Relationships Between Noise Shaping and Nested Differentiating Feedback Loops 3358 (B-4)

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Abstract
We study the application of heavy feedback in two different topologies, namely multiple-order noise shaping and nested differentiating feedback loops. Both have similar loop gain and stability considerations, although the two approaches have different implied circuit environments and areas of application. In noise shaping, emphasis is placed on the integrator characteristics of each gain stage, whereas leaky integrators or flat-gain stages with high-frequency poles form the usual basis of the nested differentiating loop concept. This short didactic paper helps in understanding the application of large amounts of feedback to control noise or distortion at baseband frequencies.

1 Introduction

The concept of nesting differentiating feedback loops (NDFL) has been introduced and promoted by Cherry [1, 2]. Generally we think of analogue circuits for its application, and the basic idea is that for the stage that creates the most distortion (usually a power output stage), the encompassing differentiating feedback stabilises the loop at high frequencies while allowing increased feedback at lower frequencies to significantly reduce system distortion.

2 Noise shaping

Our approach to the application of large amounts of feedback has come from a digital perspective, namely that of noise-shaping, as typically applied to requantizers in one-bit ADCs and DACs; for reference, see Hawksford [3]. Consider the first and second-order noise shapers shown in Figure 1. Figure 1(a) shows two topologies in which the error of the quantizer Q is modified by a single delay for the first-order shaper (there must always be at least a one-sample delay) and the filter $H(z) = z^{-1}(2 - z^{-1})$ for the second-order shaper. In each case the filtered error is subtracted retrospectively from the input sequence to enable a partial correction for the quantizer error. Figure 1(b) shows the same topologies redrawn in a different form, while Figure 1(c) shows the transformation to a delta-sigma converter topology. The digital integrator blocks are shown in the insert, and it is clear that the order of the noise shaper can be increased by simply extending the number of integrator blocks, encompassing each of them in a feedback loop from the output. It is readily shown that these noise shapers have signal transfer functions of unity (except for the simple delays, which could be removed in some of the circuits) and the quantization error (normally considered as noise) is shaped by a response

$$R(z) = (1 - z^{-1})^N \quad \ldots 1$$

where N is the order of the shaper. Such noise shaping structures have been much discussed in the literature as well [4].

3 Distortion shaping

Figure 2 illustrates a further progression of ideas. Figure 2(a) shows a third-order digital shaper for which the quantizer output-related error q[n] is explicitly drawn in, while Figure 2(b) shows an equivalent analogue circuit using real integrators, in which the quantizer is replaced by an output stage.

We can interpret this circuit as a limiting digital case in which the sampling frequency has become infinite, hence removing all delays, and where the discontinuous output quantizer is replaced with a continuous but nonlinear analog output stage. If the output stage has gain G and the integrators have respective time constants τ_{1f}, τ_{2f} and τ_{3f} for the feedback paths, τ_{1s}, τ_{2s} and τ_{3s} for the signal paths, then the transfer function of the system can be written as

$$\left[\frac{\tau_{1s}\tau_{2s}\tau_{3s}s^3}{G} + \frac{\tau_{1s}\tau_{2s}\tau_{3s}s^2}{\tau_{3f}} + \frac{\tau_{1s}\tau_{2s}s}{\tau_{3f}} + \frac{\tau_{1s}}{\tau_{1f}}\right]Y(s) = X(s) + \frac{\tau_{1s}\tau_{2s}\tau_{3s}s^3}{G}Q(s) \qquad \ldots 2$$

for which the quantizer output-related error q[n] is explicitly drawn in.

We note that the τ parameters define both the stability criteria and signal frequency response of the circuit, and that the output error Q(s) is weighted by the cube of the signal frequency. Consequently, this analogue feedback circuit can be viewed as a "distortion" shaper, where the multiple integrators by virtue of their large low-frequency gain, reduce the effects of nonlinearity at lower frequencies.

In the circuit of Figure 2(c) the feedback paths to the last two integrators have been moved forward, thus each incorporating an extra integrator, and compensating differentiators $s\tau_{2s}$ and $s\tau_{3s}$ have been inserted to keep the transfer function unaltered. An extra amplifier with gain g = 1 has also been inserted to give the circuit a more usual configuration. It is this modified circuit which we wish to compare with the NDFL circuits of Cherry [1].

In this reconfiguration, the output stage is modelled with frequency-independent gain G, but the dashed bracket associates the third integrator with the output stage so that it can display real poles. It is also possible to think of each integrator as being "leaky", of form a/(b + s), more like the actual stages of an amplifier. We might associate the first integrator, if present, with an intermediate amplifier stage, the second integrator with the voltage-gain stage, and the third integrator and output block with a more realistic output stage.

