrepreneurs and their demand for production factors in period t depend now on the credit interest rate r_{t+1}^C instead of r_{t+1}. The savings function of workers depends on the deposit interest rate r_{t+1}^D instead of r_{t+1}.

If in period $t+1$ the total payoff of entrepreneurs to banks does not cover total obligations of banks before their depositors, banks experience a deficit. Numerically it equals the aggregate deficit of all workers in the market economy above.

Definition 4.1 ***Deficit*** *in the banking system in period* $t+1$ *is*

$$d_{t+1}=(1-\eta)\,b_{t+1}-\eta r_{t+1}^D s_t^W \tag{12}$$

Banks are credible institutions and can use newly accumulated deposits to meet current withdrawals.[12] As a result, the aggregated balance sheet of banks is:

$$(1-\eta)\left(k_{t+1}-s_t^E\right)=\eta s_t^W+d_t \tag{13}$$

Since banks operate in a competitive environment, neither deposit rates r_{t+1}^D nor credit

[12] Wagner (1857) based his "theory of banking sediment" (Bodensatztheorie) upon a similar idea.

rates r^C_{t+1} differ among banks, therefore interest rates are taken as uniform in the market.

Proposition 4.1 *Competition in the banking system implies* $r^D_{t+1} = r^C_{t+1} = r_{t+1}$

The proof of the proposition follows from the fact that the expected profit of banks is equal to zero under competition in the banking system.

Now we can define a competitive equilibrium in the intermediated economy exposed to shocks:

Definition 4.2 *A (Markov) equilibrium in the shock-exposed intermediated economy in period* $t \geq 1$ *under parameters* $\{\widehat{w}_t, d_t\}$ *is an array of the price vector* $\left\{r^{C*}_{t+1}, r^{D*}_{t+1}, w^*_{t+1}\right\}$ *and of the allocation vector* $\left\{k^*_{t+1}, l^*_{t+1}, s^{E*}_t, s^{W*}_t\right\}$*, which provides that*

1. $(1-\eta)\left(k_{t+1} - s^E_t\right) = \eta s^W_t + d_t$
2. $(1-\eta)\, l_{t+1} = 1$
3. $r^C_{t+1} = r^D_{t+1} = r_{t+1}$

The last condition is the competitive outcome for credit and deposit interest rates. The link between the deposit and the credit market is given by the balance sheet equation of the banks (condition 1 in definition 4.2).

As soon as new period $t+1$ starts, the shock realization q_{t+1} determines parameters $\{\widehat{w}_{t+1}, d_{t+1}\}$ of the new equilibrium:

1. $d_{t+1} = (1-\eta)\, b_{t+1} - \eta r_{t+1} s^W_t$
   with $b_{t+1} = \min\left[r_{t+1}\left(k_{t+1} - s^E_t\right), q_{t+1} f\left(k_{t+1}, l_{t+1}\right) - e_{t+1}\right]$
   and $e_{t+1} = \min\left[w_{t+1} l_{t+1}, q_{t+1} f\left(k_{t+1}, l_{t+1}\right)\right]$
2. $\widehat{w}_{t+1} = \min\left[w_{t+1}, \frac{q_{t+1} f(k_{t+1}, l_{t+1})}{l_{t+1}}\right]$

Note that changes in the deficit level influence only the CM-line, and do not influence the LM-line, although the resulting temporary equilibrium would differ for different values of d_t. An increase in the absolute value of deficits increases the equilibrium interest rate as defined by the credit market for any wage level w_{t+1} so that the CM-line shifts upwards in (w_{t+1}, r_{t+1})-plane (straightforward from the equilibrium condition for the credit market):

$$\frac{\partial r^{CM}_{t+1}}{\partial d_t} < 0 \tag{14}$$

The sign "<" in inequality (14) is due to the fact that $d_t \leq 0$, and an increase in its absolute value corresponds to a decrease in d_t.

Lemma 4.1 *The equilibrium interest rate and the equilibrium wage rate depend on the deficit in the banking sector: the equilibrium interest rate increases and the equilibrium wage level decreases with the absolute value of the deficit:*

$$\frac{\partial r^*_{t+1}}{\partial d_t} < 0; \frac{\partial w^*_{t+1}}{\partial d_t} > 0 \tag{15}$$

The intuition behind this lemma is obvious. According to (14) and due to the independence of the labor market equilibrium of the deficit in the banking system, the equilibrium interest rate and the wage level are determined by the movement of the equilibrium point along the LM-line. Graphically, changes in the equilibrium in response to an increase in the absolute value of the deficit are the same as shown in Figure 2.

Proposition 4.2 *If $d_t = 1$ for any $t \geq 1$, then the intermediated economy replicates the market economy.*

This result ensures that if with no shocks the market economy converges to the steady state, so does the intermediated economy. Zero deficits in the banking system lead to the identity in the balance sheets of the market economy and of the intermediated one. It is important that there is no risk in any form. This allows one to neglect the crucial difference between direct debt contracts and indirect lending through deposit contracts: the debt contract presumes limited liability of the issuer and the deposit contract presumes unlimited liability of the bank under the assumption that the bank may finance deficits through borrowing from future generations.

Now assume again that in period τ the economy is in the steady state equilibrium, and the shock parameter takes the value of $q^* < 1$ in period $\tau + 1$.

