A technical report on pricing for a two sided market

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Armstrong (2006)[1] presented a pricing mechanism for a two sided market where platform e.g. night club facilitate interaction between two distinct group of agents i.e. men and women. He presents theoretical models for three platforms: a monopoly platform; a competing platform and a competing bottleneck. In section 3 he analysed the pricing strategy for a monopoly platforms. He conjectured that

> *"It is possible that the profit-maximizing outcome might involve group 1 (e.g. women), say, being offered a subsidised service, i.e., price is less than the cost of provisioning the service. This happens if the group's elasticity of demand is high and/or if the external benefit enjoyed by group 2 (e.g. men) is large."*

In this report we have elaborated the Armstrong's derivation of price in a monopoly platform which allows us to draw additional conclusions which have not been reported in the literature.

Notations:

Black font colour – Armstrong's word reproduced.

Red font colour – Our contributions.

Section 3 Monopoly Platform (From Armstrong, 2006 pp. 671-673)

This section presents the analysis for a monopoly platform. This framework does not apply to most of the examples of two-sided markets that come to mind, although there are a few applications. For instance, yellow pages directories are often a monopoly of the incumbent telephone company, shopping malls or nightclubs are sometimes far enough away from others that the monopoly paradigm might be appropriate, and sometimes there is only one newspaper or magazine in the relevant market.

[1] Armstrong, M. (2006) "Competition in two-sided markets", Rand Journal of Economics, Vol. 37, No. 3, pp. 668-691.

Suppose there are two groups of agents, denoted 1 and 2. A member of one group cares about the number of the other group who use the platform. (For simplicity, I ignore the possibility that agents care also about the number of the same group who join the platform.) Suppose that the utility of an agent is determined in the following way: if the platform attracts n_1 and n_2 members of the two groups, the utilities of a group-1 and a group-2 agent are respectively

$$u_1 = \alpha_1 . n_2 - p_1 \ ; \quad u_2 = \alpha_2 . n_1 - p_2 \ , \qquad (1)$$

where p_1 and p_2 are the platform's prices to the two groups. The parameter α_1 measures the benefit that a group-1 agent enjoys from interacting with each group-2 agent, and α_2 measures the benefit a group-2 agent obtains from each group-1 agent. Expression (1) describes how utility is determined as a function of the numbers of agents who participate. To close the demand model, I specify the numbers who participate as a function of the utilities: if the utilities offered to the two groups are u_1 and u_2, suppose the numbers of each group who join the platform are

$$n_1 = \phi_1(u_1) \ ; \ n_2 = \phi_2(u_2)$$

for some increasing functions $\phi_1(.)$ and $\phi_2(.)$.

Turning to the cost side, suppose that the platform incurs a per-agent cost f_1 for serving group-1 and per-agent cost f_2 for group-2. Therefore, the firm's profit is $\pi = n_1(p_1 - f_1) + n_2(p_2 - f_2)$. If we consider the platform to be offering utilities $\{u_1, u_2\}$ rather than prices $\{p_1, p_2\}$, then the implicit price for group-1 is $p_1 = \alpha_1 . n_2 - u_1$ (and similarly for group 2). Therefore, expressed in terms of utilities, the platform's profit is

$$\pi(u_1, u_2) = \phi_1(u_1).[\alpha_1.\phi_2(u_2) - u_1 - f_1] + \phi_2(u_2).[\alpha_2.\phi_1(u_1) - u_2 - f_2] \qquad (2)$$

Let the aggregate consumer surplus of group i be $v_i(u_i)$, where $v_i(.)$ satisfies the envelope condition $v_i^{'}(u_i) \equiv \phi_i(u_i)$. Then welfare, as measured by the unweighted sum of profit and consumer surplus, is

$$w = \pi(u_1, u_2) + v_1(u_1) + v_2(u_2) \, .$$

It is easily verified that the welfare maximizing outcome involves utilities satisfying

$$u_1 = (\alpha_1 + \alpha_2)n_2 - f_1; \qquad u_2 = (\alpha_1 + \alpha_2)n_1 - f_2$$

From expression (1), the socially optimal prices satisfy

$$p_1 = f_1 - \alpha_2 n_2 \;; \qquad p_2 = f_2 - \alpha_1 n_1 .$$

As one would expect, the optimal price for group 1, say, equals the cost of supplying service to a type-1 agent adjusted downwards by the external benefit that an extra group-1 agent brings to the group-2 agents on the platform. (There are n_2 group-2 agents on the platform, and each one benefits by α_2 when an extra group-1 agent joins.) In particular, prices should ideally be below cost if α_1 ; $\alpha_2 > 0$.

From expression (2), the profit-maximizing prices satisfy

$$\frac{\partial \pi(u_1, u_2)}{\partial u_1} = 0; \qquad \frac{\partial \pi(u_1, u_2)}{\partial u_2} = 0 \qquad \text{(i)}$$

From these two first order derivatives we can find profit maximizing prices

$$p_1 = f_1 - \alpha_2 n_2 + \frac{\phi_1(u_1)}{\phi_1^{'}(u_1)} \;; \qquad p_2 = f_2 - \alpha_1 n_1 + \frac{\phi_2(u_2)}{\phi_2^{'}(u_2)} . \qquad (3)$$

Where $\phi_1^{'}(u_1) = \frac{\partial \phi_1(u_1)}{\partial u_1}$ (ii)

Thus, the profit-maximizing price for group 1, say, is equal to the cost of providing service (f_1), adjusted downward by the external benefit to group 2 ($\alpha_2 n_2$), and adjusted upward by a factor related to the elasticity of the group's participation. These profit-maximizing prices can be obtained in the more familiar form of Lerner indices and elasticities, as recorded in the following result.