4 Discussion

The circuit of Figure 2(c) can be identified directly with NDFL circuits, even though there may be differences of small detail. The nested loops may not all take their feedback signal directly from the output, for example, although there is then a difference in implied circuit environment. In Figure 2(b), each of the integrators (which could be leaky) are regarded as similar, but in Figure 2(c) the last integrator is considered to be part of an output stage, having relatively low gain, and being rather leaky, perhaps describing the typical emitter follower output stage of an audio amplifier. Although it is customary to have a Miller capacitor across the voltage-gain stage, if this capacitor also encompasses the output stage, and feeds back to the virtual ground input of the voltage-gain stage, it becomes the $s\tau_{2f}$ differentiating feedback shown in the diagram. The effect on amplifier distortion is very beneficial, and this point has been emphasised by Cherry [2].

There are other strong parallels between NDFLs and "distortion- shaping" topologies. The order of each can in principle be increased without limit by adding more stages, giving even more feedback at lower frequencies. Another aspect is the internal stability of the loop with respect to the output stage. It is the output stage which generally is considered the dominant source of the distortion, and a high loop gain will act to reduce it. The output-stage loop gain A in the circuit of Figure 2(b) (and hence also of Figure 2(c)) can (setting the input X(s) = 0) be shown to be given by,

$$A = \frac{Q(s)}{Y(s)} - 1 = G\left[\frac{1}{s\,\tau_{3f}} + \frac{1}{s^2\,\tau_{2f}\,\tau_{2s}} + \frac{1}{s^3\,\tau_{3f}\,\tau_{2s}\,\tau_{3s}}\right] \qquad \ldots 3$$

and this is easily generalized to higher or lower order. At the highest frequencies, only the first term survives, and for good stability the phase shift must be considerably less than -180°, ideally being close to -90°, the lag of a single integrator. It is the τ_{3f} feedback path that ensures this stability at high frequencies, but the other loops allow increasing feedback at lower frequencies, reducing distortion in the process.

As an example, let us consider a 5th-order distortion shaper feedback circuit, for which G = 1, and the transfer function Y(s)/X(s) is by proper choice of τ parameters a 5th-order Butterworth unity-gain low-pass with cut off frequency of say 50 kHz. The loop gain can be written as,

$$A = \frac{s_n^5 + a\,s_n^4 + (a+2)\,s_n^3 + (a+2)\,s_n^2 + a\,s_n + 1}{s_n^5} - 1$$

$$= \frac{a}{s_n} + \frac{a+2}{s_n^2} + \frac{a+2}{s_n^3} + \frac{a}{s_n^4} + \frac{1}{s_n^5} \qquad \ldots 4$$

where, $a = 1 + 2\cos(\pi/5) + 2\cos(2\pi/5)$

and the normalized Laplace variable $s_n = s/\omega_0 = s/(2\pi\, f_0)$, and f_0 = 50 kHz. Figure 3 is a plot of equation (4), showing the basic 6 dB/octave rolloff of the loop gain at high frequencies, but with a rise of $1/f^5$ for lower frequencies. If the integrators in the circuit are leaky, the graph will look similar, but will limit near dc at some high value of gain, giving a loop gain for the output stage very similar to that indicated by Cherry [1]. In this example, unity gain occurs at 157 kHz and the attendant phase shift is - 120°, representing good stable behaviour. Note that at 20 kHz, there is already 40 dB of distortion reduction, rising in an ever-increasing way at lower frequencies.

For high orders such as discussed above, this system displays conditional stability, as pointed out by Cherry [1] and in a recent discussion of NDFL [5]. Hence it is important to consider large-signal clipping behaviour experimentally to see if the system can be provoked into self-oscillation or other bizarre behaviour. It is clear that there is no simple limit to the amount of distortion reduction at high orders, but increasing care must be taken in defining stable circuit parameters and behaviour for large signals.

5 An Overview

At first glance, it almost seems that the trivial reassignment of the feedback loops around the integrators in Figure 2(b) results in the NDFL structure of Figure 2(c). However, there are differences in realizability and circuit tradition. Cherry [1] also works out a great deal of the mathematical aspects of NDFL, with sensitivities and analysis of appropriate models.

Presumably most of this work applies also to the distortion-shaping topology as well, with appropriate measuring points or circuit associations in the two approaches. We do not in this didactic paper work out such details or attempt a mapping between the two topologies.