Proposition 4.3 *The evolution of the intermediated economy depends on the degree of the shock:*

1. *If $q^* \in [\overline{q}, 1]$, then the economy converges to the steady state with $d = 0$.*
2. *If $q^* \in \left[\underline{q}, \overline{q}\right)$, then under positive real interest rates the economy collapses in a finite number of periods, otherwise it converges to the steady state with $d = 0$ (if real interest rates are negative) or transfers deficits to future periods (if real interest rates are zero).*

3. *If* $q^* \in \left(0, \underline{q}\right)$*, the banking system is bankrupt in the period of the shock.*

The intuition behind proposition 4.3 is as follows. If $q^* \geq \overline{q}$ then old entrepreneurs repay in full and no deficits in the banking system appear. Proposition 4.2 guarantees that the intermediated economy replicates the market one, which converges to the steady state. If $q^* < \overline{q}$, then necessarily $d_{\tau+1} < 0$ since entrepreneurs default on their debts. Banks may exercise intertemporal smoothing and repay old creditors in full, covering the deficit through borrowing from the next generation of depositors. This augments deficits with factor $r_{\tau+2}$, since this is the interest rate to be paid on newly accumulated deposits. Due to competition, the net profit of banks is zero and cannot reduce the deficit. As a result, the deficit follows the development path:

$$d_{t+1} = r^*_{t+1} d_t,\ t > \tau \tag{16}$$

The resulting phase line $d_{t+1}(d_t)$ is shown in the phase plane in Fig. 3. Positive real interest rate[13] implies $r^*_{t+1} > 1$, which explains the slope of the phase line, and (15) implies that interest rate increases with higher absolute value of deficits, which explains the curvature of the phase line. Since the absolute value of deficits grows from period to period at an increasing rate, any finite level of deficits will be achieved in a finite number of periods. A soon as in n periods after the shock the level of deficits falls to $d_{\tau+n} \leq \underline{d_{\tau+n}} = -\eta s^W_{\tau+n}$. In other words, in a finite number of periods, the deficit in the banking system cannot be covered anymore with newly accumulated deposits. If $q^* < \underline{q}$, this happens immediately after the shock, since entrepreneurs fully default on their debts, and underpay workers compared to the steady state. As a result, newly accumulated deposits cannot cover the deficit.

[13] A negative real interest rate would have an effect of a subsidy provided by future generations, and would shrink the deficits. If workers might choose between depositing with the bank and investing in a durable good, negative real interest rate would be impossible. This would require changes in the utility maximization problems in the beginning of the paper.

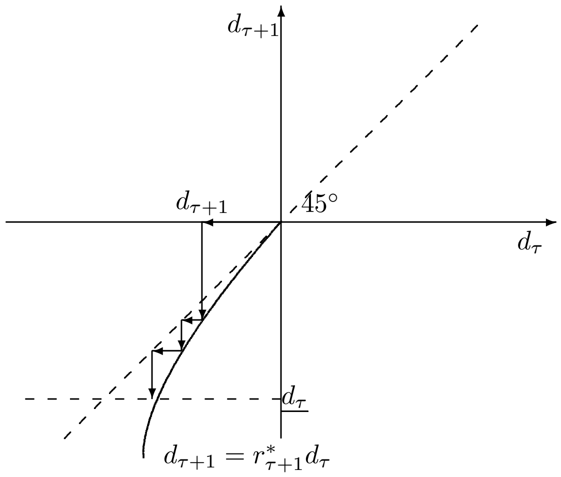

Figure 3. Evolution of the deficit in the banking system

Note that the collapse of the intermediated economy is only possible if banks are credible long lived institutions and can borrow from next generations to cover the deficit. If banks are not credible, the first young after-shock generation refuses to deposit with banks experiencing deficits. This leads to insolvency and bankruptcy of banks, and as a result, the old generation is repaid exactly in the same amount as discussed above for the market economy. The young generation can then create new banks to enable the flow of funds from young workers to young entrepreneurs. Credibility of banks is therefore harmful for the economy, since a credible banking system amplifies the impact of a single production shock until the economy collapses.[14]

An important issue here is that the population cannot observe the shock or at least are unaware about the degree of the shock (proposition 4.3 guarantees that small shocks are harmless for the intermediated economy, thus agents don't need to worry if the shock is small). As soon as wages and deposits are repaid in full no generation can observe (or have incentives to investigate) how harmful is the shock. Moreover, even if the information about deficits in the banking system is available, guarantees from a regulatory authority can be enough to make people believe that the banking system can recover. The next section determines what properties should the regulatory policy have in order to enable the recovery of the banking system.

[14] Even though a collapse of the banking system in the real world is difficult to imagine, the model predicts a tightening of credit conditions after a shock in the intermediated economy.

5 Regulation

The last result underlines the role of the competition in the banking sector, which can now collapse contrast to the non-competitive banking system of Allen and Gale (1997). Indeed, if the competition is not intense, banks are able to exploit positive profit margin, which they could use to cover the deficit. The [somewhat counterintuitive] case of negative real interest rates above allows competitive banks to recover without any regulation because deficit is reduced in this case from period to period. Similarly, a proper regulation should create facilities for deficit reduction.

Assume a regulatory measure is introduced which distorts the competitive outcome by creating an interest rate margin $\delta_{t+1}^{reg} = r_{t+1}^{C} - r_{t+1}^{D}$ in each period $t+1$ if $d_t < 0$. Such a regulatory measure might be a deposit rate ceiling, or a takeover of problematic banks by a centralized agency, or a restrictive liquidity injection. The following proposition formulates a necessary and sufficient condition for the banking system to recover.