Proposition 1 Write

$$\eta_1(p_1|n_2) = \frac{p_1 \phi_1^{'}(\alpha_1 n_2 - p_1)}{\phi_1(\alpha_1 n_2 - p_1)} \;; \quad \eta_2(p_2|n_1) = \frac{p_2 \phi_2^{'}(\alpha_2 n_1 - p_2)}{\phi_2(\alpha_2 n_1 - p_2)}$$

for a group's price elasticity of demand for a given level of participation by the other group. Then the profit-maximizing pair of prices satisfy

$$\eta_1(p_1|n_2) = \frac{p_1}{p_1 - (f_1 - \alpha_2 n_2)} \;; \; \eta_2(p_2|n_1) = \frac{p_2}{p_2 - (f_2 - \alpha_1 n_1)} \qquad (4)$$

We elaborate the derivation by considering the definition of elasticity as;

$$\eta_1(p_1|n_2)=\frac{\frac{\partial n_1}{n_1}}{\frac{\partial p_1}{p_1}}$$

[*We acknowledge that above elasticity can be defined with a negative sign on the right hand side but that can always be done by replacing* η_1 *with (-*η_1 *)*.]

$$=\frac{p_1}{n_1}\frac{\partial n_1}{\partial p_1}$$

By substituting $n_1=\phi_1(u_1)$ we get

$$\eta_1(p_1|n_2)=\frac{p_1}{\phi_1(u_1)}\frac{\partial \phi_1(u_1)}{\partial p_1}$$

For a given level of participation (n_2) by the other group, from (1) we get $\partial p_1=-\partial u_1$ and substituting ∂p_1 with $-\partial u_1$ in the above equation we get

$$\eta_1(p_1|n_2)=-\frac{p_1}{\phi_1(u_1)}\frac{\partial \phi_1(u_1)}{\partial u_1}$$

By substituting $\frac{\partial \phi_1(u_1)}{\partial u_1}$ with $\phi_1^{'}(u_1)$ we get

$$\eta_1(p_1|n_2)=-\frac{p_1\phi_1^{'}(u_1)}{\phi_1(u_1)} \qquad \text{(iii)}$$

Further substituting u_1 with $(\alpha_1.n_2-p_1)$ we get

$$\eta_1(p_1|n_2)=-\frac{p_1\phi_1^{'}(\alpha_1 n_2-p_1)}{\phi_1(\alpha_1 n_2-p_1)} \qquad \text{(iv)}$$

In a similar way we can derive $\eta_2(p_2|n_1)=-\frac{p_2\phi_2^{'}(\alpha_2 n_1-p_2)}{\phi_2(\alpha_2 n_1-p_2)}$ (v)

Therefore by replacing $\frac{\phi_1^{'}(u_1)}{\phi_1(u_1)}$ with $-\frac{\eta_1(p_1\,|\,n_2)}{p_1}$ in (3) we get

$$p_1 = f_1-\alpha_2 n_2-\frac{p_1}{\eta_1(p_1\,|\,n_2)} \qquad \text{(vi)}$$

For the convenience we may write $\eta_1(p_1 \mid n_2) = \eta_1$ (in further derivation) and by taking the third part from the right side to the left side of (vi) we get

$$p_1 = \frac{\eta_1}{(\eta_1 + 1)}(f_1 - \alpha_2 n_2) \qquad \text{(vii)}$$

Similarly we get

$$p_2 = \frac{\eta_2}{(\eta_2 + 1)}(f_2 - \alpha_1 n_1) \qquad \text{(viii)}$$

As per Armstrong's derivation, the equation (4) can be rewritten to find the profit maximizing prices as;

$$p_1 = \frac{\eta_1}{(\eta_1 - 1)}(f_1 - \alpha_2 n_2)\,; \quad p_2 = \frac{\eta_2}{(\eta_2 - 1)}(f_2 - \alpha_1 n_1) \qquad \text{(xi)}$$

If we consider the sign of elasticity as negative then by replacing η_1 with ($-\eta_1$), we get p_1 and p_2 which are same to our derivations shown in (vii) and (viii).

Armstrong conjectured that;

> *"It is possible that the profit-maximizing outcome might involves group 1, say, being offered a subsidised service, i.e., $p_1 < f_1$. From (4), this happens if the group's elasticity of demand is high and/or if the external benefit enjoyed by group 2 is large."*

From (vii) we can draw additional conclusion as follows.

1) Assuming the external benefit enjoyed by group 2 agents (i.e. $\alpha_2 n_2$) is less than the cost of provisioning the service (i.e. f_1) to a group 1 agent, we can conclude that the profit maximizing price will offer negative price to group 1 agent if demand elasticity is less than one (i.e. $|\eta_1| < 1$).
2) When the demand elasticity is greater than one (i.e. $|\eta_1| > 1$) the profit maximizing price will offer subsidy (zero or negative price) only when the external benefit enjoyed by group 2 agents (i.e. $\alpha_2 n_2$) are equal or more than the cost of provisioning service (i.e. f_1) to a group 1 agent.

This can be easily understood with numerical examples;

a) Let us assume that the demand elasticity is low say η_1 = -0.5 and external benefit is less than cost (i.e. $\alpha_2 n_2 << f_1$) then from (vii) we find $p_1 = -1.0\,(f_1 - \alpha_2 n_2)$, i.e. the price is negative.

b) When the demand elasticity is high say η_1 = -1.5, from (vii) we find the profit maximizing price $p_1 = 3.0\,(f_1 - \alpha_2 n_2)$. In this case the price will be higher than the cost (i.e. f_1) when $\alpha_2 n_2 < 0.66 f_1$. The price will approach towards cost when demand elasticity is infinity. However price can be zero or negative if the external benefit enjoyed by group 2 is equal to or greater than the cost of serving one group 1 agent.