In some ways, the distortion-shaping approach is easier to grasp initially. But NDFL is perhaps better if one is faced with a traditional class AB audio power amplifier, and wishes to improve its performance. In fact the title of one of Cherry's papers [6] suggests this. Also, when NDFL was invented, it appeared as a new result since the traditional methods of stabilising amplifiers had limitations on the amount of simple feedback that could be applied, as Bode [7] had shown. There have been engineers who have employed selected aspects of a differentiating loop all along, but the general concept of NDFL puts a firm footing on new aspects of feedback. What this paper shows is that there is another way of looking at the application of large amounts of negative feedback, and that the two are closely related.

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Figure captions

Figure 1 A progression of equivalent circuits for first-order and second-order digital noise shapers. The left column shows equivalent first-order shapers, the right column second-order. The inset in (c) shows the construction of the digital integrator block. The configurations in (c) show how to extend the order to any desired number.

Figure 2 Derivation of the nested-differentiating feedback loop (NDFL) concept. The digital third-order noise shaper in (a) is converted to analogue form in (b). By moving the feedback paths labelled τ_{2f} and τ_{3f} in (b) to the left, and adding compensating differentiators to maintain the same circuit transfer function, we achieve the NDFL representation of (c). The dotted lines in (c) encompass what we might consider a realistic output stage of a normal amplifier.

Figure 3 Loop gain as seen by the output stage, of a fifth-order noise shaper having a transfer function of a fifth-order Butterworth low-pass filter with cutoff frequency of 50 kHz. The rising loop gain at low frequencies vastly reduces distortion at these frequencies and effectively introduces "distortion shaping".