Proposition 5.1 *The banking system recovers after the shock and the economy converges to the steady state with* $d = 0$ *if and only if in any after-shock period* $d_t < 0$ *implies* $\delta_{t+1}^{reg}(d_t) > \delta_{t+1}^{crit}(d_t)$, *where*

$$\delta_{t+1}^{crit}(d_t) = \frac{\left(r_{t+1}^{C} - 1\right)(-d_t)}{\eta s_t^{W}} > 0.$$

The proof is given in Appendix C. In an economy where the government cares about the credibility of the banking system the anti-crisis intervention should create the profit margin for banks in accordance with the Proposition 5.1. If the government chooses to bankrupt banks, the Proposition still holds since only banks with $d_t = 0$ are kept open. The Proposition underlines that it is in general not enough to ensure a positive interest rate margin for banks to recover. The critical margin $\delta_{t+1}^{crit}(d_t)$ would just compensate for the growth of the deficit due to interest accrued on it during the period. If profit opportunities of banks are below this level, their balance sheets deteriorate. If the interest rate margin is higher than $\delta_{t+1}^{crit}(d_t)$, banks are able to reduce deficits. It follows that the intervention of the regulator should be prompt, otherwise the deterioration of banks' balance sheets would require higher interest rate margins to reduce the deficit.

Allen and Gale (1997) assume an intermediary to possess monopoly power, which allows it to accumulate reserves. Gersbach and Wenzelburger (2008) consider a competitive case and show that even if intermediaries enjoy positive interest rate margin, explained by a risk premium, banking system still may collapse, since the competition will shrink the margin. The model above shows that the development path of the intermediated economy differs from that of the market economy only if banks enjoy credibility and if the production shock is strong enough to create the deficit. Mavrotas and Vinogradov (2007) use this last fact to apply the model to the case of a repayment shock and analyze regulatory measures, which may improve the performance of the intermediated economy. In particular, at the examples of liquidity provision, interest rate regulation and capital requirements they show that a regulatory measure is only effective if it distorts the competitive outcome in the banking industry, namely if it provides for strictly positive expected profit opportunities for banks. This explains, why the intermediary in Allen and Gale (1997) needs market power to perform better than markets, and why risk premia in Gersbach and Wenzelburger (2008) do not solve the problem of banking collapse: competitive banks are unable to use the interest rate margin to create reserves or to cover the deficit.

6 Discussion and Conclusions

A market-based financial system provides for a quick recovery of the economy after a non-permanent negative production shock. The vulnerability of the economy to shocks depends on the level of economic development. Depending on the degree of the shock, an unregulated bank-based financial system can either replicate the market economy or collapse within a finite number of periods. This difference arises through the fact that the banking system transfers the shock into the future through its balance-sheet channel, in addition to the market channel. The balance-sheet channel allows deficit banks to borrow from future generations of depositors in order to repay the current depositors in full. Under competition, banks suffer from the zero profit margin and are unable to reduce the deficit. If banks are not credible, they are liquidated immediately after they start experiencing deficits. Absence of credibility can thus be optimal for the economy, as it prevents accumulation of deficits over periods. A regulatory measure should create conditions for a positive expected bank profit margin. In an economy with a properly regulated banking system the recovery is in general slower than in the market economy.

There is no explicit answer to the question whether banks do better than markets or vice versa. The paper stresses that not only they differ in their intertemporal risk smoothing abilities but also these smoothing abilities crucially depend on the degree of shocks and the regulatory environment. Should intertemporal smoothing be ever implemented if it is associated with future output losses? It should be noted that since the model above focused on production shocks, both economies suffer from output losses. However the model demonstrates that losses in disposable income of after-shock generations very much differ in the two economies. This creates a trade-off between the two systems. For some social welfare functions the system that provides for a short period of high losses in disposable income would be strictly preferred to the system that provides for a longer sequence of periods with of lower losses in each of them. For other social welfare functions the result may be opposite, depending on the weight the current generation has in the intertemporal social welfare function. A recession can be more harmful for the population of less developed economies, because lower income and lower savings make people less protected. At the same time, shocks that are small (insignificant) for highly capitalized economies can be severe for economies with low capitalization (see Appendix A). These two observations suggest that intertemporal risk smoothing through a properly regulated banking system may be highly desirable for less developed economies even though it is associated with longer output losses.

Although banks in the current paper use new deposits to repay old depositors (which is natural for banks as pointed out yet by Wagner, 1857), they differ from a classical Ponzi scheme in several ways. First, banks *do* credit firms, whereas a Ponzi enterprise does not need to engage in this type of activity. If banks fail to channel funds from depositors to firms, they lack a reason to exist in the framework of this paper, and thus the issue of credibility and debt rollover would not matter. Second, for a Ponzi enterprise it is crucial that it has no competitors, whereas it is the competition that drives the result in the current paper. With no competition banks would be able to cover deficits from profits, and therefore no Ponzi-bubble would appear. Third, a classical Ponzi-enterprise uses its monopolistic power to set the interest rate on the instrument it issues and the rate of growth of the outstanding stock of this instrument in such a way that the difference between the two creates risk-free profit opportunities at least for a finite number of periods (see e.g. Bhattacharya, 2003). In contrast, in the current paper banks *inherit* deficits (if any) from previous periods and thus in any given period deficits are exogenous and cannot be a choice variable. In any period each bank only can obtain the amount of deposits that is fixed by the deposit market interest rate. It is not optimal for banks to cheat and increase the absolute level of deficits because this decreases investment and thus the profit of the current period. Therefore, the optimal level of deficits is zero, as shown for the steady state. Banks are rather trapped in a market equilibrium with no profit opportunities and the outstanding stock of issued financial instruments (deposit contracts) determined by the market. Some researchers see any long enough debt rollover as a Ponzi-game (see e.g. O'Connell and Zeldes, 1988) and in this sense the above model demonstrates how such a "Ponzi-scheme" arises in a competitive equilibrium and disappears as soon as banks' profit opportunities are above competitive level. [15]

This paper did not pretend to explain the recent financial crisis. It rather focused on the question why prompt reaction of the authorities to shock-driven crises can be important if not crucial, and what features of the regulatory intervention are sufficient to reverse the crisis development. Still the example of the mortgage crisis in a predominantly market-based economy stresses the role of the coexistence of several credit channels. In an economy with multiple credit channels, the risk of the whole economy being exposed to a systemic shock is lower than in the model above. Indeed, if firms are financed through both markets and banks simultaneously, the shock is partly absorbed in financial market, where old creditors suffer from the default of borrowers. However, in a model, in which banks and financial market coexist, the assumption of no market access fails, and hence there should be a different reason for banks to exist.

Another aspect of multiple credit channels is segmented intermediation. If there are several types of intermediaries, and each of them serves a distinct sector of economy, a shock in one sector could be transferred to other sectors through non-financial markets like labor market in the model above. This issue of non-financial contagion seems to be an appealing direction for future research.

References

Allen, F., D. Gale (1997) "Financial markets, intermediaries and intertemporal smoothing", *Journal of Political Economy*, Vol. 105, No. 3, pp. 523-546

Araujo A., M.R. Pascoa, J.P. Torres-Martinez (2002) "Collateral Avoids Ponzi Schemes in Incomplete Markets", *Econometrica*, Vol. 70, No. 4, pp. 1613-1638

Arrow, K.J., G. Debreu (1954) "Existence Of An Equilibrium For A Competitive Economy", *Econometrica*, Vol. 22, No. 3, pp. 265-290

Banerji S., J. Bhattacharya, N. V. Long (2004) "Can financial intermediation induce endogenous fluctuations?" *Journal of Economic Dynamics and Control*, Vol. 28(11),

[15] An additional issue with Ponzi games is that they are known to create a problem for the existence of equilibria and optimal choices in economies with infinite horizons (e.g. Magill and Quinzii, 1994, or Araujo et al., 2002), which is not the case in the current paper as all decisions are made over a finite number of periods.

pp. 2215-2238

Barth, J.R., G. Caprio, R. Levine (2004) "Bank regulation and supervision: what works best?" *Journal of Financial Intermediation*, Vol. 13(2), pp. 205-248

Bencivenga V., B. Smith (1991) "Financial intermediation and endogenous growth", *Review of Economic Studies* 58, pp. 195-209

Benston G.W., C.W. Smith (1976) "A transaction cost approach to the theory of financial intermediation" *The Journal of Finance*, Vol. XXXI (1), pp. 215-231

Bernanke, B., M. Gertler (1987) "Banking and Macroeconomic Equilibrium," in *New Approaches to Monetary Economics*, ed. W. Barnett and K. Singleton, New York, Cambridge University Press

Bhattacharya, U. (2003) "The Optimal Design of Ponzi Schemes in Finite Economies," *Journal of Financial Intermediation*, 12, pp. 2-24

Bolton P. (2002) "Banking in Emerging Markets", *Journal of Financial Intermediation*, 11, pp. 362-365.

Chemmanur, T.J., P. Fulghieri (1994) "Reputation, Renegotiation and the Choice between Bank Loans and Publicly Traded Debt", *The Review of Financial Studies*, Vol. 7, pp. 475-506

Demirgüc-Kunt A., E. Detragiache (2000) "Does deposit insurance increase banking system stability? An empirical investigation", Proceedings of the 8th Econometric Society World Congress, Seattle, Washington, Contributed Paper No 1751

Diamond D. (1984) "Financial Intermediation and Delegated Monitoring", *Review of Economic Studies* 51, pp. 393-414

Diamond D., P. Dybvig (1983) "Bank runs, deposit insurance and liquidity", *Journal of Political Economy* 91, pp. 401-419

Diamond P. (1965) "National Debt in a Neoclassical Growth Model", *The American Economic Review*, Vol. 55, No. 5, pp. 1126 - 1150

Galí, J. (2004) "On The Role of Technology Shocks as a Source of Business Cycles: some new evidence", *Journal of the European Economic Association*, No. 2(2–3), pp. 372 – 380

Gersbach H., J. Wenzelburger (2008) "Do Risk Premia Protect from Banking Crises?", *Macroeconomic Dynamics*, Vol.12, pp. 100-111

Gorton G., A. Winton (2003) "Financial intermediation" in Handbook of the Economics and Finance, eds. G. Constantanides, M. Harris and R. Stulz, Amsterdam, North

Holland Elsevier

HALL, R. E. (1988), “Intertemporal Substitution in Consumption”, *Journal of Political Economy*, Vol. 96, pp. 339-357.

HELLMAN, T., K. MURDOCK, J. STIGLITZ (2000) "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?" *American Economic Review*, 90 (1), pp. 147-65

DE JONG, P., J. PENZER (1998) "Diagnosing Shocks in Time Series" *Journal of the American Statistical Association*, Vol.93 , pp. 796 -806

MAGILL M., M. QUINZII (1994) "Infinite Horizon Incomplete Markets," *Econometrica*, Vol. 62, No. 4, pp. 853-880

MAVROTAS, G., D. VINOGRADOV (2007) "Financial Sector Structure and Financial Crisis Burden", *Journal of Financial Stability*, 3(4), pp. 295-323

SACHS J. (1998) "Alternative approaches to financial crises in emerging markets" in *Capital flows and financial crises*, ed. Miles Kahler, Manchester University Press

WAGNER A. (1857) "Beiträge zur Lehre von den Banken", Leipzig, Voss (reprint: Vaduz, 1977)

Appendix A. Degrees of Shock

The shock in the model is determined by the shock parameter $q^* \in [0;1]$, and is measured by the output after the shock as a percentage of the output in steady state. This does not, however, mean that the shock of q^*, which is moderate or severe in one economy, would be necessarily moderate or severe in another economy. To discuss this issue, assume that the shock q^* occurs in the steady state. The severity of the shock depends on the steady state price system, namely on the wage and interest rate level.