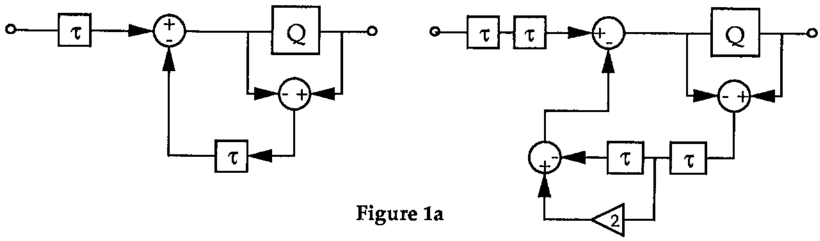


Figure 1a

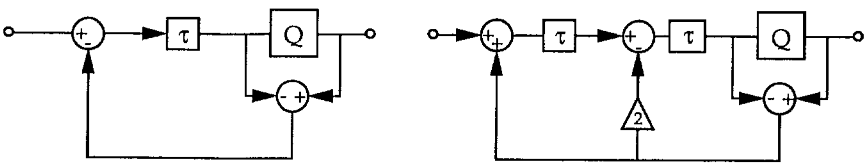


Figure 1b

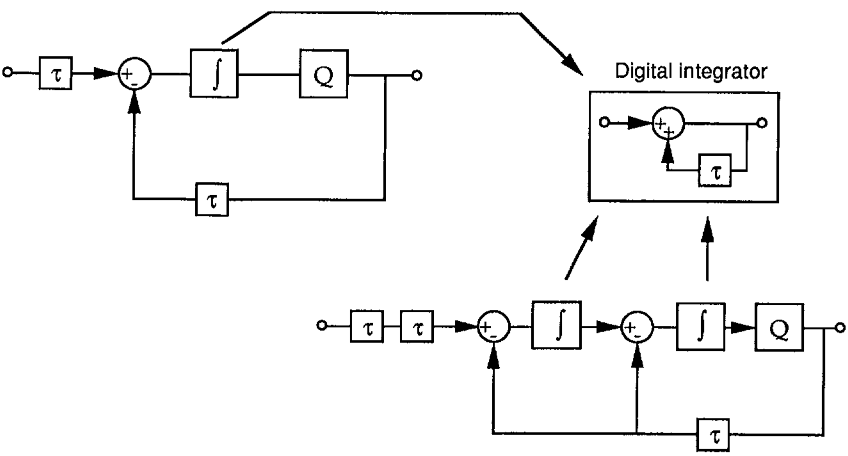


Figure 1c

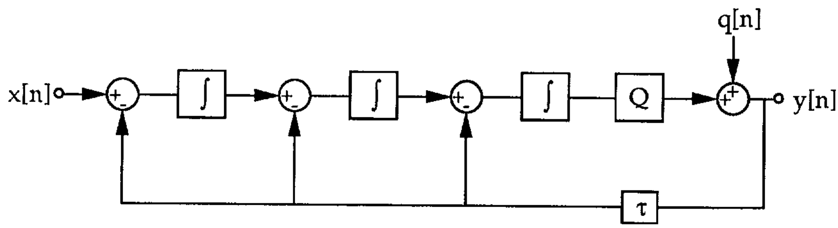


Figure 2a

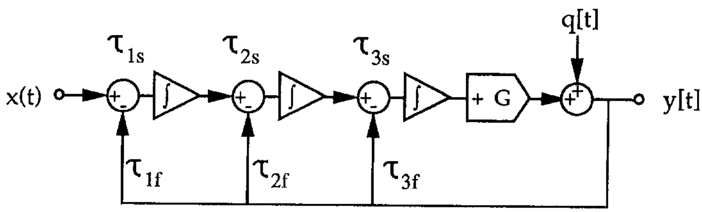
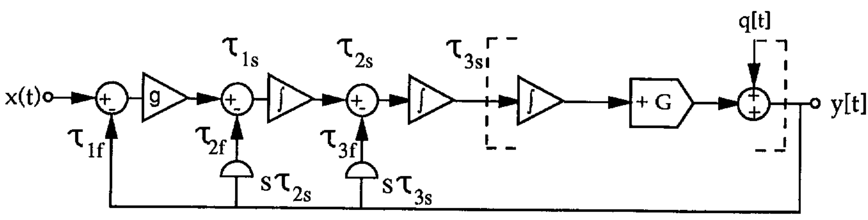


Figure 2b



(c)

Figure 2c

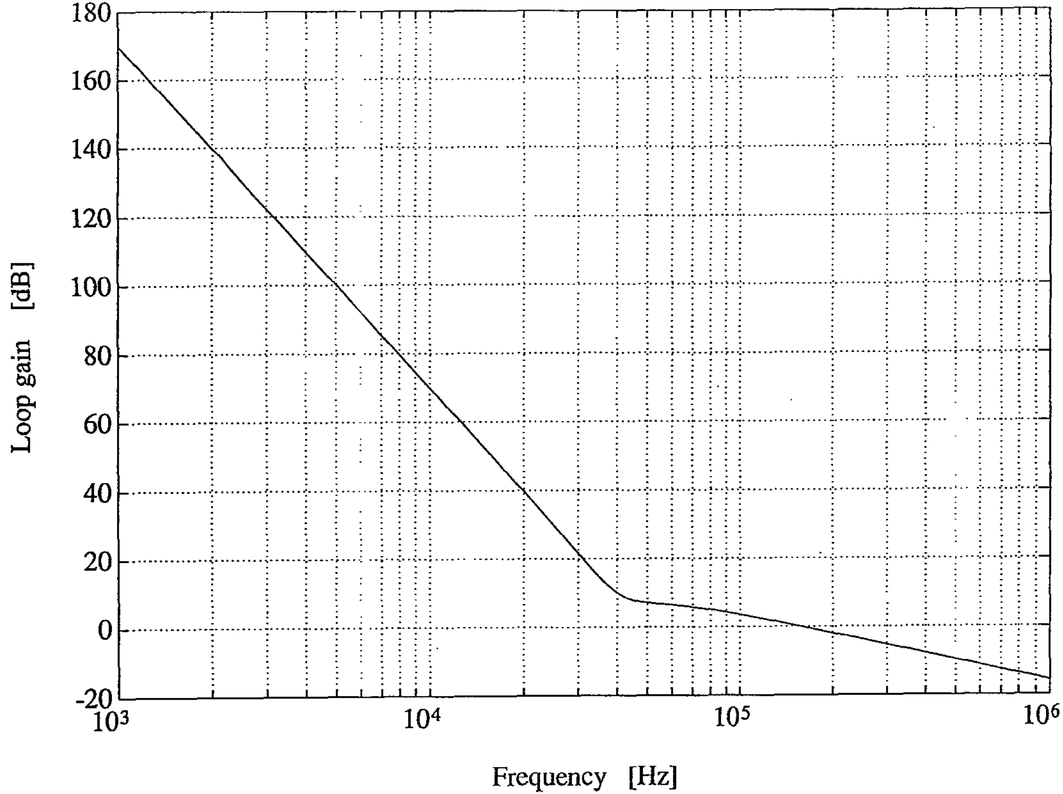

180
160
140
120
100
80
60
40
20
0
-20
Loop gain [dB]
10^3
10^4
10^5
10^6
Frequency [Hz]

Figure 3