The lower limit of the small shock is then

$$\overline{q} = \frac{\overline{r}\left(\overline{k} - \overline{s}^E\right) + \overline{w}\overline{l}}{f\left(\overline{k}, \overline{l}\right)} \tag{A-1}$$

and the lower limit of the moderate shock is

$$\underline{q} = \frac{\overline{w}\overline{l}}{f\left(\overline{k}, \overline{l}\right)} \tag{A-2}$$

with the "barred" variables referring to the steady state.

As it can be seen, an economy with a higher share of capital in production has necessarily a smaller $\underline{q}$, and hence is less vulnerable with respect to a shock: the probability that a shock of q^* is moderate, but not severe, is in highly capitalized economies higher than in less capitalized economies. Indeed, the ratio $\frac{f(\overline{k},\overline{l})}{\overline{l}}$ is the average productivity of labor $APL\left(\overline{k},\overline{l}\right)$, which increases as the capitalization of production increases. In the stationary point, profit-maximizing firms set the wage level equal to the marginal product of labor $\overline{w} = MPL\left(\overline{k},\overline{l}\right)$. Hence, equation (A-2) may be written in a form

$$\underline{q} = \frac{MPL\left(\overline{k},\overline{l}\right)}{APL\left(\overline{k},\overline{l}\right)}$$

On the one hand, the higher the average productivity of labor, the smaller the interval $\left(0,\underline{q}\right)$, which determines the area of severe shocks. On the other hand, higher capitalization leads to a higher marginal product of labor, so that the general effect may be ambiguous and depends on the substitutability between labor and capital.[16]

Equation (A-1) may be reformulated as

$$\overline{q} = \frac{\overline{r}\left(\overline{k}-\overline{s}^E\right)}{f\left(\overline{k},\overline{l}\right)} + \underline{q}$$

so that the term $\frac{\overline{r}\left(\overline{k}-\overline{s}^E\right)}{f\left(\overline{k},\overline{l}\right)}$ indicates the length of the interval $\left[\underline{q},\overline{q}\right)$ of the moderate shock. Note that $\overline{s}^E$ is the internal finance provided by entrepreneurs themselves, and $\overline{k}-\overline{s}^E$ is external borrowing. The higher the share of internal capital, the higher the probability of the shock being small. Vice versa, the higher the share of the external capital, the more vulnerable is the economy to the production shock. The above reasoning also applies to the average productivity of the borrowed capital $APK_B = \frac{f(\overline{k},\overline{l})}{\overline{k}-\overline{s}^E}$ and to the marginal productivity of capital $MPK = \overline{r}$:

$$\overline{q} - \underline{q} = \frac{MPK\left(\overline{k},\overline{l}\right)}{APK_B\left(\overline{k},\overline{l}\right)}$$

One may expect that in economies with high capitalization and low costs of capital (due to decreasing marginal productivity, high capitalization implies low MPK and therefore low equilibrium borrowing costs), the difference $\overline{q}-\underline{q}$ shrinks.

If one assumes that both $\overline{q}$ and $\overline{q}-\underline{q}$ are decreasing functions of $\overline{k}$, the following schematic representation is possible (see Fig. 4).[17] In the figure, it is shown that a shock q^* may be seen as a moderate shock for a smaller economy, whereas it is a small shock for a large (highly capitalized) economy. Moreover, it is also possible that a shock q^{**},

[16] For a Cobb-Douglas production function $f = k^\alpha l^\beta$ $(\alpha+\beta \leq 1)$ one obtains $APL = \frac{f}{l} = k^\alpha l^{\beta-1}$ and $MPL = \beta k^\alpha l^{\beta-1}$, so that $\underline{q} = \beta < 1$. If capital and labor are perfect substitutes ($f = \alpha k + \beta l$), $MPL = \beta$, and $\underline{q} = \frac{1}{\alpha k/l+\beta}$, which decreases as capitalization increases.

[17] The purpose of the diagram is only to illustrate the possibility of different treatment of the same shock by different economies. A detailed analysis of the shock-response functions is not the focus of this paper.

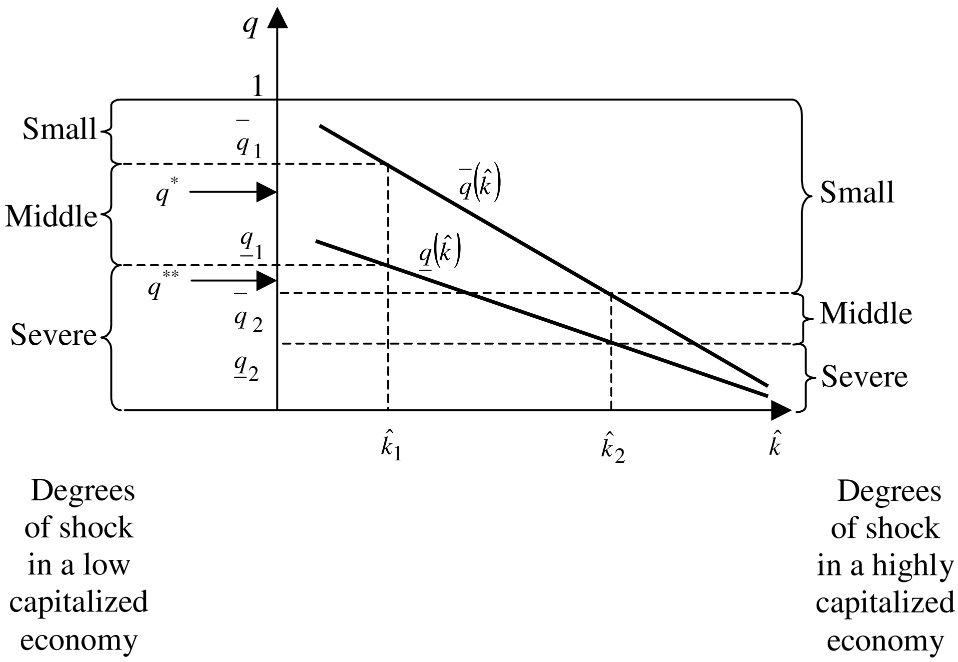

Figure 4. Degrees of shock for economies with different capitalization.

which is small for a highly developed economy, is severe for a less developed economy.[18] For example, a loss of 10% GDP (the shock parameter $q^* = 0.9$) can represent a small shock for a developed economy, but be a moderate (or even severe) shock for an underdeveloped labor-intense economy with low average productivity of labor. This discussion suggests that the results of the current paper may be of different significance for developed and underdeveloped economies.

Countries with more labor-intensive production seem to be more vulnerable to stronger shocks, whereas developed countries seem to be less vulnerable to the degrees of the shock, which may demonstrate the difference between the market-based and bank-based financial systems. This qualitative remark may be another fact in favor of establishing bank-oriented financial systems in emerging economies, due to their smaller capitalization and poorer technological development. On the contrary, in developed economies the probability of moderate shocks is lower, and the advantages of the banking system in intertemporal smoothing of exogenous negative shocks may be less noticeable.

[18] Here, the development is understood in sense of the marginal product - average product ratios introduced above. I do not focus on this issue further, since the degree of the development is not the principal issue in the analysis here. Still, it is important to note that the relevance of the analysis *may* be different for different economies.

Appendix B. Aggregate constraint

The equilibrium in the model is defined for the credit and labor markets. Imlicitly there is also a market for goods in this economy. The market for goods is in equilibrium by construction: the economy is described in real terms, so that the budget constraints guarantee equilibrium in the market for goods. To calculate the aggregate constraints we take into account that the mass of workers is η and the mass of entrepreneurs is $1-\eta$. To make the exposition brief we use the fact that workers supply exactly one unit of labor. In each period $t>1$ (the very first period in overlapping generation models differs from the rest because special assumptions need to be done about the old generation and its endowment) the aggregate output in the economy is $Y_t=(1-\eta)\,q_t f_t$ and is distributed for consumption of the old generation and consumption and savings of the young generation of this period. Consumption of the old workers c_t^{OW} equals their earnings from their savings deposited or invested in the previous period (adjusted for a possible deficit):

$$c_t^{OW}=\eta\left(r_t s_{t-1}^W+d_t^W\right)$$

Consumption of the young generation (c_t^{YW} for the consumption of young workers and c_t^{YE} for the consumption of young entrepreneurs) equals their (actual) wage earnings minus their savings:

$$c_t^{YW}+c_t^{YE}=\widehat{w}_t-\eta s_t^W-(1-\eta)\,s_t^E$$

Aggregate consumption of the old entrepreneurs c_t^{OE} is given by

$$c_t^{OE}=(1-\eta)\max\left[q_t f_t-r_t\left(k_t-s_{t-1}^E\right)-w_t l_t,0\right]$$

By substituting $\frac{1-\eta}{\eta}b_t-r_t s_{t-1}^W$ for d_t^W in c_t^{OW} and summing up the above variables we can write the aggregate consumption $C_t=c_t^{OW}+c_t^{YW}+c_t^{YE}+c_t^{OE}$ plus aggregate savings $S_t=\eta s_t^W+(1-\eta)\,s_t^E$ in the economy in period t as

$$C_t+S_t=(1-\eta)\,b_t+\widehat{w}_t+(1-\eta)\max\left[q_t f_t-r_t\left(k_t-s_{t-1}^E\right)-w_t l_t,0\right]$$

Recall that $\widehat{w}_t=\min\left[w_t,\frac{q_t f_t}{l_t}\right]$ and $b_t=\min\left[r_t\left(k_t-s_{t-1}^E\right),q_t f_t-e_t\right]$, where $e_t=\min\left[w_t l_t,q_t f_t\right]=\widehat{w}_t l_t$. Substituting these into the above formula yields:

$$\begin{aligned}C_t+S_t &= (1-\eta)\min\left[r_t\left(k_t-s_{t-1}^E\right),q_t f_t-\widehat{w}_t l_t\right]+\widehat{w}_t+\\ &\quad +(1-\eta)\max\left[q_t f_t-r_t\left(k_t-s_{t-1}^E\right)-w_t l_t,0\right]\end{aligned}$$

We only need to recall that since the whole young generation supplies exactly one unit of labor, the amount of productive labor per entrepreneur is $l_t = \frac{1}{1-\eta}$, which results in

$$C_t + S_t = \min\left[(1-\eta) r_t \left(k_t - s_{t-1}^E\right), (1-\eta) q_t f_t - \widehat{w}_t\right] + \widehat{w}_t + \\ + \max\left[(1-\eta)\left(q_t f_t - r_t\left(k_t - s_{t-1}^E\right)\right) - w_t, 0\right]$$

If the shock is small we obtain

$$C_t + S_t = (1-\eta) r_t \left(k_t - s_{t-1}^E\right) + w_t + (1-\eta)\left(q_t f_t - r_t\left(k_t - s_{t-1}^E\right)\right) - w_t = \\ = (1-\eta) q_t f_t = Y_t$$

If the shock is moderate, the equation turns into

$$C_t + S_t = (1-\eta) q_t f_t - w_t + w_t = Y_t$$

Finally, if the shock is severe,only young generation is (partially) paid in amount $\widehat{w}_t = \frac{q_t f_t}{l_t} = (1-\eta) q_t f_t$, therefore

$$C_t + S_t = \widehat{w}_t = (1-\eta) q_t f_t = Y_t$$

Appendix C. Proofs

PROOF of Proposition 3.1

Proof.

Existence

Preferences and production technology satisfy the assumptions of the competitive equilibrium existence theorem (Arrow and Debreu, 1954):[19]

1. the set of available consumption vectors (c_t, c_{t+1}) for each generation t is closed, convex and bounded from below
2. the preferences of consumers of each generation t are represented by continuous, monotonically increasing, quasi-concave utility functions of (c_t, c_{t+1})
3. the initial endowment of the individuals is strictly positive at least in one component (in the model, each individual in each generation is endowed with one unit of labor, which is converted into $\widehat{w}_t > 0$ units of initial endowment in goods)
4. production technologies belong to a part of each generation and are given by a continuous strictly increasing and concave production functions with no output at zero input.

[19] Arrow and Debreu (1954) consider multiproduct technologies with an assumption that in the absence of factor restrictions, the production of any good may be increased without a decrease in the production of other goods. The model in the current paper is based upon a one-product technology.

Uniqueness

Consider first labor market (LM). The LM-equilibrium condition is

$$l\left(r_{t+1}, w_{t+1}\right) = \frac{1}{1-\eta} \tag{C-1}$$

Function $l\left(r_{t+1}, w_{t+1}\right)$ decreases in both interest rate and wage level ($\frac{\partial l}{\partial r_{t+1}} < 0$, $\frac{\partial l}{\partial w_{t+1}} < 0$). The implicit function theorem guarantees that equation (C-1) defines a unique function $r_{t+1}\left(w_{t+1}\right)$ with $\frac{\partial r_{t+1}}{\partial w_{t+1}} < 0$. This means that for any given interest rate established in the credit market, there will always exist only one equilibrium wage level in the labor market.

Consider now the equilibrium in the credit market (CM):

$$(1-\eta)\left(k_{t+1} - s_t^E\right) = \eta s_t^W \tag{C-2}$$

Optimal choice of the entrepreneurs implies $\frac{\partial k}{\partial r_{t+1}} < 0$, $\frac{\partial k}{\partial w_{t+1}} < 0$, $\frac{\partial s_t^E}{\partial w_{t+1}} < 0$ and $\frac{\partial s_t^E}{\partial r_{t+1}} > 0$. Optimal choice of the workers implies $\frac{\partial s_t^W}{\partial w_{t+1}} < 0$ and $\frac{\partial s_t^W}{\partial r_{t+1}} > 0$. The sum and the difference of differentiable functions are differentiable. Hence, equation (C-2) also implicitly yields a differentiable function $r_{t+1}\left(w_{t+1}\right)$ which is unique.

Combining $r_{t+1}^{CM}\left(w_{t+1}\right)$, defined in the credit market, with $r_{t+1}^{LM}\left(w_{t+1}\right)$, defined in the labor market, we obtain an equilibrium interest rate and wage level, which exist. Assume that there are several equilibria and choose the one with the smallest $w_{t+1} = \underline{w}$. Consider now the difference $r_{t+1}^{CM}\left(w_{t+1}\right) - r_{t+1}^{LM}\left(w_{t+1}\right)$, which is zero in the equilibrium chosen. This difference increases as w_{t+1} increases (see Lemma 6.1 below):

$$\frac{\partial r_{t+1}^{CM}}{\partial w_{t+1}} - \frac{\partial r_{t+1}^{LM}}{\partial w_{t+1}} > 0 \tag{C-3}$$

Hence, the difference $r_{t+1}^{CM}\left(w_{t+1}\right) - r_{t+1}^{LM}\left(w_{t+1}\right)$ is positive for any $w_{t+1} > \underline{w}$. This means there are no equilibria with $w_{t+1} > \underline{w}$. Because of the choice of $\underline{w}$, there are also no equilibria with $w_{t+1} < \underline{w}$, This proves the uniqueness of the equilibrium point ($w_{t+1}^* = \underline{w}$, $r_{t+1}^* = r_{t+1}^{LM}\left(\underline{w}\right) = r_{t+1}^{CM}\left(\underline{w}\right)$)

■

Lemma 6.1 *Equilibrium gap $r_{t+1}^{CM}\left(w_{t+1}\right) - r_{t+1}^{LM}\left(w_{t+1}\right)$ increases in w_{t+1}*

Proof. The slope of the equilibrium line in the credit market can be found through implicit differentiation:

$$\frac{\partial r_{t+1}}{\partial w_{t+1}} = -\frac{\eta \frac{\partial s_t^W}{\partial w_{t+1}} - (1-\eta)\left(\frac{\partial k}{\partial w_{t+1}} - \frac{\partial s_t^E}{\partial w_{t+1}}\right)}{\eta \frac{\partial s_t^W}{\partial r_{t+1}} - (1-\eta)\left(\frac{\partial k}{\partial r_{t+1}} - \frac{\partial s_t^E}{\partial r_{t+1}}\right)} \tag{C-4}$$

The denominator in this fraction is always positive. Since (C-4) is valid for any value of parameter $\eta \in [0,1]$, we can check, whether it is positive or negative for its upper and lower limits:

$$\left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{CM}_{\eta=0} = -\frac{\frac{\partial k}{\partial w_{t+1}} - \frac{\partial s_t^E}{\partial w_{t+1}}}{\frac{\partial k}{\partial r_{t+1}} - \frac{\partial s_t^E}{\partial r_{t+1}}} < 0 \tag{C-5}$$

$$\left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{CM}_{\eta=1} = -\frac{\frac{\partial s_t^W}{\partial w_{t+1}}}{\frac{\partial s_t^W}{\partial r_{t+1}}} > 0 \tag{C-6}$$

This means that for all possible functions k, s^E and s^W, setting $\eta = 0$ (and close to it) guarantees that CM-line is monotonically decreasing in w_{t+1}; and setting $\eta = 1$ (and close to it) guarantees that CM-line is monotonically increasing in w_{t+1} for any given wage parameter w_t.

Furthermore, for each set of functions k, s^E and s^W, the slope of the CM-line monotonically increases in η as η changes from 0 to 1:

$$\frac{\partial}{\partial \eta}\left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{CM} = -\frac{\frac{\partial s_t^W}{\partial r_{t+1}}\left(\frac{\partial k}{\partial w_{t+1}} - \frac{\partial s_t^E}{\partial w_{t+1}}\right) - \frac{\partial s_t^W}{\partial w_{t+1}}\left(\frac{\partial s_t^E}{\partial r_{t+1}} - \frac{\partial k}{\partial r_{t+1}}\right)}{\left[\eta\frac{\partial s_t^W}{\partial r_{t+1}} - (1-\eta)\left(\frac{\partial k}{\partial r_{t+1}} - \frac{\partial s_t^E}{\partial r_{t+1}}\right)\right]^2} > 0 \tag{C-7}$$

This ensures that at any point w_{t+1} the derivative $\frac{\partial r_{t+1}}{\partial w_{t+1}}$ is always bounded by (C-5) from below and by (C-6) from above. Since $\left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{LM} < \left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{CM}_{\eta=0}$ (for proof see Lemma 6.2 below), the slope of the LM-line is smaller than the smallest possible slope of the CM-line in any point w_{t+1}, so that the gap $r_{t+1}^{CM} - r_{t+1}^{LM}$ increases in w_{t+1}.[20]

■

Lemma 6.2 *The slopes of the LM- and CM-lines are related with*

$$\left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{LM} < \left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{CM}_{\eta=0} \tag{C-8}$$

Proof. The slope of the LM-line is given by

$$\left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{LM} = -\frac{\frac{\partial l}{\partial w_{t+1}}}{\frac{\partial l}{\partial r_{t+1}}} \tag{C-9}$$

The slope of the CM-line is given by

$$\left\{\frac{\partial r_{t+1}}{\partial w_{t+1}}\right\}^{CM} = -\frac{\eta\frac{\partial s_t^W}{\partial w_{t+1}} - (1-\eta)\left(\frac{\partial k}{\partial w_{t+1}} - \frac{\partial s_t^E}{\partial w_{t+1}}\right)}{\eta\frac{\partial s_t^W}{\partial r_{t+1}} - (1-\eta)\left(\frac{\partial k}{\partial r_{t+1}} - \frac{\partial s_t^E}{\partial r_{t+1}}\right)} \tag{C-10}$$

[20] It suffices to consider the derivative of this gap.

Choose $\eta = 0$. We need to prove that

$$-\frac{l_w}{l_r} < -\frac{k_w - s_w^E}{k_r - s_r^E} \quad \text{(C-11)}$$

where k_w, l_w, s_w^E, k_r, l_r, and s_r^E denote derivatives of the respective functions. This last condition is fulfilled as soon as

$$l_w k_r - l_w s_r^E > k_w l_r - s_w^E l_r \quad \text{(C-12)}$$

Consider the properties of factor demands[21]:

$$l_w = \frac{f_{kk}}{f_{kk} f_{ll} - (f_{kl})^2} < 0 \quad \text{(C-13)}$$

$$k_w = l_r = \frac{-f_{kl}}{f_{kk} f_{ll} - (f_{kl})^2} < 0 \quad \text{(C-14)}$$

$$k_r = \frac{f_{ll}}{f_{kk} f_{ll} - (f_{kl})^2} < 0 \quad \text{(C-15)}$$

Hence

$$k_w l_r - l_w k_r = \frac{(f_{lk})^2 - f_{kk} f_{ll}}{\left[f_{kk} f_{ll} - (f_{kl})^2\right]^2} < 0 \quad \text{(C-16)}$$

Combining this with $s_w^E < 0$ and $s_r^E > 0$, we obtain

$$-l_w s_r^E > k_w l_r - l_w k_r - s_w^E l_r \quad \text{(C-17)}$$

The latter inequality is true, since the left-hand side is positive and the right-hand side is negative. This proves C-12 and consequently the statement of the Lemma.

■

PROOF of Proposition 5.1

Proof.

The dynamics of deficits is given by

$$d_{t+1} = (1 - \eta)\, r_{t+1}^C \left(k_{t+1} - s_t^E\right) - \eta r_{t+1}^D s_t^W \quad \text{(C-18)}$$

Substitute for $r_{t+1}^D = r_{t+1}^C - \delta_{t+1}^{reg}(d_t)$:

$$d_{t+1} = r_{t+1}^C \left[(1 - \eta)\left(k_{t+1} - s_t^E\right) - \eta s_t^W\right] + \delta_{t+1}^{reg}(d_t)\, \eta s_t^W \quad \text{(C-19)}$$

Since $(1 - \eta)\left(k_{t+1} - s_t^E\right) - \eta s_t^W = d_t$, the latter implies $d_{t+1} > d_t$ if and only if

$$\delta_{t+1}^{reg}(d_t) > \frac{\left(r_{t+1}^C - 1\right)(-d_t)}{\eta s_t^W} = \delta_{t+1}^{crit}(d_t) > 0 \quad \text{(C-20)}$$

■

[21] The denominator $f_{kk} f_{ll} - (f_{kl})^2$ is positive due to the concavity assumption.